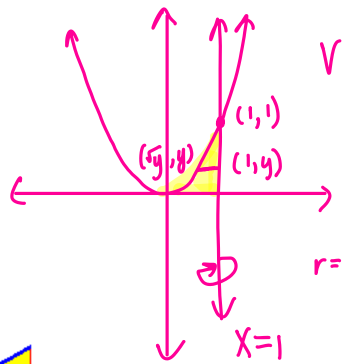
Name: ______________________________________ Date: ______________

AP Calculus AB Volumes of Solids of Revolution Ms. Loughran

Do Now:

Find the volume of the solid formed when the region bounded by $y = x^2$, the x-axis, and $x = 1$ is revolved about the line $x = 1$.



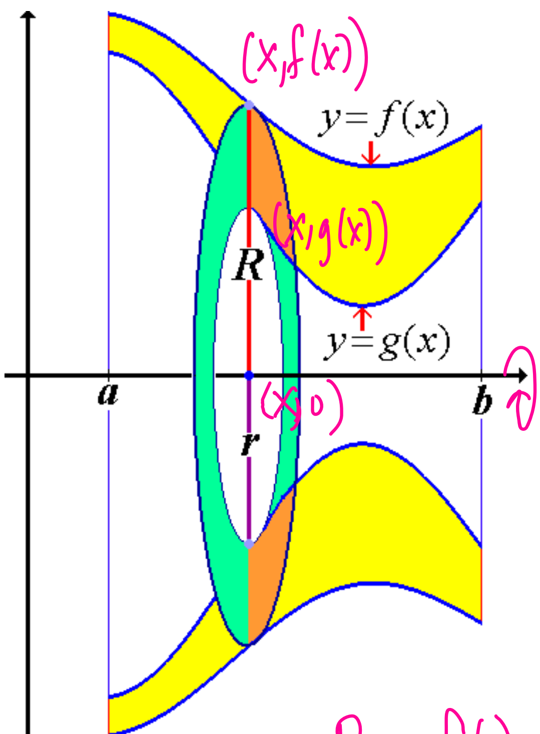
$$V = \pi \int_0^1 (1-\sqrt{y})^2 \, dy$$

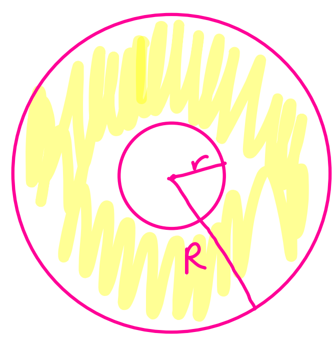
$$\pi \int_0^1 (1 - 2\sqrt{y} + y) \, dy$$

$r = 1 - \sqrt{y}$

$$\pi \left[y - \frac{4}{3} y^{3/2} + \frac{y^2}{2} \Big|_0^1 \right]$$

$$\pi \left[1 - \frac{4}{3} + \frac{1}{2} \right]$$





$A_{\text{shaded region}} = \pi R^2 - \pi r^2$

$= \pi (R^2 - r^2)$

$* \; R^2 - r^2 \neq (R-r)^2$

$R = f(x) - 0 = f(x)$

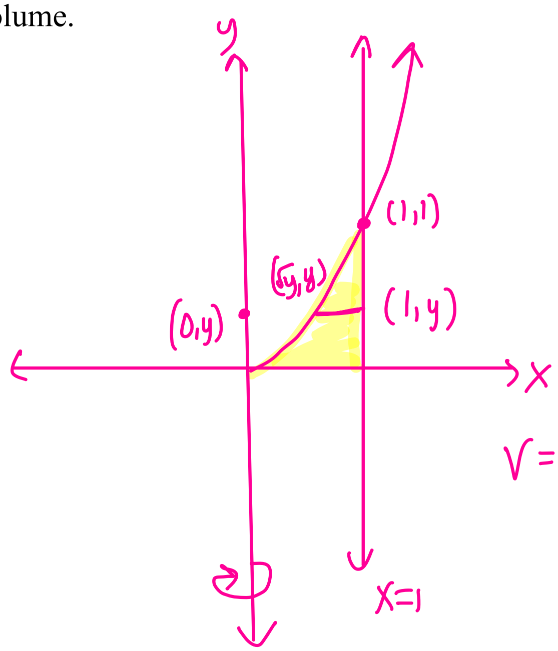
$r = g(x) - 0 = g(x)$

$$V = \pi \int_a^b \Big(\overset{R^2}{(f(x))^2} - \overset{r^2}{(g(x))^2} \Big) \, dx$$

* length of the strip is not used in this method

$x^2 = y$

1. The region bounded by $x = \sqrt{y}$, the x-axis and $x = 1$ is revolved about the y-axis. Find the volume.



$R = 1 - 0 = 1$

$r = \sqrt{y} - 0 = \sqrt{y}$

$$V = \pi \int_0^1 1^2 - (\sqrt{y})^2 \, dy$$

$$\pi \left[y - \frac{y^2}{2} \Big|_0^1 \right]$$

$$\pi \left[1 - \frac{1}{2} - 0 \right] = \frac{\pi}{2}$$

2. Find the volume of the solid that results when the region bounded by $y = x$ and $y = x^2$ is revolved about the x-axis.

