Do Now: 2010 AB 6

6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

(a) $f(1) = 2$, $(1,2)$

$\left.\frac{dy}{dx}\right|_{(1,2)} = 1(2)^3 = 8$

$y - 2 = 8(x-1)$

(b) $y - 2 = 8(1.1-1)$

$y = 8(1.1-1) + 2 = 2.8$

$\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = 2^3(1+3(1)^2(2)^2) > 0$

Since $\frac{d^2y}{dx^2} > 0$ at $(1,2)$ the curve (y) is concave up at $(1,2)$. Therefore $f(1.1)$ is an underapproximation

(c) $\frac{dy}{dx} = xy^3$

$\int \frac{dy}{y^3} = \int x\,dx$

$\frac{y^{-2}}{-2} = \frac{x^2}{2} + C$

$\frac{1}{-2y^2} = \frac{x^2}{2} + C$

$(1,2)$

$-\frac{1}{8} = \frac{1}{2} + C$

$-\frac{5}{8} = C$

$-\frac{1}{2y^2} = \frac{x^2}{2} - \frac{5}{8}$

$\cancel{-2}\cdot -\frac{1}{\cancel{2}y^2} = \frac{4x^2-5}{\cancel{8}_4} \cdot -2$

$\frac{1}{y^2} = \frac{5-4x^2}{4}$

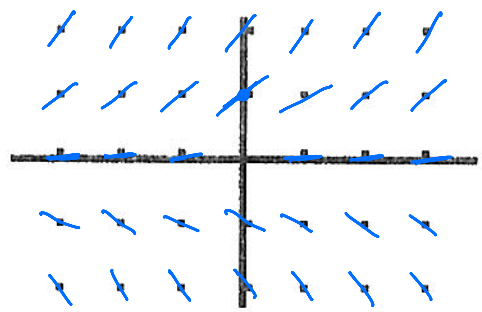
$y^2 = \frac{4}{5-4x^2}$

$y = \cancel{\pm}\sqrt{\frac{4}{5-4x^2}}$

Name:__________________________________ Date: ________
AP Calculus AB: Slope Fields (Directional Fields) Ms. Loughran

A slope field is a picture of the family of general solutions of a differential equation. This picture is created by using small segments of tangent lines which approximate the curves at each point.

Consider $\frac{dy}{dx} = y$. In words, any solution to this differential equation has the property that at any point in the plane, the slope of the curve equals the y-coordinate there. Let's draw a picture.



Within the slope field, the ghosts of the solution curves are lurking. The slope field is a set of signposts pointing you in the direction you should go at each point.

Algebraic solution:

$$\frac{dy}{dx} = y$$

$$y\,dx = dy$$

$$\int dx = \int \frac{dy}{y}$$

$$x + c = \ln|y|$$

$$e^{x+c} = e^{\ln|y|}$$

$$e^x \cdot e^c = |y|$$

$$Ke^x = |y|$$

$$y = \pm Ke^x$$

don't need to write the table

point	$\frac{dy}{dx}$
(1,1)	-1
(2,1)	-2
(3,1)	-3
(-1,1)	1
(-2,1)	2
(-3,1)	(3)

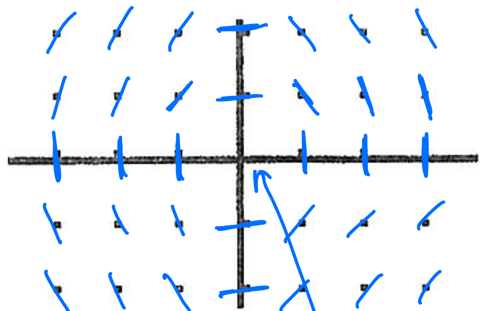
Let's try another one.

$$\frac{dy}{dx} = -\frac{x}{y}$$

look for

$\frac{dy}{dx} = 0$, $x=0, y\neq 0$ (horizontal tan lines)

$\frac{dy}{dx}$ undefined $y=0, x\neq 0$ (vertical tan line)



$\frac{0}{0}$ is indeterminate form, not differentiable at that point

What does the solution of this differential equation appear to be?

Let's find it.

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\int y\,dy = \int -x\,dx$$

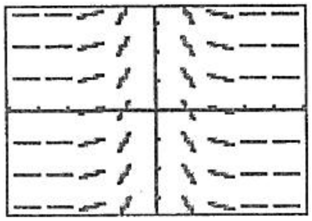
$$\frac{y^2}{2} = \frac{-x^2}{2} + C$$

$$y^2 = -x^2 + 2C$$

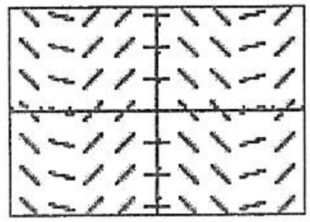
$$x^2 + y^2 = 2C \leftarrow \text{circle}$$

Match each slope field with the equation that the slope field could represent.

