Name: ______________________________________ Date: __________

AP Calculus AB Ms. Loughran

1. Given $f(x) = x^4 - 5x^3 + 2x^2 + 8x$. Write an equation of the:
   (a) tangent line to the graph of $f(x)$ at $x = 1$.
   (b) normal line to the graph of $f(x)$ at $x = 1$.

a) $f'(x) = 4x^3 - 15x^2 + 4x + 8$

$f'(1) = 4(1)^3 - 15(1) + 4(1) + 8 = 1$

$f(1) = 1^4 - 5(1)^3 + 2(1) + 8 = 6$

$y - 6 = x - 1$

b) $y - 6 = -(x - 1)$

2. Given $g(x) = \dfrac{x+1}{x+2}$, write an equation of the tangent line to $y = g(x)$ at $x = 1$.

$$g'(x) = \lim_{h \to 0} \frac{\frac{(x+h+1)}{(x+h+2)}(x+h+2)(x+2) - \frac{(x+1)}{(x+2)}(x+h+2)(x+2)}{h(x+h+2)(x+2)}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + \cancel{xh} + \cancel{x} + \cancel{2x} + 2h + \cancel{2} - (\cancel{x^2} + \cancel{xh} + \cancel{2x} + \cancel{x} + h + \cancel{2})}{h(x+h+2)(x+2)}$$

$$\lim_{h \to 0} \frac{\cancel{h}^{\,1}}{\cancel{h}(x+h+2)(x+2)} = \frac{1}{(x+2)^2}$$

$g'(1) = \dfrac{1}{(1+2)^2} = \dfrac{1}{9}$

$g(1) = \dfrac{1+1}{1+2} = \dfrac{2}{3}$

$y - \frac{2}{3} = \frac{1}{9}(x - 1)$

or $9y - 6 = x - 1$

$\frac{dy}{dx} = y'$

$\frac{d^2y}{dx^2} = y''$

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PC: Higher Order Derivatives Ms. Loughran

1. Given: $y = 6x^3 - 4x^2 + 2x$. Find $\frac{d^2y}{dx^2}$.

$\frac{dy}{dx} = 18x^2 - 8x + 2$

second derivative

$\frac{d^2y}{dx^2} = 36x - 8$

2. Given: $y = \frac{x^{-4}}{8} + \frac{3x^{-2}}{2}$. Find y". ← second derivative

$y = \frac{1}{8}x^{-4} + \frac{3}{2}x^{-2}$

$y' = -\frac{1}{2}x^{-5} - 3x^{-3}$

$y'' = \frac{5}{2}x^{-6} + 9x^{-4} = \frac{5}{2x^6} + \frac{9}{x^4}$

3. Given: $y = 3x^4 - 2x^2$. Find $\frac{d^2y}{dx^2}$ at $x = 1$.

↖ evaluate the second derivative at $x=1$

$\frac{dy}{dx} = 12x^3 - 4x$

$\frac{d^2y}{dx^2} = 36x^2 - 4$

$\left.\frac{d^2y}{dx_2}\right|_{x=1} = 36(1)^2 - 4 = 32$

Split it up so you can use the power rule

4. Given: $y = \frac{x^2+3}{2x}$. Find $\frac{d^2y}{dx^2}$

$y = \frac{x^{\cancel{2}}}{2\cancel{x}} + \frac{3}{2x}$

$y = \frac{1}{2}x + \frac{3}{2}x^{-1}$

$\frac{dy}{dx} = \frac{1}{2} - \frac{3}{2}x^{-2}$

$\frac{d^2y}{dx^2} = 3x^{-3}$ or $\frac{3}{x^3}$

Find the first four derivatives of the function.

