Do Now: #s 9 and 32 from the last page of the Limits Practice packet (Introductory Limits Problems)

9) $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

$(\cancel{x}+h-\cancel{x})(x+h+x)$

$\lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$

32) $\lim_{x \to 1} \left(\frac{(x-1)}{(\sqrt{x^2+3} - 2)} \cdot \frac{(\sqrt{x^2+3}+2)}{(\sqrt{x^2+3})+2} \right)$

$x^2 + 3 - 4$

$x^2 - 1$

$\lim_{x \to 1} \frac{\cancel{(x-1)}(\sqrt{x^2+3}+2)}{(x+1)\cancel{(x-1)}} = \frac{4}{2} = 2$

Name: ______________________________ Date: ____________

AP Calculus: Continuity Ms. Loughran

Intuitive definition:

A function is continuous if it can be drawn without lifting the pencil from the paper.

Mathematical definition of continuity at a point:

A function is continuous at $x = a$, if and only if:

1) $f(a)$ exists
2) $\lim_{x \to a} f(x)$ exists (Remember this means $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$)
3) $\lim_{x \to a} f(x) = f(a)$

1. For what value(s) of x is each of the following functions discontinuous?

   (a) $f(x) = \dfrac{x}{x+1}$

   @ $x = -1$

   (b) $f(x) = \dfrac{x+1}{x^2 - 4x + 3}$

   $(x-3)(x-1)$

   @ $x = 3, 1$

2. Given: $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$, $x > 1$ or $x < 1$

   $f(1) = 2$

   Is the function continuous at $x = 1$?

   $\lim_{x \to 1} \dfrac{\overbrace{x^2 - 1}^{(\cancel{x-1})(x+1)}}{\cancel{x-1}} = \lim_{x \to 1} x+1 = 2$

   $f(1) = \lim_{x \to 1} f(x)$ so yes

3. For $x \neq 2$, $f(x) = \dfrac{x^2 + 3x - 10}{x - 2}$. What value should be assigned to $f(2)$ to make $f(x)$ continuous at $x = 2$?

   $\lim_{x \to 2} \dfrac{\overbrace{x^2 + 3x - 10}^{(x+5)(\cancel{x-2})}}{\cancel{x-2}} = \lim_{x \to 2} x+5 = 7$

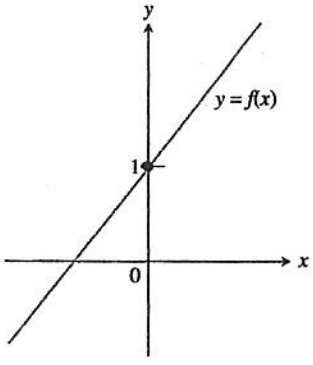
   $f(2) = \lim_{x \to 2} f(x)$

   $f(2) = 7$

Name: ______________________________ Date: ________

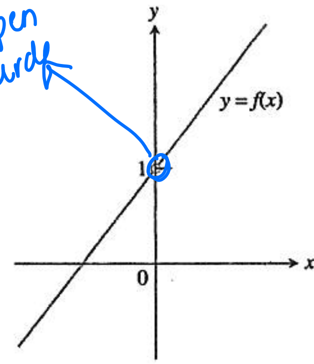
AP Calculus AB Continuity

Decide whether or not each graph is continuous at x = 0.



(a)

yes

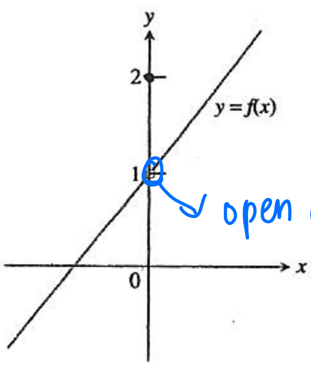


(b)

open circle

not continuous
@ $x = 0$

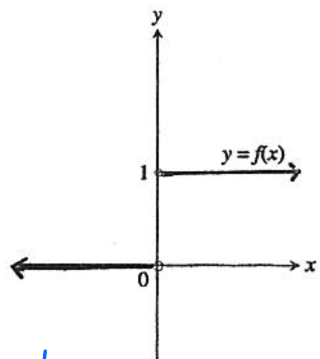
removeable
discontinuity



(c)

