Name: _____ Date: _____

AP Calculus AB Ms. Loughran

Do Now:

Evaluate each of the following.

1. $\lim_{x\to 9}\frac{3-\sqrt{x}}{9-x}$ $= \frac{1}{6}$

$(3-\sqrt{x})(3+\sqrt{x})$

2. $\lim_{x\to 25}\frac{5-\sqrt{x}}{x-25}$ $= -\frac{1}{10}$

$(\sqrt{x}+5)(\sqrt{x}-5)$

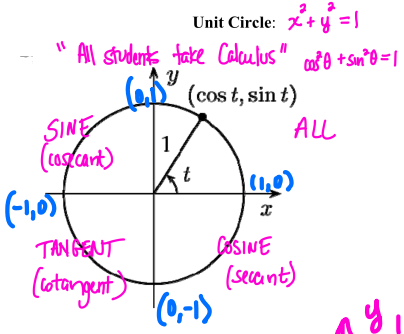
3. $\lim_{x\to 2^-}\frac{3}{x-2}$ $= -\infty$

1.9	2
$-\infty$	VA

4. $\lim_{x\to 0}\frac{1-\cos(4x)}{x}$

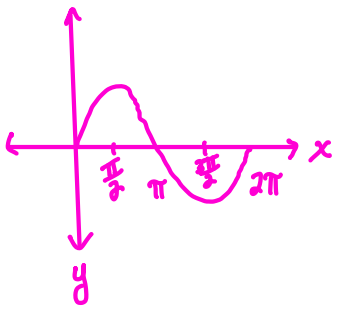
5. $\lim_{x\to -\infty}\frac{\sqrt{3x^2+4}}{x+1}$ $= -\sqrt{3}$

Continuing from yesterday:

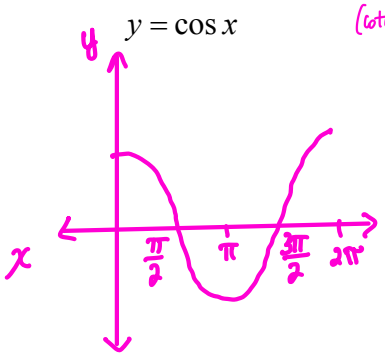


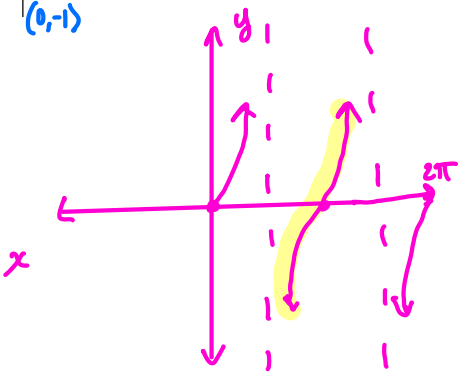
Graphs

$y = \sin x$



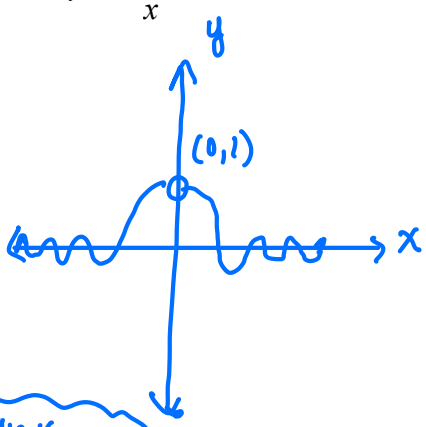
$y = \cos x$





More Graphs

$y = \frac{\sin x}{x}$

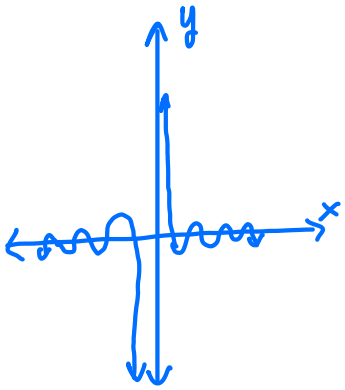


$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to \infty} \frac{\sin x}{x} = 0$

$\lim_{x\to -\infty} \frac{\sin x}{x} = 0$

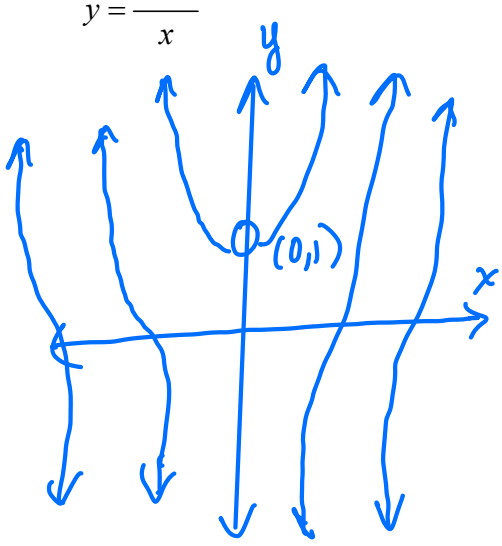
$y = \frac{\cos x}{x}$



$\lim_{x\to 0} \frac{\cos x}{x} = \text{dne}$

$\lim_{x\to \pm\infty} \frac{\cos x}{x} = 0$

$y = \frac{\tan x}{x}$



$\lim_{x\to 0} \frac{\tan x}{x} = 1$

$\lim_{x\to \pm\infty} \frac{\tan x}{x}$ dne

Summary of Trig Limits:

$\lim_{x\to 0} \frac{\sin kx}{x} = k$

$\lim_{x\to 0} \frac{\sin kx}{kx} = 1$

$\lim_{x\to 0} \frac{\sin kx}{\sin mx} = \frac{k}{m}$

Remember that the above would also apply when using tan instead of sin.

