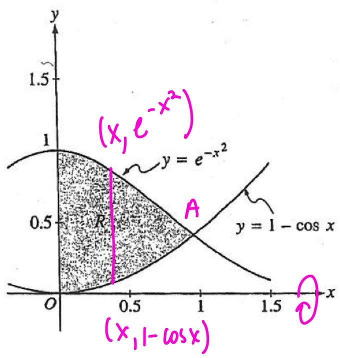
Do Now: From the Area and Volume packet (posted on 03-18)
2000 AB 1 parts b and c

2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



$A = .941...$

1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(b) $R = e^{-x^2} - 0 = e^{-x^2}$

$r = 1 - \cos x - 0 = 1 - \cos x$

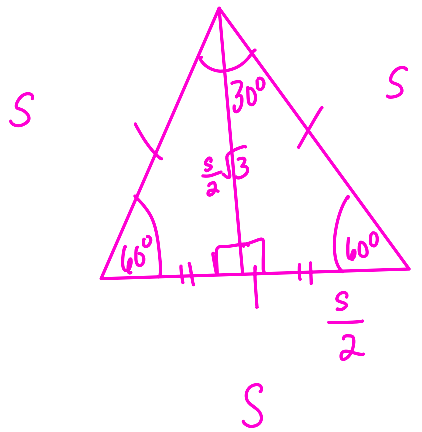
$$V = \pi \int_0^A \left((e^{-x^2})^2 - (1-\cos x)^2 \right) dx$$

$= 1.747$ or 1.746

(c) $\text{side} = e^{-x^2} - (1 - \cos x)$

$$V = \int_0^A \left(e^{-x^2} - (1-\cos x) \right)^2 dx = .461$$

Let's find a general formula for finding the area of an equilateral triangle.

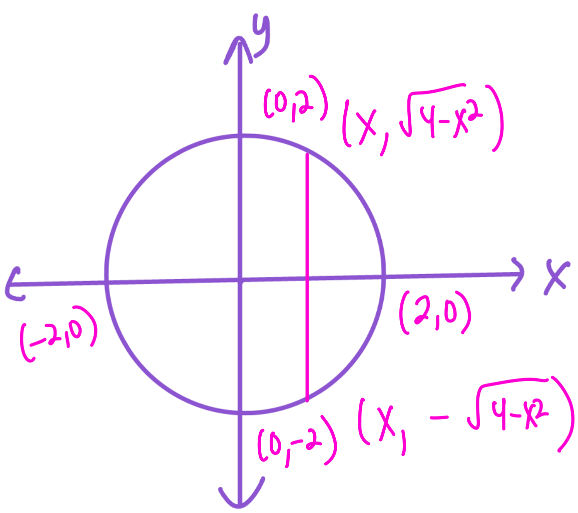


$$A_{\Delta} = \frac{1}{2}bh$$

$$A_{eq.\Delta} = \frac{1}{2}s \cdot \frac{s}{2}\sqrt{3}$$

$$= \frac{s^2\sqrt{3}}{4}$$

3. Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 4$ with the indicated cross sections taken perpendicular to the x-axis:
   (a) squares
   (b) equilateral triangles
   (c) isosceles right triangles with hypotenuse in bounded region



$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4-x^2}$$

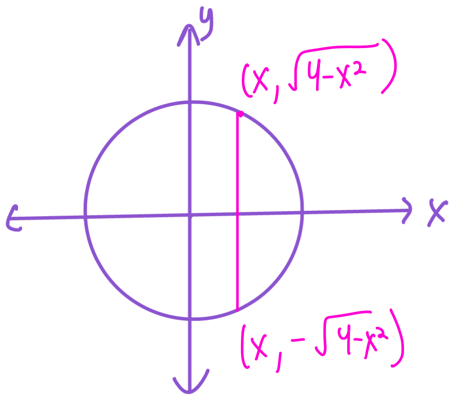
$$\text{side} = \sqrt{4-x^2} - (-\sqrt{4-x^2})$$

