

Name: _____

Date: _____

AP Calculus AB

Ms. Loughran

Do Now:

1. Given: $g(t) = \begin{cases} t^2 & , t \geq 0 \\ t-2 & , t < 0 \end{cases}$

Find: (a) $\lim_{t \rightarrow 0^-} g(t) = -2$

(b) $\lim_{t \rightarrow 0^+} g(t) = 0^2 = 0$

(c) $\lim_{t \rightarrow 0} g(t)$ dne

* 2. Evaluate: $\lim_{x \rightarrow \infty} \frac{x \sin x + 2 \sin x}{2x^2}$

3. Evaluate: $\lim_{x \rightarrow +\infty} \frac{1}{x} = 0$

4. Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\sqrt{x+4}-2}{x} \right) \left(\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} \right) = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+4}+2)} = \frac{1}{4}$

5. Evaluate: $\lim_{x \rightarrow -\infty} \frac{\sqrt{5x^2-2}}{x+3} = -\frac{\sqrt{5}}{1} = -\sqrt{5}$

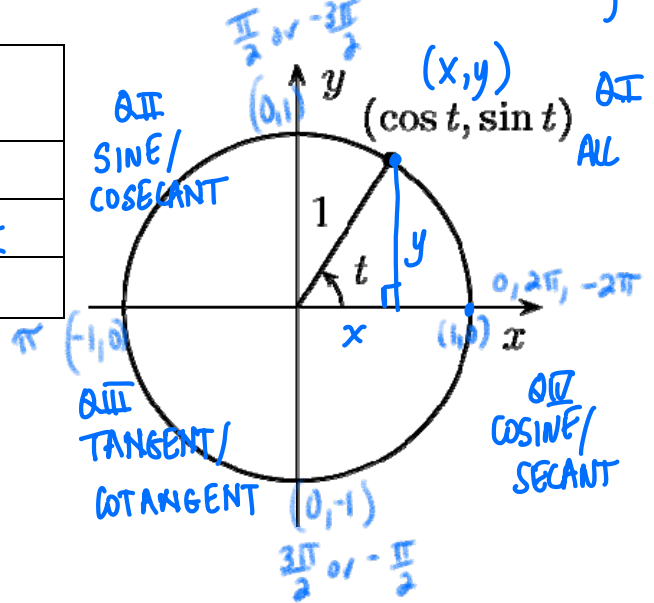
* check the sign when approaching $-\infty$ and there is a radical involved

Name: _____
 AP Calc: Important Trig Stuff

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"ALL STUDENTS TAKE CALCULUS"
 Unit Circle: $x^2 + y^2 = 1$

θ	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin \theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan \theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$



Quotient Identities:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \cos \theta \neq 0$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}, \sin \theta \neq 0$$

Reciprocal Trig Functions:

$$\csc \theta = \frac{1}{\sin \theta}, \sin \theta \neq 0$$

$$\sec \theta = \frac{1}{\cos \theta}, \cos \theta \neq 0$$

$$\cot \theta = \frac{1}{\tan \theta}, \tan \theta \neq 0$$

Pythagorean Identities:

$$* \cos^2 \theta + \sin^2 \theta = 1 *$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

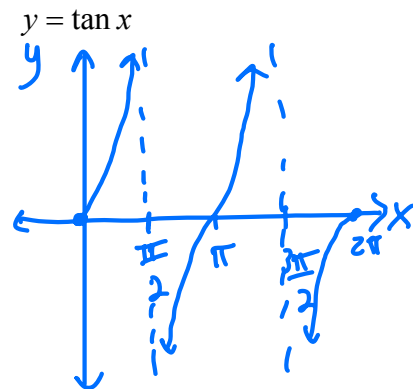
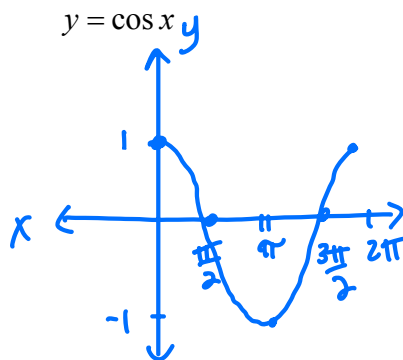
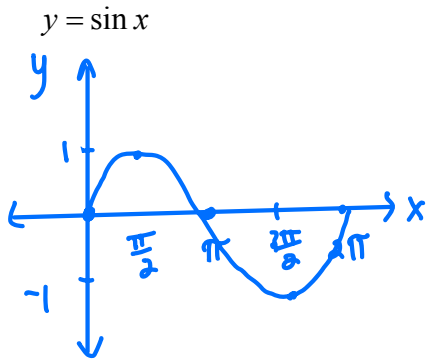
$$\cot^2 \theta + 1 = \csc^2 \theta$$

