

Calculus AB

Section I

Part A

2003

1. If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} = 2(x^3 + 1) \cdot 3x^2$

- (A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

2. $\int_0^1 e^{-4x} dx =$

- (A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4} - 1$ (D) $\frac{1}{4} - \frac{e^{-4}}{4}$ (E) $4 - 4e^{-4}$
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3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f . Which of the following statements must be true?

- (A) $f(0) = 2$
 (B) $f(x) \neq 2$ for all $x \geq 0$
 (C) $f(2)$ is undefined.
 (D) $\lim_{x \rightarrow 2} f(x) = \infty$
 (E) $\lim_{x \rightarrow \infty} f(x) = 2$

4. If $y = \frac{2x+3}{3x+2}$, then $\frac{dy}{dx} = \frac{6x+4 - (6x+9)}{(3x+2)^2} = \frac{(3x+2)(2) - (2x+3)(3)}{(3x+2)^2}$
- (A) $\frac{12x+13}{(3x+2)^2}$ (B) $\frac{12x-13}{(3x+2)^2}$ (C) $\frac{5}{(3x+2)^2}$ (E) $\frac{2}{3}$
- (D) $\frac{-5}{(3x+2)^2}$

5. $\int_0^{\pi/4} \sin x \, dx =$

- (A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2} - 1$ (D) $-\frac{\sqrt{2}}{2} + 1$ (E) $\frac{\sqrt{2}}{2} - 1$
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6. $\lim_{x \rightarrow \infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

- (A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1
-

8. $\int x^2 \cos(x^3) dx =$

(A) $-\frac{1}{3} \sin(x^3) + C$

(B) $\frac{1}{3} \sin(x^3) + C$

(C) $-\frac{x^3}{3} \sin(x^3) + C$

(D) $\frac{x^3}{3} \sin(x^3) + C$

(E) $\frac{x^3}{3} \sin\left(\frac{x^4}{4}\right) + C$

9. If $f(x) = \ln(x + 4 + e^{-3x})$, then $f'(0)$ is

(A) $-\frac{2}{5}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{2}{5}$

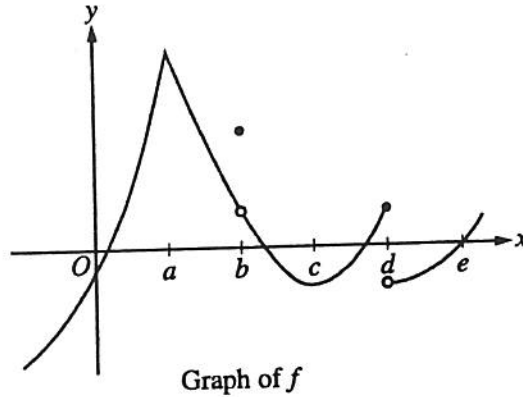
(E) nonexistent

$$f'(x) = \frac{1}{x+4+e^{-3x}} \cdot (-3e^{-3x})$$

$$f'(0) = \frac{1}{0+4+e^0} (-3e^0) = -\frac{2}{5}$$

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Part A



13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$ $x^2 2 \cos(2x) + 2x \sin 2x$
 $2x(x \cos(2x) + \sin 2x)$

- (A) $2x \cos 2x$
 (B) $4x \cos 2x$
 (C) $2x(\sin 2x + \cos 2x)$
 (D) $2x(\sin 2x - x \cos 2x)$
 (E) $2x(\sin 2x + x \cos 2x)$

15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?
- (A) $(-\infty, -1]$ only
(B) $(-\infty, 0)$
(C) $[-1, 0)$ only
(D) $(0, \sqrt[3]{2}]$
(E) $[\sqrt[3]{2}, \infty)$

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16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

$$m = \frac{7 - (-2)}{1 - (-2)} = \frac{9}{3} = 3$$

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

- (A) $y = 5x - 3$
- (B) $y = x^2 + 1$
- (C) $y = x^2 + 3x$
- (D) $y = x^2 + 3x - 2$
- (E) $y = 2x^2 + 3x - 3$

$$f(x) = \begin{cases} x + 2 & \text{if } x \leq 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

$x+2 = 4x-7$ @ $x=3$
 $5 = 5$

20. Let f be the function given above. Which of the following statements are true about f ?

- I. $\lim_{x \rightarrow 3} f(x)$ exists. ✓
- II. f is continuous at $x = 3$. ✓
- III. f is differentiable at $x = 3$.