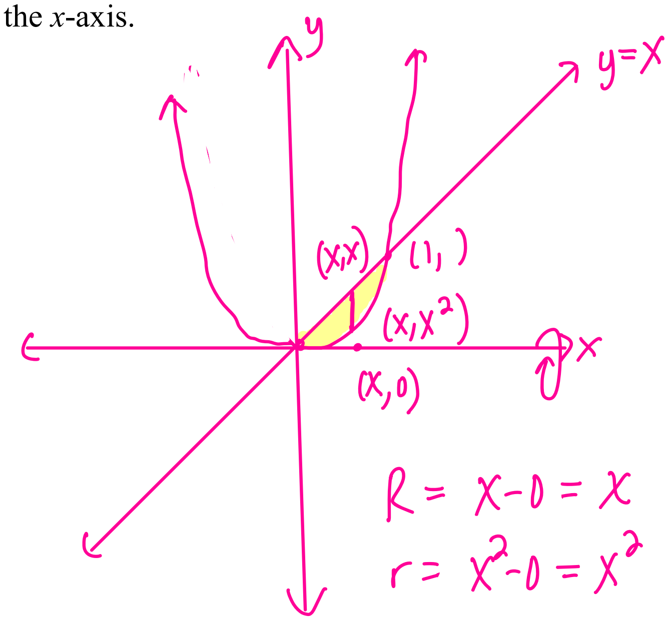
Pts of int

$x = x^2$

$0 = x^2 - x$

$0 = x(x-1)$

$x = 0, 1$



$R = x - 0 = x$

$r = x^2 - 0 = x^2$

$$V = \pi \int_0^1 x^2 - (x^2)^2 \, dx = \pi \left[\frac{x^3}{3} - \frac{x^5}{5} \Big|_0^1 \right]$$

$$\pi \left[\frac{1}{3} - \frac{1}{5} - 0 \right]$$

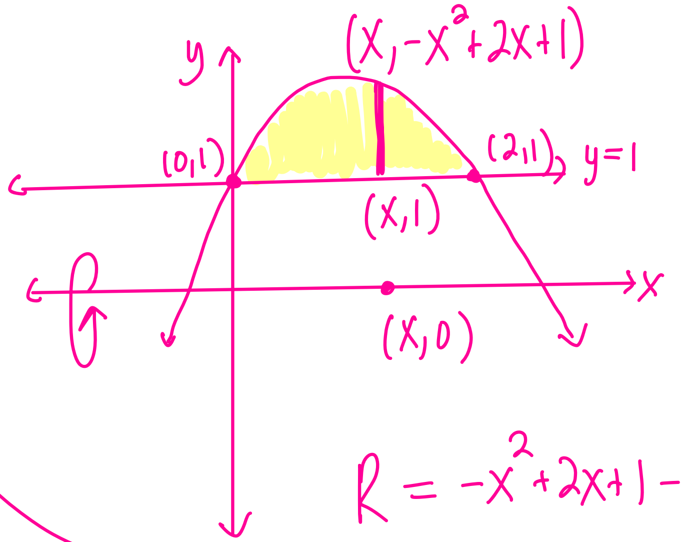
$(-x^2+2x+1)^2 - 1$

$x^4 - 2x^3 - x^2 - 2x^3 + 4x^2 + 2x - x^2 + 2x + \cancel{1} - \cancel{1}$

$x^4 - 4x^3 + 2x^2 + 4x$

y-int: $(0,1)$

3. Consider the area bounded by the graphs of $y = -x^2 + 2x + 1$ and $y = 1$. Find the volume generated if this area is rotated about the x-axis.



Pts of int.

$-x^2+2x+1 = 1$

$-x^2+2x = 0$

$-x(x-2) = 0$

$x = 0, 2$

$R = -x^2+2x+1-0 = -x^2+2x+1$

$r = 1-0 = 1$

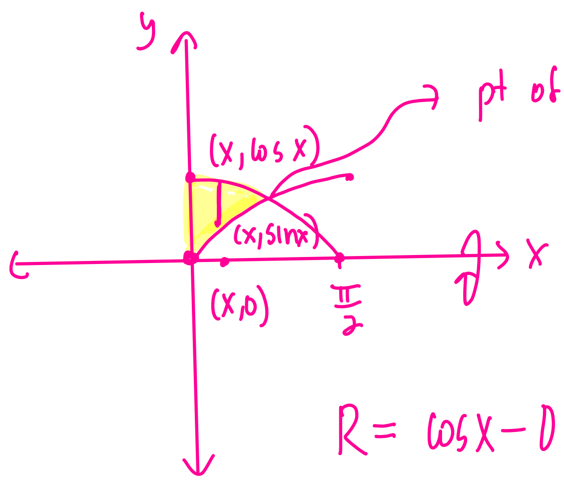
$$V = \pi \int_0^2 (-x^2+2x+1)^2 - 1^2 \, dx = \pi \int_0^2 (x^4 - 4x^3 + 2x^2 + 4x)\,dx$$

$$\pi \left[\frac{x^5}{5} - x^4 + \frac{2x^3}{3} + 2x^2 \Big|_0^2 \right]$$

$$\pi \left[\frac{32}{5} - 16 + \frac{16}{3} + 8 \right]$$

$$\frac{56\pi}{15}$$

4. The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved around the x-axis to form a solid. Find its volume.



pt of int: $x = \frac{\pi}{4}$

$\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$

$R = \cos x - 0 = \cos x$

$r = \sin x - 0 = \sin x$

$$V = \pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)\,dx = \pi \int_0^{\frac{\pi}{4}} \cos 2x \, dx = \pi \left[\frac{1}{2} \sin 2x \Big|_0^{\frac{\pi}{4}} \right]$$

$* \cos^2 x - \sin^2 x = \cos 2x *$

$$= \pi \left[\frac{1}{2} \sin \frac{\pi}{2} - \frac{1}{2} \sin 0 \right]$$

$$\pi \left[\frac{1}{2}(1) - \frac{1}{2}(0) \right] = \frac{\pi}{2}$$