Do Now:

2018 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

5. Let f be the function defined by $f(x) = e^x \cos x$.

(a) Find the average rate of change of f on the interval $0 \le x \le \pi$.

f'

a) $\frac{1}{\pi - 0}\int_0^{\pi} f'(x)\,dx = \frac{1}{\pi}\left(f(\pi) - f(0)\right)$

$\frac{1}{\pi}\left(e^{\pi}\cos\pi - e^{0}\cos 0\right)$

$\frac{1}{\pi}\left(-e^{\pi} - 1\right)$

(b) What is the slope of the line tangent to the graph of f at $x = \frac{3\pi}{2}$?

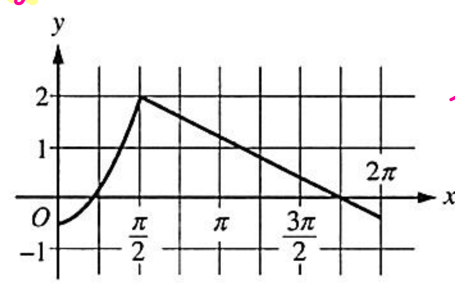
$f'(x) = -e^x \sin x + e^x \cos x$ $\quad f'\left(\frac{3\pi}{2}\right) = -e^{3\pi/2}(-1) + \cancel{e^{3\pi/2}(0)} = e^{3\pi/2}$

(c) Find the absolute minimum value of f on the interval $0 \le x \le 2\pi$. Justify your answer.

(d) Let g be a differentiable function such that $g\left(\frac{\pi}{2}\right) = 0$. The graph of g', the derivative of g, is shown below. Find the value of $\lim_{x \to \pi/2} \frac{f(x)}{g(x)}$ or state that it does not exist. Justify your answer.

$\frac{0}{0}$ L'Hopitals

$f\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}} \cos\frac{\pi}{2} = 0$



Graph of g'

$\lim_{x \to \frac{\pi}{2}} \frac{f'(x)}{g'(x)}$

$\lim_{x \to \frac{\pi}{2}} \frac{-e^x \sin x + e^x \cos x}{g'(x)} = \frac{-e^{\frac{\pi}{2}}\sin\frac{\pi}{2} + \cancel{e^{\frac{\pi}{2}}\cos\frac{\pi}{2}}}{2}$

$= \frac{-e^{\frac{\pi}{2}}}{2}$

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c)

$f'(x) = 0$

$-e^x \sin x + e^x \cos x = 0$

$e^x(-\sin x + \cos x) = 0$

$e^x \neq 0$ | $-\sin x + \cos x = 0$

$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

candidate test

$f(0) = e^0 \cos 0 = 1$

$f\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{4}} \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} e^{\frac{\pi}{4}}$

$f\left(\frac{5\pi}{4}\right) = e^{\frac{5\pi}{4}} \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} e^{\frac{5\pi}{4}}$

$f(2\pi) = e^{2\pi} \cos 2\pi = e^{2\pi}$

abs min: $-\frac{\sqrt{2}}{2} e^{\frac{5\pi}{4}}$

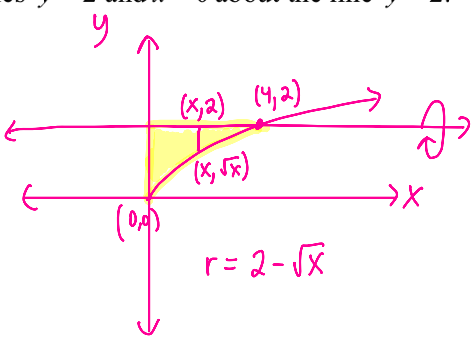
DAY 2

Do Now:

1. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the line $y = 2$.

$\sqrt{x} = 2$

$x = 4$



$y = 2$

$r = 2 - \sqrt{x}$

$$V = \pi \int_0^4 \left(2-\sqrt{x}\right)^2 dx$$

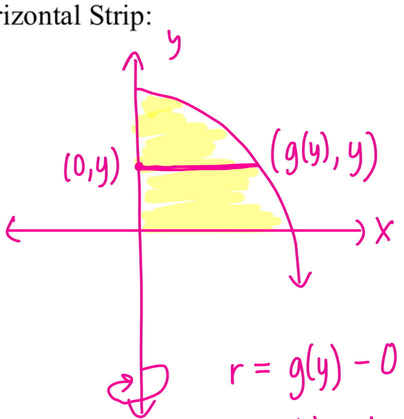
$$\pi \int_0^4 \left(4 - 4\sqrt{x} + x\right) dx$$

$$\pi \left[4x - \frac{8}{3}x^{3/2} + \frac{x^2}{2} \Big| _0^4 \right]$$

$$\pi \left[4(4) - \frac{8}{3}(4)^{3/2} + \frac{4^2}{2} \right]$$

$$\pi \left[4(4) - \frac{8}{3}(8) + \frac{4^2}{2} \right]$$

Horizontal Strip:

