Name:____ Date:_______

AP Calculus: Limits of Difference Quotients

Do Now:

1. $\lim\limits_{x\to 0}\dfrac{3-3\cos x}{x}$
2. $\lim\limits_{x\to 0}\dfrac{\cos x\tan x}{x}$
3. $\lim\limits_{x\to \frac{1}{2}} x\sec \pi x$

We did these in the Do Now on 09-15, so instead do the question below

4. For $x \neq 2$, $f(x)=\dfrac{x^2+3x-10}{x-2}$. What value should be assigned to $f(2)$ to make $f(x)$ continuous at $x=2$?

5. For each of the following, find a value for the constant *k*, that will make the function continuous.

(a) $f(x)=\begin{cases}7x-2, & x\le 1\\ kx^2, & x>1\end{cases}$

(b) $f(x)=\begin{cases}x-k, & x\le 2\\ 2x-3, & x>2\end{cases}$

(c) $f(x)=\begin{cases}\dfrac{\sin x}{x}, & x\neq 0\\ k, & x=0\end{cases}$

(d) $f(x)=\begin{cases}\dfrac{4-\sqrt{x}}{x-16}, & x\neq 16\\ k, & x=16\end{cases}$

Find $\dfrac{f(x+h)-f(x)}{h}$ for $f(x)=3x^2$

$$\frac{3(x+h)^2-3x^2}{h}=\frac{3(x^2+2xh+h)^2-3x^2}{h}=\frac{\cancel{3x^2}+6xh+3h^2-\cancel{3x^2}}{h}$$

$$=\frac{\cancel{h}(6x+3h)}{\cancel{h}}$$

Classwork

For each of the following functions, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

1. $f(x) = 3x^2$

$$\lim_{h\to 0} 6x + 3h = 6x$$

4. $f(x) = 1 - x - x^2$

$$\lim_{h\to 0} \frac{1-(x+h)-(x+h)^2-(1-x-x^2)}{h}$$

$$\lim_{h\to 0} \frac{1-x-h-(x^2+2xh+h^2)-1+x+x^2}{h}$$

$$\lim_{h\to 0} \frac{\cancel{1}-\cancel{x}-h-\cancel{x^2}-2xh-h^2\cancel{-1}+\cancel{x}+\cancel{x^2}}{h}$$

$$\lim_{h\to 0} \frac{-h-2xh-h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(-1-2x-h)}{\cancel{h}} = -1-2x$$

5. $f(x) = \sqrt{x}$

$$\lim_{h\to 0}\left(\frac{(\sqrt{x+h}-\sqrt{x})}{h}\cdot\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}\right)$$

$$\lim_{h\to 0} \frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}} = \frac{\sqrt{x}}{2x}$$

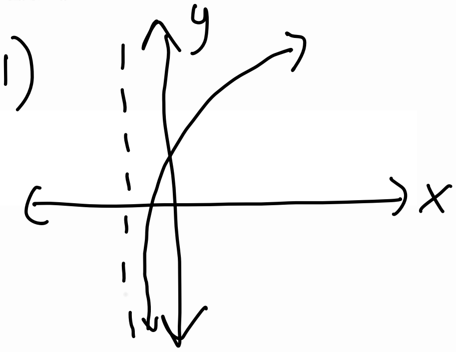
2. $f(x) = \frac{2}{x}$

$$\lim_{h\to 0}\left(\frac{\frac{2}{\cancel{x+h}}^{\,x\cancel{(x+h)}} - \frac{2}{\cancel{x}}^{\,\cancel{x}(x+h)}}{h\,x(x+h)}\right)$$

$$\lim_{h\to 0}\frac{\cancel{2x}-\cancel{2x}-2h}{xh(x+h)} = \lim_{h\to 0}\frac{-2\cancel{h}}{x\cancel{h}(x+h)}$$

$$\lim_{h\to 0}\frac{-2}{x(x+h)} = \frac{-2}{x^2}$$

$y = \ln(x+1)$



Homework 09-18

AP Calc: Continuity

1. For what value(s) of x is each of the following functions discontinuous?

(a) $f(x) = \dfrac{1}{(x+2)^2}$ at $x = -2$

(b) $g(x) = \dfrac{x+1}{x^2 - 4x + 3}$

at $x = 1$
and $x = 3$

(c) $h(x) = \sqrt[3]{2x-1}$ none

(d) $f(x) = \ln(x+1)$ none

For each of the following, find a value for the constant k, that will make the function continuous.

2. $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2kx, & x \geq 3 \end{cases}$ $k = \frac{4}{3}$

3. $g(x) = \begin{cases} 4 - x^2, & x < -1 \\ kx^2 - 1, & x \geq -1 \end{cases}$ $k = 4$

4. $h(x) = \begin{cases} \dfrac{\sin x}{2x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ $k = \frac{1}{2}$

5. $f(x) = \begin{cases} \dfrac{5 - \sqrt{x}}{x - 25}, & x \neq 25 \\ k, & x = 25 \end{cases}$ $k = -\frac{1}{10}$

6. Given $g(x) = \begin{cases} 1, & x \leq -1 \\ -x, & -1 < x < 0 \\ 1, & x = 0 \\ -x, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$ Is g continuous at x = -1? 0? 1? Explain.

$g(-1) = 1$
$\lim_{x \to -1^-} g(x) = 1$ $\lim_{x \to -1^+} g(x) = 1$
at $x = -1$?
continuous because $\lim_{x \to -1} g(x) = g(-1)$

at $x = 0$?
not continuous because $\lim_{x \to 0} g(x) \neq g(0)$

$g(1) = 1$
$\lim_{x \to 1^-} g(x) = -1$ $\lim_{x \to 1^+} g(x) = 1$
at $x = 1$?
not continuous because $\lim_{x \to 1} g(x)$ DNE

In exercises 7 and 8, use the graph of the function with domain $-1 \le x \le 3$.

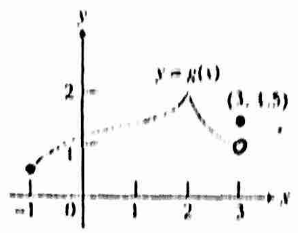
7. Determine

(a) $\lim_{x \to 3^-} g(x)$. = 1 (b) $g(3)$. = 1.5

(c) whether $g(x)$ is continuous at $x = 3$. no because $\lim_{x \to 3^-} g(x) \neq g(3)$

(d) the points of discontinuity of $g(x)$.

x = 3



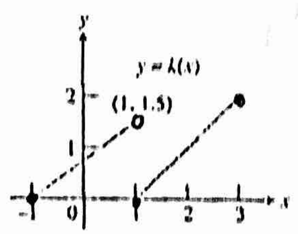
8. Determine

1.5 → (a) $\lim_{x \to 1^-} k(x)$. (b) $\lim_{x \to 1^+} k(x)$. = 0 (c) $k(1)$. = 0

(d) whether $k(x)$ is continuous at $x = 1$. no because $\lim_{x \to 1} k(x)$ ONE

(e) the points of discontinuity of $k(x)$.

x = 1



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(d) $f(x) = \begin{cases} \frac{4-\sqrt{x}}{x-16}, & x \neq 16 \\ k, & x = 16 \end{cases}$

Classwork

For each of the following functions, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

1. $f(x) = 3x^2$
2. $f(x) = \frac{2}{x}$
3. $f(x) = \frac{x}{x+3}$
4. $f(x) = 1 - x - x^2$
5. $f(x) = \sqrt{x}$
6. $f(x) = 4 - 2x^2 + 5x$
7. $f(x) = \frac{1}{2x+1}$
8. $f(x) = \sqrt{2x}$