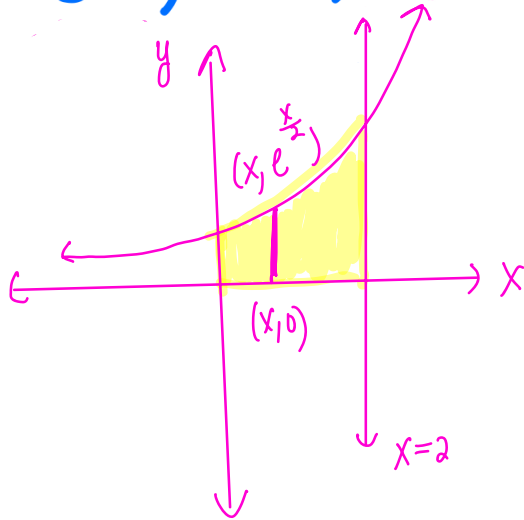


① Find the area of the curve bounded by  $y = e^{\frac{x}{2}}$ ,  $x=0$ ,  $x=2$  and  $y=0$ .

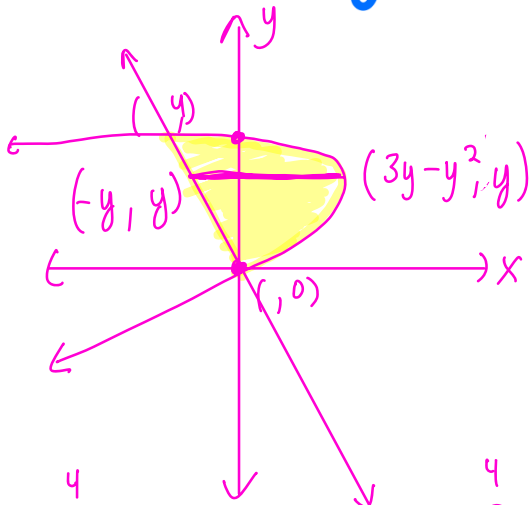


$$A = \int_0^2 e^{\frac{x}{2}} dx$$

$$2e^{\frac{x}{2}} \Big|_0^2$$

$$2e - 2e^0 = 2e - 2$$

② Find the area bounded by the curve  $x = 3y - y^2$  and the line  $x = -y$ .



$$0 = 3y - y^2$$

$$0 = y(3 - y)$$

$$y = 0, 3$$

pts of int:

$$-y = 3y - y^2$$

$$0 = 4y - y^2$$

$$0 = y(4 - y)$$

$$y = 0 \mid y = 4$$

$$A = \int_0^4 (3y - y^2 - (-y)) dy = \int_0^4 (4y - y^2) dy$$

$$= 2y^2 - \frac{y^3}{3} \Big|_0^4 = 32 - \frac{64}{3} - 0 = 32 - \frac{64}{3}$$

③ Rewrite the limit of the Riemann sum as a definite integral.

$$a) \lim_{n \rightarrow \infty} \sum_{i=1}^n \ln\left(2 + \frac{5i}{n}\right) \cdot \frac{5}{n} \rightarrow \Delta x = \frac{b-a}{n}$$

$$f(x) = \ln x$$

$$a = 2$$

$$5 = b - a$$

$$5 = b - 2$$

$$7 = b$$

$$\int_2^7 \ln x \, dx$$

$$b) \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \sin\left(\frac{\pi}{2} + \frac{\pi i}{n}\right) \right] \cdot \frac{\pi}{n} \rightarrow b - a$$

$$f(x) = \sin x$$

$$a = \frac{\pi}{2}$$

$$b - \frac{\pi}{2} = \pi$$

$$b = \frac{3\pi}{2}$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin x \, dx$$

④ Rewrite the definite integral as a limit of a Riemann sum

$$a) \int_0^3 e^x dx$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n e^{\frac{3i}{n}} \cdot \frac{3}{n}$$

$$f(a + \Delta x i)$$

$$f\left(0 + \frac{3i}{n}\right)$$

$$f\left(\frac{3i}{n}\right) = e^{\frac{3i}{n}}$$

$$b) \int_0^{\pi} \cos x dx$$

$$\Delta x = \frac{\pi}{n}$$

$$a = 0$$

$$f\left(0 + \frac{\pi}{n} i\right)$$

$$f\left(\frac{\pi i}{n}\right) = \cos\left(\frac{\pi i}{n}\right)$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \cos\left(\frac{\pi i}{n}\right) \cdot \frac{\pi}{n}$$

Calculator questions?

Questions from last exam, particularly FTC Pt II free response?

$$c) \int_{\frac{\pi}{2}}^{\pi} \sin x dx$$

$$a = \frac{\pi}{2}$$

$$\Delta x = \frac{\pi - \frac{\pi}{2}}{n} = \frac{\frac{\pi}{2}}{n} = \frac{\pi}{2n}$$

$$f\left(\frac{\pi}{2} + \frac{\pi i}{2n}\right)$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(\frac{\pi}{2} + \frac{\pi i}{2n}\right) \cdot \frac{\pi}{2n}$$