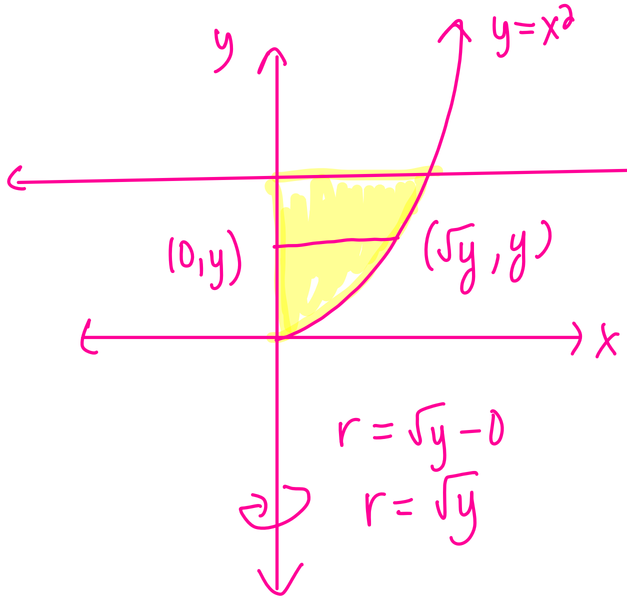


$r = g(y) - 0$

$r =$ right most curve $-$ left most curve

$$V = \pi \int_{\text{lowest } y\text{-value}}^{\text{highest } y\text{-value}} \left(g(y)\right)^2 dy$$

1. Find the volume of the solid found by revolving the region bounded by the curve $y = x^2$, the y-axis and $y = 1$ about the y-axis.