$\lim_{x\to 0} \frac{\cos kx}{x} = \text{dne}$

When "splitting" trig limits, never:

① $0 \cdot \pm\infty$

② dne · dne

* ③ $0 \cdot$ dne

If we split a limit to

dne · a limit that does exist ⇒ dne

* if $0 \cdot$ dne $= 0$

↑ bounded

ex. $\lim_{x\to\infty} \frac{\sin x}{x} = 0$

$\lim_{x\to\infty} \frac{1}{x} \cdot \sin x$

$\lim_{x\to\infty} \frac{1}{x} \cdot \lim_{x\to\infty} \sin x = 0$

$0 \cdot$ dne

↑ bounded b/w -1 and 1

Do Now from yesterday

2. Evaluate: $\lim_{x\to\infty} \frac{x\sin x + 2\sin x}{2x^2}$

$\lim_{x\to\infty} \left(\frac{x\sin x}{2x^2} + \frac{2\sin x}{2x^2} \right)$

$\lim_{x\to\infty} \frac{x\sin x}{2x^2} + \lim_{x\to\infty} \frac{2\sin x}{2x^2}$

$\lim_{x\to\infty} \frac{\sin x}{2x} + \lim_{x\to\infty} \left(\frac{\sin x}{x} \cdot \frac{1}{x} \right)$

$\frac{1}{2} \cdot \frac{\sin x}{x}$ $\quad 0 \cdot 0$

$0 + 0 = 0$

Do Now from today

4. $\lim_{x\to 0} \left(\frac{1-\cos(4x)}{x} \right) \cdot \frac{(1+\cos(4x))}{(1+\cos(4x))}$ $\quad 1-\cos^2(4x)$

$\lim_{x\to 0} \frac{\sin^2(4x)}{x(1+\cos(4x))}$

$\lim_{x\to 0} \left(\frac{\sin(4x)}{x} \cdot \frac{\sin(4x)}{1+\cos(4x)} \right)$ $\quad 0$

$4 \cdot 0 = 0$

Homework 09-09

AP Calc: Limits involving Trig

Evaluate each of the following.

1 1. $\lim_{x\to\frac{\pi}{4}} \sin 2x$

dne 2. $\lim_{x\to\frac{\pi}{2}} \tan x$

3 3. $\lim_{x\to 0} \frac{\sin 3x}{x}$

$\frac{5}{3}$ 4. $\lim_{x\to 0} \frac{\tan 5x}{3x}$

1 5. $\lim_{x\to 0} \frac{\sin x \cos x}{x}$

0 6. $\lim_{x\to 0} \frac{\sin^2 x}{x}$

∞ 7. $\lim_{x\to 0} \frac{\cos x}{x^2}$

dne 8. $\lim_{x\to\infty} \frac{\tan x}{x}$

4 9. $\lim_{x\to 0} \frac{3\sin 4x}{\sin 3x}$

-1 10. $\lim_{x\to 0} \frac{\sin x}{2x^2 - x}$

2 11. $\lim_{x\to 0} \frac{x+\sin x}{x} = \lim_{x\to 0}\left(\frac{x}{x} + \frac{\sin x}{x}\right)$

$1+1=2$

1 12. $\lim_{x\to\infty} x(\sin\frac{1}{x})$

0 13. $\lim_{x\to 0} x(\sin\frac{1}{x})$

0 14. $\lim_{x\to 0} \frac{1-\cos x}{x}$

dne 15. $\lim_{x\to 0} \frac{\sin x}{x^2}$

∞ 16. $\lim_{x\to 0} \frac{\sin x}{x^3}$

$\frac{1}{2}$ 17. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

0 18. $\lim_{x\to 0} \frac{x}{\cos x} = \frac{0}{1} = 0$

∞ 19. $\lim_{x\to\infty} x^2 \sin(\frac{1}{x})$

$\frac{1}{2}$ 20. $\lim_{x\to 0} \frac{\sin x}{x(x+2)} = \lim_{x\to 0}\left(\frac{\sin x}{x} \cdot \frac{1}{x+2}\right)$

$1 \cdot \frac{1}{2} = \frac{1}{2}$

5 21. $\lim_{x\to\infty} \frac{5x+\sin x}{x} = \lim_{x\to\infty}\left(\frac{5x}{x} + \frac{\sin x}{x}\right)$

$5+0=5$