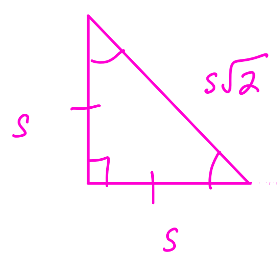
$$= 2\sqrt{4-x^2}$$

$$V = \int_{-2}^{2} \frac{\left(2\sqrt{4-x^2}\right)^2\sqrt{3}}{4}\,dx$$

$$\frac{\sqrt{3}}{4}\int_{-2}^{2} 4(4-x^2)\,dx = \frac{\sqrt{3}}{\cancel{4}}\cancel{4}\int_{-2}^{2} 4-x^2\,dx$$

$$\sqrt{3}\left[4x - \frac{x^3}{3}\Big|_{-2}^{2}\right] = \sqrt{3}\left[4(2) - \frac{2^3}{3} - \left(4(-2) - \frac{(-2)^3}{3}\right)\right]$$





$A = \frac{1}{2}s^2$

$$\text{hypotenuse} = \sqrt{4-x^2} - \left(-\sqrt{4-x^2}\right)$$

$$\text{hyp} = 2\sqrt{4-x^2}$$

$$\text{side} = \frac{2\sqrt{4-x^2}}{\sqrt{2}}$$

$$V = \int_{-2}^{2} \frac{1}{2}\left(\frac{2\sqrt{4-x^2}}{\sqrt{2}}\right)^2 dx$$

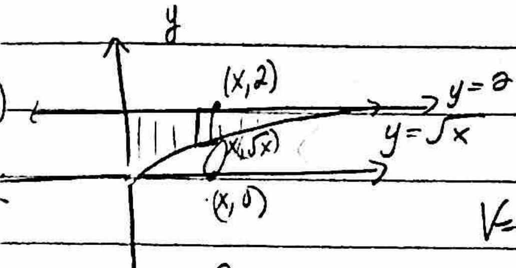
$$V = \frac{1}{2}\int_{-2}^{2} \frac{4\left(4-x^2\right)}{2}\, dx$$

$$V = \cancel{\frac{1}{2}} \cdot \cancel{2}\int_{-2}^{2} 4-x^2\, dx$$

$$4x - \frac{x^3}{3}\Big|_{-2}^{2}$$

$$4(2) - \frac{2^3}{3} - \left(4(-2) - \frac{(-2)^3}{3}\right)$$

Homework 03-27



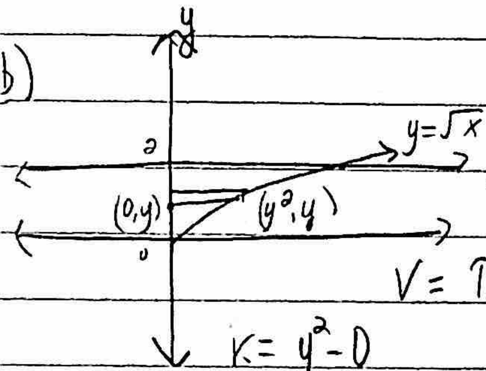
$R = 2-0 = 2$

$r = \sqrt{x} - 0 = \sqrt{x}$

$$V = \pi \int_0^4 \left[(2)^2 - (\sqrt{x})^2\right] dx = \pi \int_0^4 (4-x)\,dx = \pi \cdot \left(4x - \frac{x^2}{2}\right)\Big]_0^4$$

$$\pi\left(\overset{16}{4(4)} - \frac{4^2}{2} - 0\right) = \boxed{8\pi}$$

b)



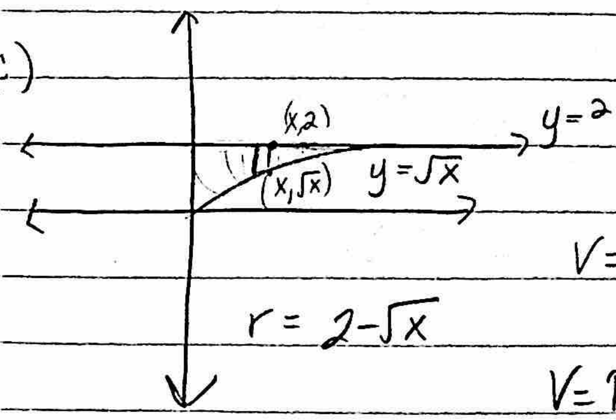
$r = y^2 - 0$

$r = y^2$

$$V = \pi \int_0^2 (y^2)^2\,dy = \pi \int_0^2 y^4\,dy = \pi \cdot \frac{y^5}{5}\Big]_0^2$$

$$\pi\left(\frac{(2)^5}{5}\right) = \boxed{\frac{32\pi}{5}}$$

c)