Double Angle Formulas:

$$* \sin 2\theta = 2 \sin \theta \cos \theta *$$

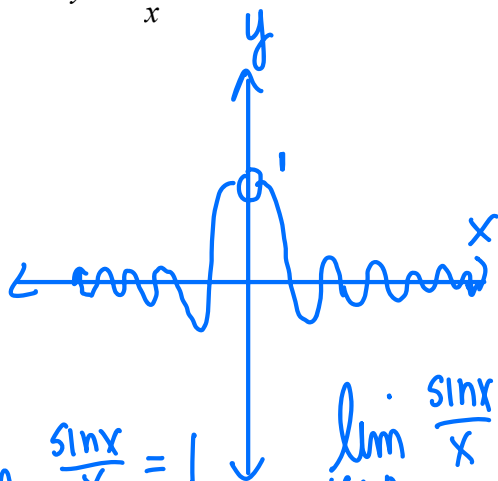
$$\cos 2\theta = \begin{cases} 2\cos^2 \theta - 1 \\ 1 - 2\sin^2 \theta \\ \cos^2 \theta - \sin^2 \theta \end{cases}$$

Graphs one cycle of each

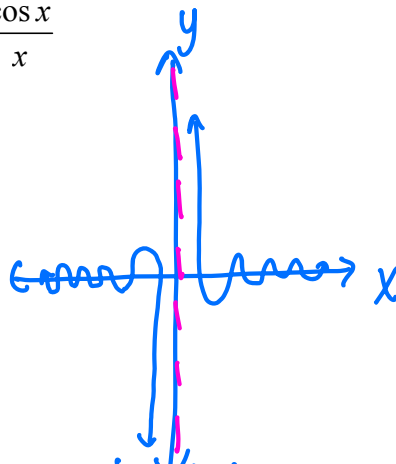


More Graphs

$y = \frac{\sin x}{x}$



$y = \frac{\cos x}{x}$



$y = \frac{\tan x}{x}$

$$\lim_{x \rightarrow 0^-} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow \pm\infty} \frac{\sin x}{x} = 0$$

$$\lim_{x \rightarrow 0^-} \frac{\cos x}{x} = -\infty$$

$$\lim_{x \rightarrow 0^+} \frac{\cos x}{x} = \infty$$

$$\lim_{x \rightarrow 0} \frac{\cos x}{x} = \text{dne}$$

$$\lim_{x \rightarrow \pm\infty} \frac{\cos x}{x} = 0$$

Summary of Trig Limits:

$$\lim_{x \rightarrow 0} \frac{\sin kx}{x} = k$$

$$\lim_{x \rightarrow 0} \frac{\sin kx}{kx} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin kx}{\sin mx} = \frac{k}{m}$$

Remember that the above would also apply when using **tan** instead of sin.

$$\lim_{x \rightarrow 0} \frac{\cos kx}{x} = \text{dne}$$

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{\tan 2x} = \frac{3}{2}$$

Homework 09-08

Homework 09-06

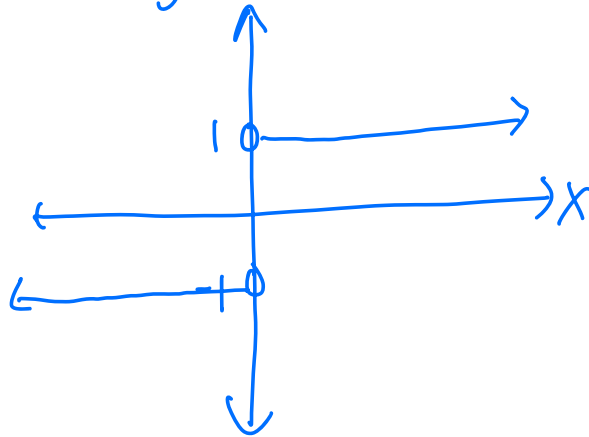
Name _____

Limits Review B

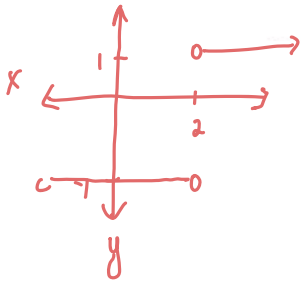
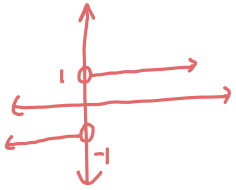
Directions: Find the limit (if it exists).

