Name:______________________________________ Date:__________

AP Calc AB: More Practice with Derivative Definition Ms. Loughran

The function f' defined by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

is called the derivative of *f* with respect to *x*.

The derivative of a function *f* can be interpreted either as a function whose value at *x* is the slope of the tangent line to the graph of $y = f(x)$ at *x*, or alternatively, it can be interpreted as a function whose value at *x* is the instantaneous rate of change of *y* with respect to *x* at the point *x*.

The **normal line** to a curve at a point is the line perpendicular to the tangent line at that point.

1. Given $f(x) = 2x^2 - 7x + 1$.

(a) Find $f'(x)$. $= 4x-7$

(b) Find $f'(-3)$. $= 4(-3)-7 = -19$

(c) Write an equation of the tangent line of $f(x)$ at $x = -3$.

a) $$f'(x) = \lim_{h \to 0} \frac{2\overbrace{(x+h)^2}^{(x^2+2xh+h^2)} - 7(x+h) + 1 - (2x^2 - 7x + 1)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{7x} - 7h + \cancel{1} - \cancel{2x^2} + \cancel{7x} - \cancel{1}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 7)}{\cancel{h}} = 4x-7$$

c) $f(-3) = 2(-3)^2 - 7(-3) + 1 = 40$ $(-3, 40)$

$y - 40 = -19(x+3)$

2. Given $f(x) = 4x - 3x^2$.

(a) Find $f'(x)$. $4 - 6x$

(b) Write an equation of the tangent line to $f(x)$ when $x = -1$.

(c) Find a value of x when $f(x)$ will have a horizontal tangent line.

c) slope $= 0$

$f'(x) = 0$

$4 - 6x = 0$

$x = \frac{4}{6}$ or $\frac{2}{3}$

a) $f'(x) = \lim_{h \to 0} \dfrac{4(x+h) - 3(x+h)^2 - 4x + 3x^2}{h}$

$\lim_{h \to 0} \dfrac{\cancel{4x} + 4h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{4x} + \cancel{3x^2}}{h} = \lim_{h \to 0} \dfrac{\cancel{h}(4 - 6x - 3h)}{\cancel{h}} = 4 - 6x$

b) $f(-1) = 4(-1) - 3(-1)^2 = -7$ $\quad f'(-1) = 4 - 6(-1) = 10$

$y + 7 = 10(x + 1)$

3. Given $f(x) = 3x^2 - 4$

(a) Find $f'(x)$.

(b) Find $f'(-2)$, $f'(0)$, and $f'(3)$.

(c) Write an equation of the tangent line at $x = -2$.

(d) Write an equation of the normal line at $x = -2$.

a) $f'(x) = \lim_{h \to 0} \dfrac{3(x+h)^2 - 4 - 3x^2 + 4}{h}$

$\lim_{h \to 0} \dfrac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 4} - \cancel{3x^2} \cancel{+ 4}}{h} = \lim_{h \to 0} \dfrac{\cancel{h}(6x + 3h)}{\cancel{h}} = 6x$

b) $f'(-2) = 6(-2) = -12$ $\quad f'(0) = 6(0) = 0$ $\quad f'(3) = 6(3) = 18$

c) $f(-2) = 3(-2)^2 - 4 = 8$ $\quad y - 8 = -12(x + 2)$

d) $y - 8 = \frac{1}{12}(x + 2)$