Do Now:

Evaluate: ① $\lim_{x\to 5^-} \frac{3}{x-5}$ ② $\lim_{x\to 0} \frac{1-\cos(4x)}{x}$ ③ $\lim_{x\to -4} \frac{x^3+64}{x+4}$ ④ $\lim_{x\to 10^+} \frac{|x-10|}{10-x}$ ⑤ $\lim_{x\to 4} \frac{\sqrt{x-3}-1}{x-4}$ ⑥ $\lim_{x\to -\infty} \frac{\sqrt{3x^2+4}}{x+1}$ ⑦ $\lim_{x\to 0} \frac{(x+3)^{-1}-3^{-1}}{x}$ ⑧ $\lim_{x\to \infty} \frac{-2x-x^4}{-x^2+5x-1}$ ⑨ $\lim_{x\to -\infty} x\left(1-\cos\left(\frac{1}{x}\right)\right)$ ⑩ $\lim_{x\to 4} \frac{\sin(x-4)}{x^2-x-12}$

⑪ Find the constants a and b such that the function is continuous on the entire real line.

a) $f(x)=\begin{cases} \frac{4\sin x}{x} & x<0 \\ a-2x & x\geq 0 \end{cases}$

b) $g(x)=\begin{cases} 2 & x\leq -1 \\ ax+b & -1<x<3 \\ -2 & x\geq 3 \end{cases}$

$\lim_{x\to 0^-} \frac{4\sin x}{x} = \lim_{x\to 0^+} a-2x$

$4 = a-2(0)$

$4 = a$

$\lim_{x\to -1^-} 2 = \lim_{x\to -1^+} ax+b$

$2 = a(-1)+b$

$2 = -a+b$ $\quad b=a+2$

$\lim_{x\to 3^-} ax+b = \lim_{x\to 3^+} -2$

$3a+b=-2$

$3a+a+2=-2$

$4a+2=-2$

$4a=-4$ $\quad a=-1$

$b=a+2$

$b=-1+2$

$b=1$

① $\lim_{x \to 5^-} \frac{3}{x-5}$

4.9	5
$\pm$	VA
$-\infty$	

② $\lim_{x \to 0} \left(\frac{(1-\cos(4x))}{x} \cdot \frac{(1+\cos(4x))}{(1+\cos(4x))} \right)$ $\quad 1-\cos^2(4x)$

$\lim_{x \to 0} \left(\frac{\sin^2(4x)}{x(1+\cos(4x))} \right)$

$4 \cdot 0$

$\lim_{x \to 0} \left(\frac{\sin(4x)}{x} \cdot \frac{\sin(4x)}{1+\cos(4x)} \right) = 0$ $\quad \sin(4x) \to 0$, $\cos(4x) \to 1$

③ $\lim_{x \to -4} \frac{x^3+64}{x+4}$ $\quad (x+4)(x^2-4x+16)$

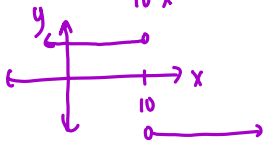
$= 16 - 4(-4) + 16$

$= 48$

④ $\lim_{x \to 10^+} \frac{|x-10|}{10-x} = -1$ $\quad .1$, $-.1$

10	10.1
holes	

$$\frac{|x-10|}{10-x} = \begin{cases} \frac{x-10}{10-x} = -1 & x-10 > 0,\ x > 10 \\ -\frac{(x-10)}{10-x} = 1 & x < 10 \end{cases}$$



⑤ $\lim_{x\to 4}\left(\frac{(\sqrt{x-3}-1)}{x-4}\cdot\frac{(\sqrt{x-3}+1)}{(\sqrt{x-3}+1)}\right)$ $x-3-1$

$\lim_{x\to 4}\frac{x-4}{(x-4)(\sqrt{x-3}+1)} = \frac{1}{2}$

⑥ $\lim_{x\to -\infty}\frac{\sqrt{3x^2+4}}{x+1} = -\sqrt{3}$

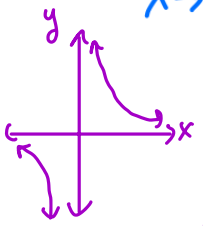
⑦ $\lim_{x\to 0}\frac{(x+3)^{-1}-3^{-1}}{x}$

$\lim_{x\to 0}\left(\frac{\frac{1}{x+3}-\frac{1}{3}}{x}\right)$ $\frac{3(x+3)}{3(x+3)}$ $\frac{3(x+3)}{3(x+3)}$

$\lim_{x\to 0}\frac{3-(x+3)}{3x(x+3)} = \lim_{x\to 0}\frac{-x}{3x(x+3)} = \frac{-1}{9}$

⑧ $\lim_{x\to\infty}\frac{-2x-x^4}{-x^2+5x-1} = \infty$

(9) $\lim_{x \to -\infty} x\left(1-\cos\left(\frac{1}{x}\right)\right)$



$u = \frac{1}{x}$

$\lim_{u \to 0^-} \frac{1}{u}(1-\cos u)$

$\lim_{u \to 0^-} \frac{1-\cos u}{u} = 0$

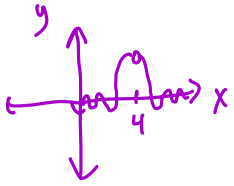
(10) $\lim_{x \to 4} \frac{\sin(x-4)}{x^2-x-12}$

$\frac{\sin(x-4)}{(x-4)(x+3)}$

$1 \cdot \frac{1}{7}$

$\lim_{x \to 4} \left(\frac{\sin(x-4)}{x-4} \cdot \frac{1}{x+3}\right) = \frac{1}{7}$

transformation of $y = \frac{\sin x}{x}$ shifted right 4



1	1	2	-11	8
		1	3	-8
	1	3	-8	0

(13) $\lim_{x \to 1} \frac{x^3+2x^2-11x+8}{x^2-6x+5}$

$\frac{(x-1)(x^2+3x-8)}{(x-1)(x-5)}$

$= \frac{1^2+3(1)-8}{-4} = \frac{-4}{-4} = 1$