- 17 1. $\lim_{x \rightarrow 2} (3x^2 + 5)$
- 0 3. $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^2 + 1} = 0$
plug in
- 12 5. $\lim_{x \rightarrow -9} \frac{x^2 + 6x - 27}{x + 9}$
- $\frac{1}{4}$ 7. $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4}$
- $\frac{1}{7}$ 9. $\lim_{x \rightarrow 3} \frac{x + 2}{x^3 + 8}$
- 7 11. $\lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{x - 5}$
- $\frac{1}{4}$ 13. $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$ * Do Now #4
- $\frac{1}{6}$ 15. $\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x}$
- 0 17. $\lim_{x \rightarrow 3} \sqrt{9 - x^2}$
19. $\lim_{x \rightarrow \infty} \frac{2x}{(x+2)^2} = 0$
 * When approaching $\pm \infty$ if the degree of the numerator < the degree of the den. the limit = 0
21. $\lim_{x \rightarrow \infty} \frac{4+x}{3} = +\infty$
 * When approaching $\pm \infty$ if the degree of num > the degree of den, then the limit $\pm \infty$, you have to plug in to figure out if it is + or -
23. $\lim_{x \rightarrow \infty} \frac{3x+2}{x+1} = 3$
 * When approaching $\pm \infty$ if the degree of num = the degree of den, then the limit is the ratio of the leading coefficients
- 0 25. $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x^2-9}}$
- 3 2. $\lim_{x \rightarrow 4} \frac{x^2 - 5x + 4}{x - 4}$
- 1 4. $\lim_{x \rightarrow 1} \frac{x^2 - x - 2}{x - 3}$ *plug in*
- 2 6. $\lim_{x \rightarrow 1} \frac{1 - \sqrt{2x^2 - 1}}{x - 1} \cdot \frac{(1 + \sqrt{2x^2 - 1})}{(1 + \sqrt{2x^2 - 1})}$
 $\lim_{x \rightarrow 1} \frac{1 - 2x^2 + 1}{(x-1)(1 + \sqrt{2x^2 - 1})} = \frac{-4}{2} = -2$
- dne 8. $\lim_{x \rightarrow 2} \frac{x-2}{|x-2|}$
 $\frac{1.9}{|-1.9|} = \frac{2.1}{|2.1|} = 1$
- 3 10. $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1}$
- 1 12. $\lim_{t \rightarrow \infty} \frac{1}{2t} - \frac{t}{t+1} = \lim_{t \rightarrow \infty} \frac{1}{2t} - \lim_{t \rightarrow \infty} \frac{t}{t+1} = 0 - 1 = -1$
- 1 14. $\lim_{x \rightarrow \infty} \frac{3x+5}{4-3x} = \frac{3}{-3} = -1$
- $\frac{1}{3}$ 16. $\lim_{x \rightarrow \infty} \frac{x^2 + 2x}{3x^2 - 6x + 1}$
- 12 18. $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$
20. $\lim_{x \rightarrow 2^+} \frac{x}{\sqrt{x^2 - 4}}$
 $\frac{2}{\sqrt{0^+}} = \frac{2}{0^+} = +\infty$
 if $\lim_{x \rightarrow 2^-} \frac{x}{\sqrt{x^2 - 4}} = \frac{1.9}{\sqrt{0^-}} = \frac{1.9}{0^-} = -\infty$ dne
22. $\lim_{x \rightarrow 3} \frac{x-3}{x^3 - 27} = \frac{1}{27}$
 $(x-3)(x^2+3x+9)$
24. $\lim_{x \rightarrow \infty} \frac{4x^3 - 5x^2 - 1}{(x+2)^2}$

$$f(x) = \frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1 & x > 0 \\ \frac{-x}{x} = -1 & x < 0 \end{cases}$$



$$f(x) = \frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1 & x > 0 \\ -\frac{x}{x} = -1 & x < 0 \end{cases}$$



$$8. \lim_{x \rightarrow 2} \frac{x-2}{|x-2|} \text{ dne}$$

$$8. \lim_{x \rightarrow 2} \frac{x-2}{|x-2|} \text{ dne}$$

table method

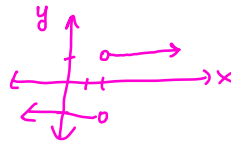
1.9	2	2.1
-1	↑	1

we

$\frac{x}{|x|}$

↑
transformation

$$\frac{x-2}{|x-2|} = \begin{cases} \frac{x-2}{x-2} = 1 & x-2 > 0, x > 2 \\ -\frac{x-2}{x-2} = -1 & x < 2 \end{cases}$$



$$2 - 2x^2$$

$$1 - 2x^2 + 1$$

$$1 - (2x^2 - 1)$$

$$6. \lim_{x \rightarrow 1} \frac{1 - \sqrt{2x^2 - 1}}{x - 1} \cdot \frac{(1 + \sqrt{2x^2 - 1})}{(1 + \sqrt{2x^2 - 1})}$$

$$(x-1)(1 + \sqrt{2x^2 - 1})$$

$$\lim_{x \rightarrow 1} \frac{2(1-x)(1+x)}{(x-1)(1 + \sqrt{2x^2 - 1})} = \frac{-4}{2} = -2$$