Definite Integral as the Limit of a Riemann Sum

$\int_0^4 x^3 dx$ <p style="text-align: right; color: red;"> <math>a=0</math>  <math>b=4</math>  <math>\Delta x = \frac{4-0}{n} = \frac{4}{n}</math>  <math>f(x) = x^3</math>  <math>f(0 + \frac{4k}{n})</math> </p>	$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{4k}{n}\right)^3 \cdot \frac{4}{n}$
$\int_2^4 x^3 dx$	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(2 + \frac{2i}{n}\right)^3 \cdot \left(\frac{2}{n}\right)$ <p style="text-align: right; color: red;"> <math>\Delta x = \frac{2}{n} = \frac{b-a}{n} = \frac{b-2}{n}</math> </p>
$\int_2^6 x^3 dx$ <p style="text-align: right; color: red;"> <math>\Delta x = \frac{6-2}{n} = \frac{4}{n}</math>  <math>a=2</math> </p>	$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{4k}{n}\right)^3 \cdot \frac{4}{n}$ <p style="text-align: right; color: red;"> <math>2 = b-a</math>  <math>4 = b</math> </p>
$\int_4^6 x^3 dx$	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(4 + \frac{2i}{n}\right)^3 \cdot \left(\frac{2}{n}\right)$ <p style="text-align: right; color: red;"> <math>a=4</math>  <math>\frac{b-a}{n} = \frac{2}{n}</math>  <math>b-4=2</math>  <math>b=6</math> </p>
$\int_2^6 (x+2)^3 dx$ <p style="text-align: right; color: red;"> <math>\Delta x = \frac{6-2}{n} = \frac{4}{n}</math>  <math>a=2</math>  <math>f\left(2 + \frac{4k}{n}\right) = \left(2 + \frac{4k}{n} + 2\right)^3</math> </p>	$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(4 + \frac{4k}{n}\right)^3 \left(\frac{4}{n}\right)$
$\int_0^{\pi} \sin x dx$	$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\sin\left(\frac{\pi i}{n}\right)\right] \cdot \left(\frac{\pi}{n}\right)$ <p style="text-align: right; color: red;"> <math>\frac{\pi}{n} = \frac{b-a}{n}</math>  <math>\pi = b-0</math>  <math>\pi = b</math> </p>

$$\int_{\pi/2}^{\pi} \sin x \, dx$$

$\Delta x = \frac{\pi - \frac{\pi}{2}}{n} = \frac{\frac{\pi}{2}}{n} = \frac{\pi}{2n}$   
 $a = \frac{\pi}{2}$   
 $\sin\left(\frac{\pi}{2} + \frac{\pi k}{2n}\right)$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \sin\left(\frac{\pi}{2} + \frac{\pi k}{2n}\right) \cdot \left(\frac{\pi}{2n}\right)$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin x \, dx$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \sin\left(\frac{\pi}{2} + \frac{\pi i}{n}\right) \right] \cdot \left(\frac{\pi}{n}\right)$$

$a = \frac{\pi}{2}$   
 $\frac{\pi}{n} = \frac{b - \frac{\pi}{2}}{n}$

$$\int_{\pi/4}^{3\pi/4} \sin x \, dx$$

$\Delta x = \frac{3\pi/4 - \pi/4}{n} = \frac{\pi/2}{n} = \frac{\pi}{2n}$   
 $a = \frac{\pi}{4}$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(\frac{\pi}{4} + \frac{\pi i}{2n}\right) \cdot \left(\frac{\pi}{2n}\right)$$

$\frac{\pi}{2n} = \frac{b - \frac{\pi}{4}}{n}$   
 $\frac{\pi}{2} = b$

$$\int_{\frac{5}{4}}^{\frac{17}{4}} \sqrt{4x} \, dx$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{5 + \frac{12i}{n}} \cdot \left(\frac{3}{n}\right)$$

$\Delta x = \frac{3}{n}$   
 $f(x) = \sqrt{x}$   
 $a + \Delta x k = \frac{5}{4} + \frac{3k}{n} = 5 + \frac{12k}{n}$   
 $\frac{b - \frac{5}{4}}{n} = \frac{3}{n}$   
 $b - \frac{5}{4} = 3$   
 $b = \frac{17}{4}$   
 $4(a + \frac{3k}{n}) = 5 + \frac{12k}{n}$   
 $4a = 5$   
 $a = \frac{5}{4}$

$$\int_1^5 \sqrt{x} \, dx$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{1 + \frac{4i}{n}} \cdot \left(\frac{4}{n}\right)$$

$a = 1$   
 $\Delta x = \frac{4}{n}$   
 $\frac{4}{n} = \frac{b - 1}{n}$   
 $b = 5$

$$\int_2^5 \sqrt{1 + 4x} \, dx$$

$\Delta x = \frac{5 - 2}{n} = \frac{3}{n}$   
 $a = 2$   
 $\sqrt{1 + 4\left(2 + \frac{3k}{n}\right)}$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{9 + \frac{12k}{n}} \cdot \left(\frac{3}{n}\right)$$

$$\sqrt{1 + 8 + \frac{12k}{n}}$$

$$\sqrt{9 + \frac{12k}{n}}$$

$$\int_1^4 \sqrt{x} \, dx$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{1 + \frac{3i}{n}} \cdot \left(\frac{3}{n}\right)$$

$a_1 + \Delta x i$   
 $a=1$

$\frac{3}{n} = \frac{b-1}{n}$   
 $b=4$

$$\int_1^e \ln x \, dx$$

$\Delta x = \frac{e-1}{n}$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \ln\left(1 + \frac{(e-1)k}{n}\right) \left(\frac{e-1}{n}\right)$$

$$\int_1^{e+1} \ln x \, dx$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \ln\left(1 + \frac{e \cdot i}{n}\right) \right] \cdot \left(\frac{e}{n}\right)$$

$a + \Delta x i$   
 $a=1$

$\frac{e}{n} = \frac{b-1}{n}$   
 $b=e+1$

$$\int_e^{2e} \ln x \, dx$$

$\Delta x = \frac{2e-e}{n}$   
 $\Delta x = \frac{e}{n}$   
 $a=e$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \ln\left(e + \frac{ek}{n}\right) \left(\frac{e}{n}\right)$$