$$y = x^2$$
$$\pm\sqrt{y} = x$$

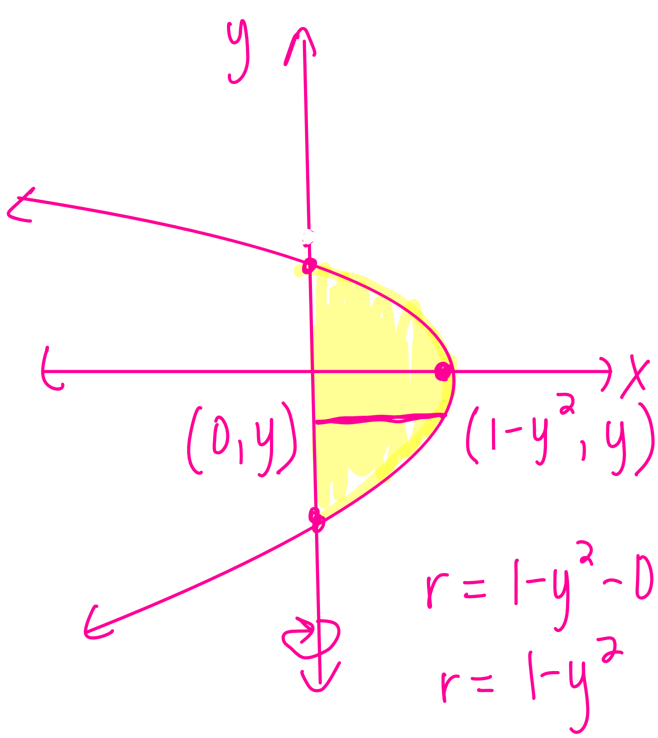
$$V = \pi \int_0^1 (\sqrt{y})^2 \, dy$$

$$\pi \left[\frac{y^2}{2} \Big|_0^1 \right] = \frac{\pi}{2}$$

2. Find the volume of the region bounded by $x = 1 - y^2$ and the y-axis revolved around the y-axis.

x-int: $x = 1 - 0^2 = x = 1$

y-int: $0 = 1 - y^2, \; y = \pm 1$



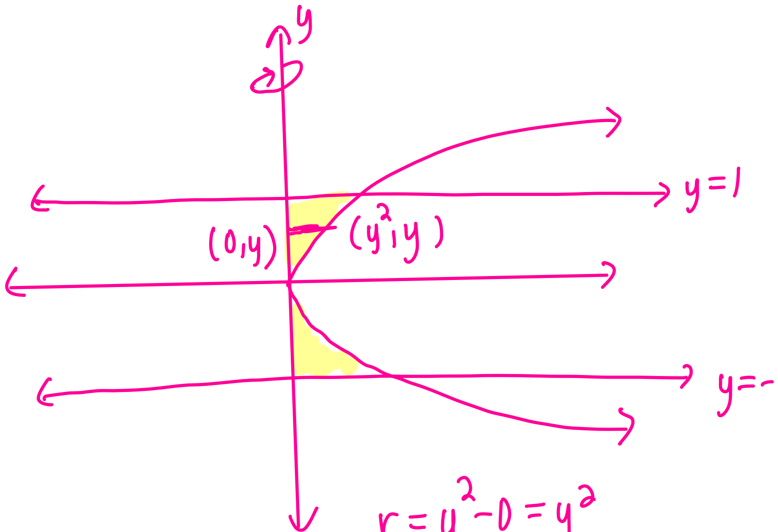
$$V = \pi \int_{-1}^{1} (1-y^2)^2 \, dy$$

$$V = \pi \int_{-1}^{1} (1 - 2y^2 + y^4) \, dy$$

$$\pi \left[y - \frac{2y^3}{3} + \frac{y^5}{5} \Big|_{-1}^{1} \right] = \frac{16\pi}{15}$$

$y = \pm\sqrt{x}$

3. Find the volume of the region bounded by $x = y^2$ and the y-axis from $y = -1$ and $y = 1$ revolved around the y-axis.

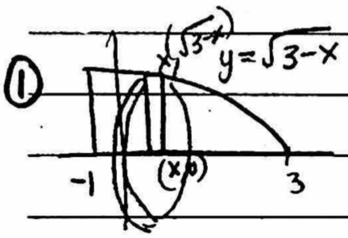


$r = y^2 - 0 = y^2$

$$V = \pi \int_{-1}^{1} (y^2)^2 \, dy = \pi \left[\frac{y^5}{5} \Big|_{-1}^{1} \right]$$

$$\pi \left[\frac{1}{5} - \left(-\frac{1}{5} \right) \right] = \frac{2\pi}{5}$$

Homework 03-20

① 

$r=\sqrt{3-x}-0$

$r=\sqrt{3-x}$

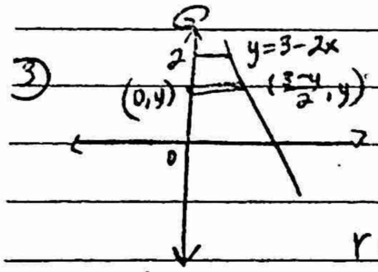
$V=\pi\int_{-1}^{3}\left(\sqrt{3-x}\right)^2dx=\pi\cdot\int_{-1}^{3}3-x\,dx$

$\pi\cdot\left[3x-\frac{x^2}{2}\right]_{-1}^{3}$

$\pi\cdot\left(3(3)-\frac{3^2}{2}-\left(3(-1)-\frac{(-1)^2}{2}\right)\right)$

$\pi\left(9-\frac{9}{2}+3+\frac{1}{2}\right)$

$\pi\left(9-\frac{8}{2}+3\right)=8\pi$

③ 

Solve for x in terms of y:

$\frac{2x}{2}=\frac{3-y}{2}$

$x=\frac{3-y}{2}$

$r=\frac{3-y}{2}-0$

$r=3-y/2$

y-values

$V=\pi\int_{0}^{2}\left(\frac{3-y}{2}\right)^2dy$

$\left(\frac{1}{2}(3-y)\right)^2$

$\left(\frac{1}{4}\right)(3-y)^2$ pull out

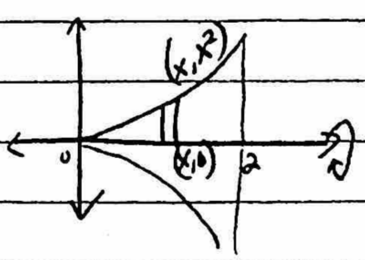
$V=\frac{\pi}{4}\int_{0}^{2}(3-y)^2\,dy$