not continuous
at $x = 0$

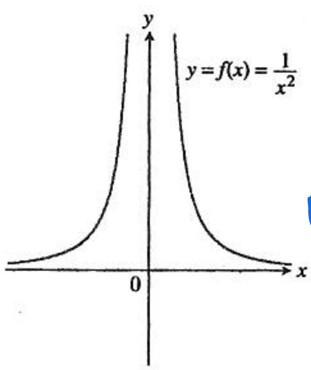
open circle we could make it continuous by redefining $f(0)$ letting $f(0) = 1$ instead of $f(0) = 2$



(d)

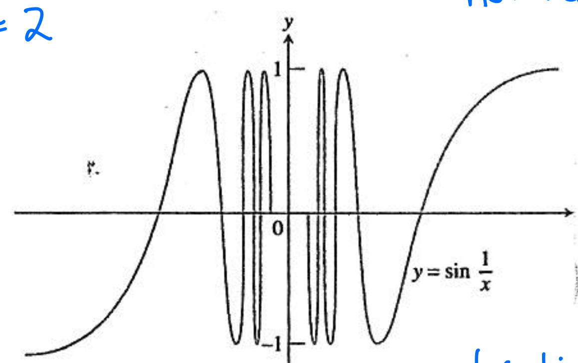
not
continuous
at $x = 0$

jump
discontinuity
not removeable



(e)

not
continuous
at $x = 0$
infinite discontinuity
not removeable



(f)

not continuous
at $x = 0$

oscillating discontinuity
not removeable

4. For each of the following, find a value for the constant *k*, that will make the function continuous.

a) $f(x)=\begin{cases} 7x-2 & \text{if } x\le 1 \\ kx^2 & \text{if } x>1 \end{cases}$ $f(1)=5$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$\lim_{x\to 1^-} 7x-2 = \lim_{x\to 1^+} kx^2$

$7(1)-2 = k(1)^2$

$5 = k$

b) $f(x)=\begin{cases} x-k & \text{if } x\le 2 \\ 2x-3 & \text{if } x>2 \end{cases}$ more formally $\lim_{x\to 2^-} x-k = \lim_{x\to 2^+} 2x-3$

$x-k = 2x-3$ @ $x=2$

$2-k = 2(2)-3$

$2-k = 1$

$-k = -1$

$k = 1$

c) $f(x)=\begin{cases} \dfrac{\sin x}{x} & \text{if } x\ne 0 \\ k & \text{if } x=0 \end{cases}$ $\cancel{\frac{\sin x}{x} = k \text{ @ } x=0}$

$\lim_{x\to 0} \frac{\sin x}{x} = k$

$1 = k$

d) $f(x)=\begin{cases} \dfrac{4-\sqrt{x}}{x-16} & \text{if } x\ne 16 \\ k & \text{if } x=16 \end{cases}$ $f(16)=k$

$\lim_{x\to 16} \frac{\cancel{4-\sqrt{x}}}{x-16} \quad \frac{}{(\cancel{\sqrt{x}-4})(\sqrt{x}+4)} = \lim_{x\to 16} \frac{-1}{\sqrt{x}+4} = \frac{-1}{8}$

$k = -\frac{1}{8}$

transformation of $\frac{1}{x}$

moves it right one

5. Is $f(x)=\dfrac{1}{x-1}$ continuous at $x=1$? $x=2$?

continuous at $x=1$? $f(1)$ is undefined

continuous at $x=2$? $f(2)=1$

$$\left.\begin{array}{l} f(2)=1 \\ \lim\limits_{x\to 2} \dfrac{1}{x-1}=1 \end{array}\right\} = \text{yes}$$

6. Given the function $f(x)=|x-3|$. Prove that f is continuous at $x=3$. Graph $f(x)$.

transformation of $|x|$, moves it right 3

$$\left.\begin{array}{l} f(3)=0 \\ \lim\limits_{x\to 3} f(x)=0 \end{array}\right\} = \text{continuous}$$