- (A) None
- (B) I only
- (C) II only
- (D) I and II only
- (E) I, II, and III

$$f'(x) = \begin{cases} 1 & x \leq 3 \\ 4 & x > 3 \end{cases}$$

$1 \neq 4$

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Part A

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

- (A) $y = 7x - 3$
 (B) $y = 7x + 7$
 (C) $y = 7x + 11$
 (D) $y = -5x - 1$
 (E) $y = -5x - 5$

$$f(-1) = 4(-1)^3 - 5(-1) + 3 = -4 + 5 + 3 = 4$$

$$f'(x) = 12x^2 - 5$$

$$f'(-1) = 12(-1)^2 - 5 = 7$$

$$y - 4 = 7(x + 1)$$

$$y = 7x + 11$$

25. A particle moves along the x -axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

- (A) $t = 1$ only
 (B) $t = 3$ only
 (C) $t = \frac{7}{2}$ only
 (D) $t = 3$ and $t = \frac{7}{2}$
 (E) $t = 3$ and $t = 4$

$$v(t) = 6t^2 - 42t + 72$$

$$v(t) = 6(t^2 - 7t + 12)$$

$$v(t) = 6(t-3)(t-4)$$

$$0 = 6(t-3)(t-4)$$

$$t = 3, 4$$

26. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$?

- (A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

(3,2)

$$6y \frac{dy}{dx} - 4x = -2x \frac{dy}{dx} - 2y$$

$$6(2) \frac{dy}{dx} - 4(3) = -2(3) \frac{dy}{dx} - 2(2)$$

$$12 \frac{dy}{dx} - 12 = -6 \frac{dy}{dx} - 4$$

$$18 \frac{dy}{dx} = 8 \quad \frac{dy}{dx} = \frac{8}{18} = \frac{4}{9}$$

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

- (A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

$$x^3 + x = 2$$

$$x^3 + x - 2 = 0$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

$$g'(2) = \frac{1}{f'(1)} = \frac{1}{4}$$

$$f'(x) = 3x^2 + 1$$

$$f'(1) = 3(1)^2 + 1 = 4$$

$g: (2, 1)$

$f: (1, 2)$

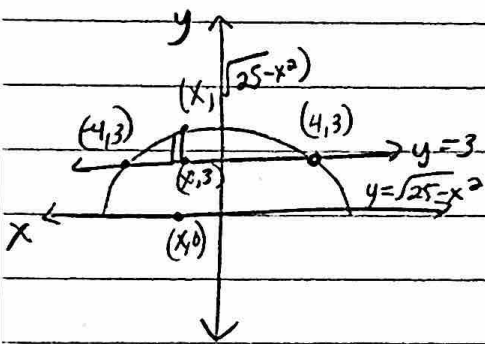
$$x^2 + x + 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(2)}}{2(1)} \leftarrow \text{imaginary}$$

Homework 03-26

⑨ $y = \sqrt{25-x^2}$, $y=3$

$x^2 + y^2 = 25$



$R = \sqrt{25-x^2} - 0 = \sqrt{25-x^2}$

$r = 3 - 0 = 3$

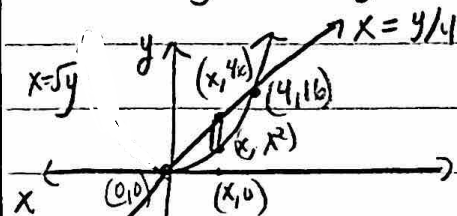
$V = \pi \int_{-4}^4 ((\sqrt{25-x^2})^2 - (3)^2) dx$

$V = \pi \int_{-4}^4 (25-x^2-9) dx = \pi \int_{-4}^4 16-x^2 = \pi \cdot \left[16x - \frac{x^3}{3} \right]_{-4}^4$

$\pi \left[16(4) - \frac{4^3}{3} - \left(16(-4) - \frac{(-4)^3}{3} \right) \right]$
 $\pi \left(64 - \frac{64}{3} + 64 - \frac{64}{3} \right)$
 $\pi \left(128 - \frac{128}{3} \right) = \frac{256\pi}{3}$

$\rightarrow x^2 = y$ $4x = y$

⑬ $x = \sqrt{y}$ $x = y/4$



$R = 4x - 0 = 4x$

$r = x^2 - 0 = x^2$

$V = \pi \int_0^4 ((4x)^2 - (x^2)^2) dx$

$V = \pi \int_0^4 (16x^2 - x^4) dx = \pi \cdot \left[\frac{16x^3}{3} - \frac{x^5}{5} \right]_0^4$

$= \pi \left[\frac{16(4)^3}{3} - \frac{(4)^5}{5} - 0 \right] = \pi \left[\frac{1024}{3} - \frac{1024}{5} \right]$
 $\pi \left[\frac{5120}{15} - \frac{3072}{15} \right] = \frac{2048\pi}{15}$

