Name: ______________________________ Date: ________________

AP Calculus Definition of a Derivative Ms. Loughran

Do Now:

1. What is the slope of the function $f(x) = 2x - 3$ at $(0,-3)$? at $(-1,-5)$? at $(2,1)$?

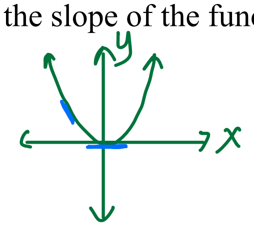
   2 2 2

   linear function

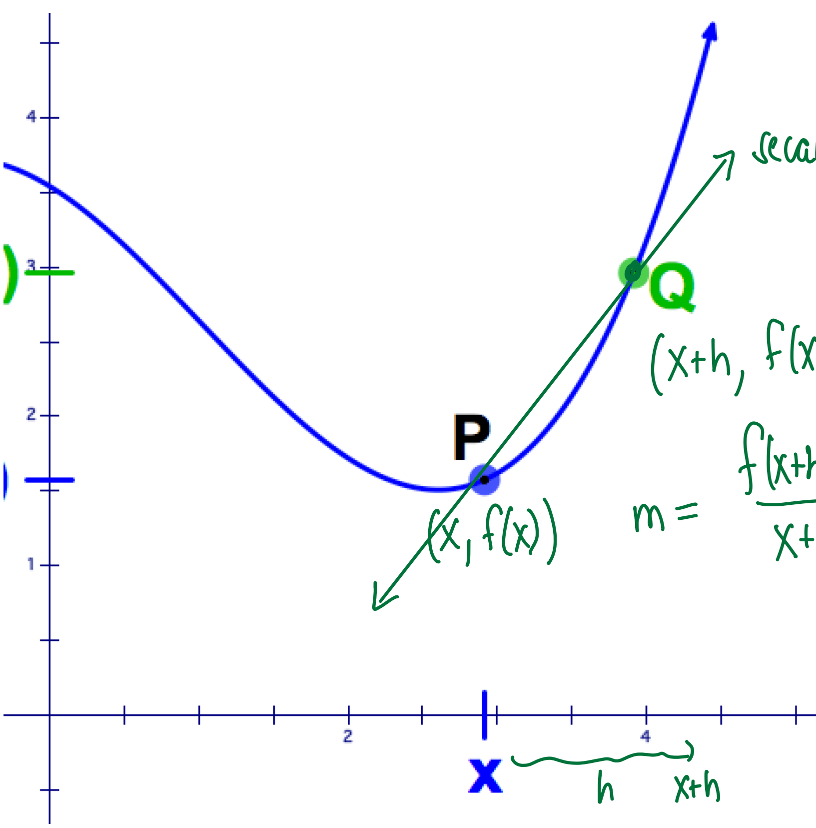
   $f(x) = mx + b$

   slope $= m$

2. What is the slope of the function $f(x) = x^2$ at $(0,0)$? at $(-2,4)$? at $(3,9)$?



Let's investigate a general curve:



difference quotient

$$m = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

the slope of the tangent line at P, it is the slope of f(x) at P - it is the derivative of f(x)

Because my drawing skills are sub sub subpar. I created a little clip.

https://youtu.be/fDLGzk9bPP8

The function $f\,'$ defined by the formula

$$f\,'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

is called the derivative of *f* with respect to *x*.

The derivative of a function *f* can be interpreted either as a function whose value at *x* is the slope of the tangent line to the graph of $y = f(x)$ at *x*, or alternatively, it can be interpreted as a function whose value at *x* is the instantaneous rate of change of *y* with respect to *x* at the point *x*.

The **normal line** to a curve at a point is the line perpendicular to the tangent line at that point.

Let's revisit #2 from the Do Now.

$$f(x) = x^2$$

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

$f'(x)$

slope producing formula

@ $(0,0)$

$$f'(0) = 2(0) = 0$$

Equation of the tangent line:

point slope

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 0(x - 0)$$

$$y = 0$$

$y=0$

@ $(-2,4)$

$$f'(-2) = 2(-2) = -4$$

Equation of the tangent line:

$$y - 4 = -4(x+2)$$

@ $(3,9)$

$$f'(3) = 2(3) = 6$$

Equation of the tangent line:

$$y - 9 = 6(x-3)$$

Equation of the NORMAL line:

$$y - 9 = -\frac{1}{6}(x-3)$$

Homework 09-19

(3) $\frac{3}{(x+3)^2}$

(6) $-4x+5$

(7) $\frac{-2}{(2x+1)^2}$

(8) $\frac{1}{\sqrt{2x}}$ or $\frac{\sqrt{2x}}{2x}$

(1) Given $f(x) = x^2 + 4x + 9$, write an equation of the tangent line to $f(x)$ at $x = 1$.

(1) need pt
(2) need slope of $f(x)$ at $x=1$, $f'(1)$

$f(1) = 1^2 + 4(1) + 9 = 14$, $(1, 14)$

$$\lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) + 9 - (x^2 + 4x + 9)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{4x} + 4h \cancel{+9} - \cancel{x^2} - \cancel{4x} - \cancel{9}}{h} = \lim_{h \to 0} \frac{2xh + h^2 + 4h}{h}$$

$$= \lim_{h \to 0} 2x + h + 4 = 2x + 4 = f'(x)$$

$f'(1) = 2(1)+4 = 6$

$y - 14 = 6(x-1)$

② Given $g(x) = 2x^2 - 3x + 5$. Write the equation of the normal line to $g(x)$ at $x = 1$.

$g(1) = 2(1)^2 - 3(1) + 5 = 4$ $(1,4)$

$$\lim_{h\to 0} \frac{2\overbrace{(x+h)^2}^{(x^2+2xh+h^2)} - 3(x+h) + 5 - 2x^2 + 3x - 5}{h}$$

$$\lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h \cancel{+5} - \cancel{2x^2} \cancel{+3x} \cancel{-5}}{h}$$

$$\lim_{h\to 0} \frac{4xh + 2h^2 - 3h}{h} = \lim_{h\to 0} 4x + 2h - 3 = 4x - 3$$

$g'(1) = 4(1) - 3 = 1$

$y - 4 = -(x-1)$

$m_{\text{normal line}} = -1$

③ Given $f(x) = -x^2 + 2x - 3$, find the equation of the tangent line to $f(x)$ at $x = 2$.

$$f(2) = -(2)^2 + 2(2) - 3 = -3 \qquad (2, -3)$$

$$\lim_{h \to 0} \frac{-(x+h)^2 + 2(x+h) - 3 + x^2 - 2x + 3}{h}$$

$$\lim_{h \to 0} \frac{\cancel{-x^2} - 2xh - h^2 + \cancel{2x} + 2h \cancel{-3} \cancel{+x^2} \cancel{-2x} \cancel{+3}}{h}$$

$$\lim_{h \to 0} \frac{-2xh - h^2 + 2h}{h} = \lim_{h \to 0} -2x - h + 2 = -2x + 2$$

$$f'(2) = -2(2) + 2 = -2$$

$$y + 3 = -2(x - 2)$$