$V=\pi/4\cdot\int_{0}^{2}9-6y+y^2\,dy$

$V=\pi/4\left[9y-3y^2+\frac{1}{3}y^3\right]_{0}^{2}$

$V=\pi/4\left(18-12+8/3\right)=13\pi/6$

⑤ $y=x^2,\ x=0,\ x=2,\ y=0$



$r=x^2-0$

$r=x^2$

x-values

$V=\pi\int_{0}^{2}(x^2)^2\,dx$

$V=\pi\int_{0}^{2}x^4\,dx=\pi\left(\frac{x^5}{5}\right]_{0}^{2}\Big)$

$\pi\cdot\left(32/5-0\right)=32\pi/5$

⑦ $y=x^2,\ x=0,\ y=1$

y-values

⑦ $y=\sqrt{\cos x}$, $x=\pi/4$, $x=\pi/2$, $y=0$

$r=\sqrt{\cos x}-0$

$r=\sqrt{\cos x}$

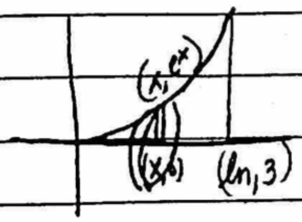
$$V=\pi\int_{\pi/4}^{\pi/2}\left(\sqrt{\cos x}\right)^2dx$$

$$V=\pi\int_{\pi/4}^{\pi/2}\cos x\,dx=\pi\ \sin x\Big]_{\pi/4}^{\pi/2}$$

$$\pi\cdot\left(\sin\pi/2-\sin\pi/4\right)$$

$$\pi\left(1-\sqrt{2}/2\right)=\frac{(2-\sqrt{2})\pi}{2}$$

⑪ $y=e^x$ $y=0$, $x=0$, $x=\ln 3$



$r=e^x-0$

$r=e^x$

$$V=\pi\int_0^{\ln 3}\left(e^x\right)^2dx$$

$$V=\pi\cdot\int_0^{\ln 3}e^{2x}dx=\pi/2\cdot e^{2x}\Big]_0^{\ln 3}$$

$u=2x$

$du=2dx$

$du/2=dx$

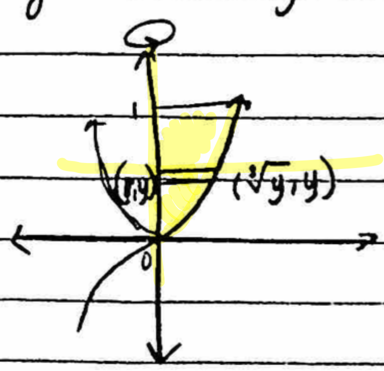
$e^u du$

$$\pi/2\cdot\left(e^{2\ln 3}-e^{2\cdot 0}\right)$$

$$\pi/2\left(e^{\ln 3^2}-1\right)$$

$$\pi/2\,(9-1)=4\pi$$

⑮ $y=x^3$, $x=0$, $y=1$



y-values

$$V=\pi\int_0^1\left(\sqrt[3]{y}\right)^2dy$$

$r=\sqrt[3]{y}-0$

$r=\sqrt[3]{y}$

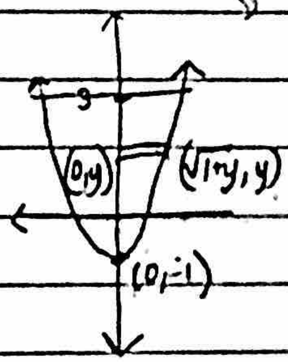
$$V=\pi\int_0^1 y^{2/3}dy=\pi\left(3/5\,y^{5/3}\right)\Big]_0^1$$

$$\pi\cdot 3/5(1)^{5/3}=3\pi/5$$

(17) $x=\sqrt{1+y}$, $x=0$, $y=3$

$\rightarrow x^2 = 1+y$

$x^2-1=y$



t.p. $-b/2a = \frac{0}{2a}$

$r=\sqrt{1+y}-0$

$r=\sqrt{1+y}$

$$V=\pi\int_{-1}^{3}\left(\sqrt{1+y}\right)^2 dy$$

$$V=\pi\int_{-1}^{3}(1+y)\,dy$$

$$V=\pi\cdot\left[y+\tfrac{1}{2}y^2\right]_{-1}^{3}$$

$$V=\pi\left(3+\tfrac{9}{2}-\left(-1+\tfrac{1}{2}\right)\right)$$

$$V=\pi\left(3+9/2+1-\tfrac{1}{2}\right)$$

$$V=\pi(3+4+1)=8\pi$$