$\sqrt{y} = y/4$

$4\sqrt{y} = y$

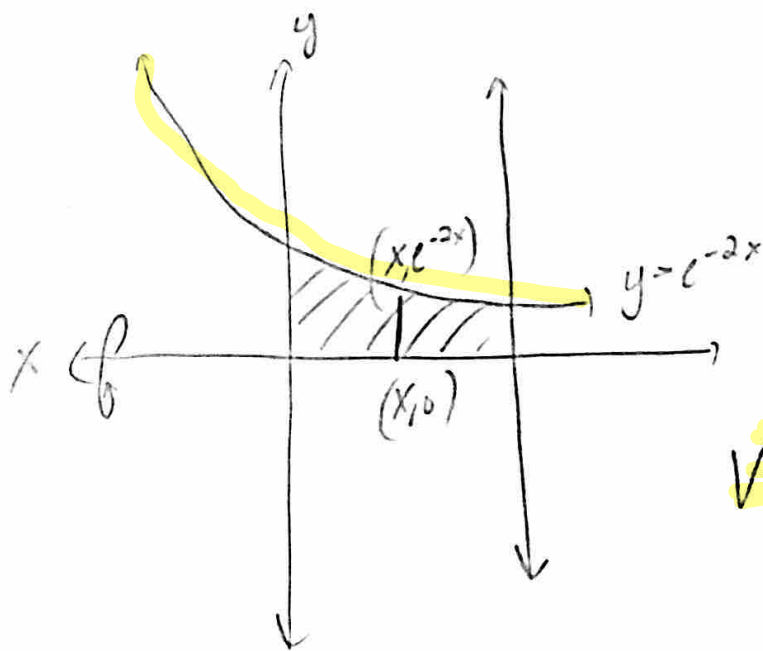
$16y = y^2$

$y^2 - 16y = 0$

$y(y-16) = 0$

$y=0$ $y=16$
 $x=0$ $x=4$

(12)

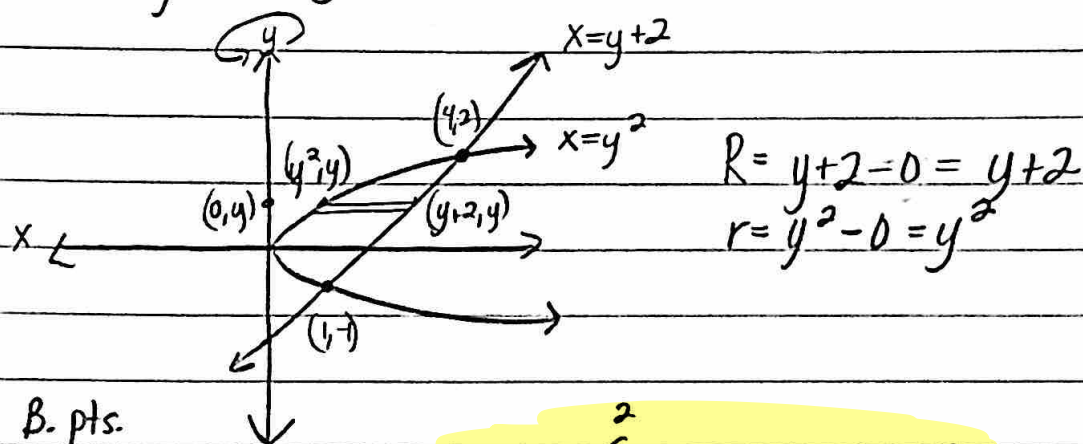


$$r = e^{-2x}$$

$$V = \pi \int_0^1 (e^{-2x})^2 dx$$

$$\approx \left[-\frac{1}{4} e^{-4x} \right]_0^1 = \pi \left[-\frac{1}{4} e^{-4} - \left(-\frac{1}{4} e^0 \right) \right] = \pi \int_0^1 e^{-4x} dx = \left(-\frac{1}{4e^4} + \frac{1}{4} \right) \pi$$

② $X=y^2, X=y+2$



B. pts.

$$y^2 = y + 2$$

$$y^2 - y - 2 = 0$$

$$(y-2)(y+1) = 0$$

$$y = 2 \quad y = -1$$

$$x = 4 \quad x = 1$$

$$V = \pi \int_{-1}^2 [(y+2)^2 - (y^2)^2] dy$$

$$V = \pi \int_{-1}^2 [y^2 + 4y + 4 - y^4] dy$$

$$V = \pi \left[\frac{y^3}{3} + 2y^2 + 4y - \frac{y^5}{5} \right]_{-1}^2$$

$$V = \pi \left(\frac{2^3}{3} + 2(2)^2 + 4(2) - \frac{2^5}{5} - \left(\frac{(-1)^3}{3} + 2(-1)^2 + 4(-1) - \frac{(-1)^5}{5} \right) \right)$$

$$V = \pi \left(\frac{8}{3} + 8 + 8 - \frac{32}{5} - \left(-\frac{1}{3} + 2 - 4 + \frac{1}{5} \right) \right)$$

$$V = \pi \left(\frac{8}{3} + 16 - \frac{32}{5} + \frac{1}{3} - 2 + 4 - \frac{1}{5} \right)$$

$$V = \pi \left(18 + 3 - \frac{33}{5} \right)$$

$$V = \pi \left(21 - \frac{33}{5} \right) = \pi \left(\frac{105}{5} - \frac{33}{5} \right)$$

$$= \frac{72\pi}{5}$$