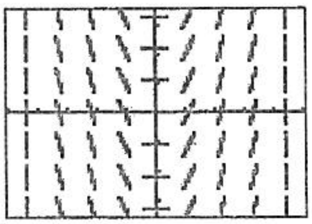
(A)



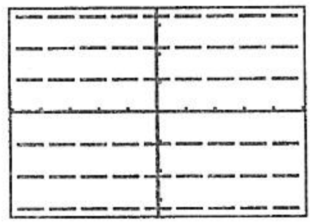
(B)



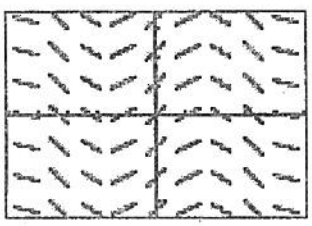
(C)



(D)



(E)



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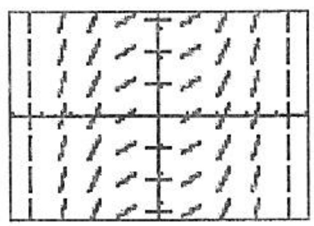
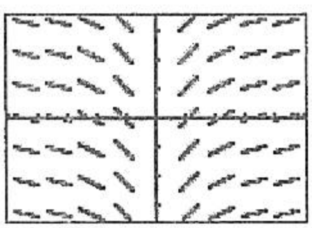
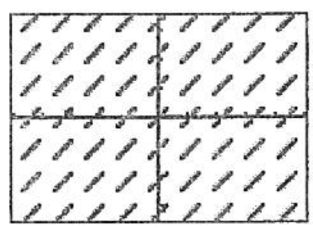


(G)



(H)



7. $y = 1$ D

8. $y = x$ H

9. $y = x^2$ C

10. $y = \frac{1}{6}x^3$ F

11. $y = \frac{1}{x^2}$ A

12. $y = \sin x$ E

13. $y = \cos x$ B

14. $y = \ln|x|$ G

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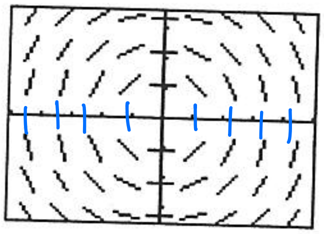
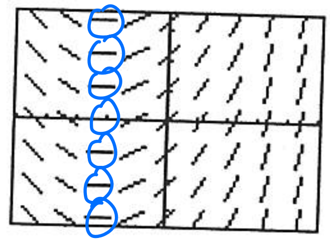
Match the slope fields with their differential equations.

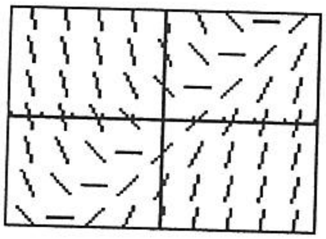
(A)



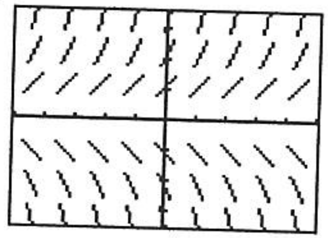
(B)



(C)



(D)



15. $\frac{dy}{dx} = \frac{1}{2}x + 1$ B

$\frac{dy}{dx} = 0$ $\frac{1}{2}x + 1 = 0$, $\frac{1}{2}x = -1$, $x = -2$

16. $\frac{dy}{dx} = y$ D

$\frac{dy}{dx} = 0$, $y = 0$

17. $\frac{dy}{dx} = x - y$ C

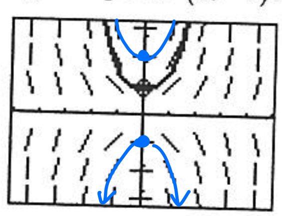
$\frac{dy}{dx} = 0$ $x - y = 0$, $x = y$

18. $\frac{dy}{dx} = -\frac{x}{y}$ A

$\frac{dy}{dx} = 0$, $x = 0, y \neq 0$

$\frac{dy}{dx}$ und $y = 0, x \neq 0$

19. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.
(a) Sketch the solution curve through the point (0, 2).
(b) Sketch the solution curve through the point $(0, -1)$.



20. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.
(a) Sketch the solution curve through the point (0, 1).
(b) Sketch the solution curve through the point $(-3, 0)$.