$r = 2 - \sqrt{x}$

$$V = \pi \int_0^4 (2-\sqrt{x})^2\,dx$$

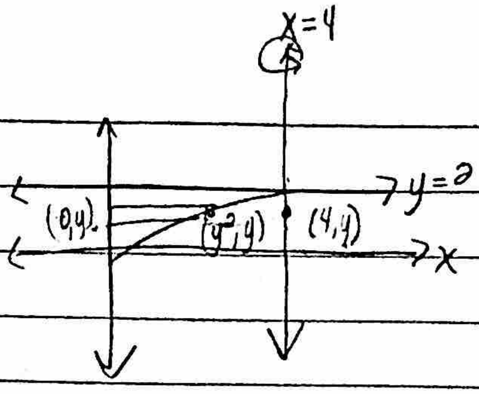
$$V = \pi \int_0^4 (4 - 4\sqrt{x} + x)\,dx$$

$$V = \pi \cdot 4x - \frac{8x^{3/2}}{3} + \frac{x^2}{2}\Big|_0^4$$

$$\pi\left(\left(4(4) - \frac{8(4)^{3/2}}{3} + \frac{4^2}{2}\right) - 0\right) = \pi\left(16 - \frac{64}{3} + 8\right)$$

$$= \pi\left(24 - \frac{64}{3}\right) = \boxed{\frac{8\pi}{3}}$$

(d)



$$V = \pi \int_0^2 \left[(4)^2 - (4-y^2)^2\right] dy$$

$$V = \pi \int_0^2 (16 - (16 - 8y^2 + y^4)) dy$$

$R = 4-0 = 4$

$r = 4 - y^2$

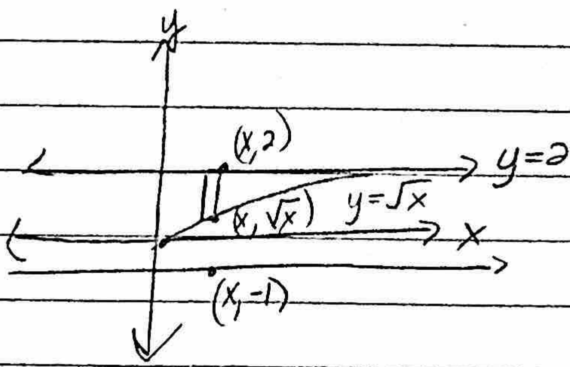
$$= \pi \int_0^2 (8y^2 - y^4) dy$$

$$\pi \cdot \frac{8}{3}y^3 - y^5/5 \Big|_0^2$$

$$\pi\left(\frac{8}{3}(2)^3 - (2)^5/5\right) = \pi\left(\frac{64}{3} - 32/5\right)$$

$$= \frac{224\pi}{15}$$

(e)



$$V = \pi \int_0^4 \left[(3)^2 - (\sqrt{x}+1)^2\right] dx$$

$$V = \pi \int_0^4 9 - (x + 2\sqrt{x} + 1)\, dx$$

$R = 2-(-1) = 3$

$r = \sqrt{x} - (-1) = \sqrt{x} + 1$

$$V = \pi \int_0^4 (8 - x - 2\sqrt{x})\, dx$$

$-\int 2\sqrt{x}$

$-2\int x^{\frac{1}{2}}$

$-2 \cdot \frac{2}{3} x^{3/2}$

$\frac{-4}{3} x^{3/2}$

$$V = \pi \cdot \left(8x - \frac{x^2}{2} - \frac{4x^{3/2}}{3}\right)\Big|_0^4$$

$$V = \pi\left(8(4) - \frac{(4)^2}{2} - \frac{4(4)^{3/2}}{3}\right)$$

$$V = \pi\left(32 - 8 - 32/3\right) = \pi\left(24 - \frac{32}{3}\right)$$

$$= \frac{40\pi}{3}$$