5. $y = x^4 + x^3 - 2x^2 + x - 5$

$y' = 4x^3 + 3x^2 - 4x + 1$

$y'' = 12x^2 + 6x - 4$

$y''' = 24x + 6$

y'''' or $y^{IV} = 24$

6. $y = x^2 + x + 3$

$y' = 2x+1$
$y'' = 2$
$y''' = 0$
$y^{iv} = 0$

7. $y = x^{-1} + x^2$

$y' = -x^{-2} + 2x$
$y'' = 2x^{-3} + 2$
$y''' = -6x^{-4}$
$y^{iv} = 24x^{-5}$ or $\frac{24}{x^5}$

8. $y = \frac{x+1}{x} = \frac{x}{x} + \frac{1}{x} = 1 + x^{-1}$

$y' = -x^{-2}$
$y'' = 2x^{-3}$
$y''' = -6x^{-4}$
$y^{iv} = 24x^{-5}$ or $\frac{24}{x^5}$

Homework 09-27

8. If $f(x) = 7$ for all real numbers x, find

(a) $f(10)$. (b) $f(0)$.

(c) $f(x + h)$. (d) $\lim_{x \to 0} \frac{f(x) - f(a)}{x - a}$.

9. Find the derivatives of these functions with respect to x.

(a) $f(x) = \pi$ (b) $f(x) = \pi^2$ (c) $f(x) = \pi^{15}$

10. Find the derivatives of these functions with respect to x using the definition of the derivative.

(a) $f(x) = \frac{x}{\pi}$ (b) $f(x) = \frac{\pi}{x}$

Section 3.3 Exercises

In Exercises 1–6, find dy/dx.

1. $y = -x^2 + 3$ **2.** $y = \frac{x^3}{3} - x$

3. $y = 2x + 1$ **4.** $y = x^2 + x + 1$

5. $y = \frac{x^3}{3} + \frac{x^2}{2} + x$ **6.** $y = 1 - x + x^2 - x^3$

In Exercises 7–12, find the horizontal tangents of the curve.

7. $y = x^3 - 2x^2 + x + 1$ **8.** $y = x^3 - 4x^2 + x + 2$

9. $y = x^4 - 4x^2 + 1$ **10.** $y = 4x^3 - 6x^2 - 1$

11. $y = 5x^3 - 3x^5$ **12.** $y = x^4 - 7x^3 + 2x^2 + 15$

13. Let $y = (x + 1)(x^2 + 1)$. Find dy/dx **(a)** by applying the Product Rule, and **(b)** by multiplying the factors first and then differentiating.

14. Let $y = (x^2 + 3)/x$. Find dy/dx **(a)** by using the Quotient Rule, and **(b)** by first dividing the terms in the numerator by the denominator and then differentiating.

In Exercises 15–22, find dy/dx. (You can support your answer graphically.)

15. $(x^3 + x + 1)(x^4 + x^2 + 1)$ **16.** $(x^2 + 1)(x^3 + 1)$

17. $y = \frac{2x + 5}{3x - 2}$ **18.** $y = \frac{x^2 + 5x - 1}{x^2}$

19. $y = \frac{(x - 1)(x^2 + x + 1)}{x^3}$ **20.** $y = (1 - x)(1 + x^2)^{-1}$

21. $y = \frac{x^2}{1 - x^3}$ **22.** $y = \frac{(x + 1)(x + 2)}{(x - 1)(x - 2)}$

23. Suppose u and v are functions of x that are differentiable at $x = 0$, and that $u(0) = 5$, $u'(0) = -3$, $v(0) = -1$,

... is the slope of the line tangent

to the

$= -1$

$\frac{1}{\sqrt{x}}$

...nction.

$+ 3$

...ent to the

...nts where

...? At what

(1) $\frac{dy}{dx} = -2x$

(2) $\frac{dy}{dx} = x^2 - 1$

(3) $\frac{dy}{dx} = 2$

(4) $\frac{dy}{dx} = 2x + 1$

(5) $\frac{dy}{dx} = x^2 + x + 1$

(6) $\frac{dy}{dx} = -1 + 2x - 3x^2$

(7) at $x = \frac{1}{3}, 1$

$y = (\frac{1}{3})^3 - 2(\frac{1}{3})^2 + \frac{1}{3} + 1$

$y = (1)^3 - 2(1)^2 + 1 + 1$

(11) $x = 0, \pm 1$

$y = 5(0)^3 - 3(0)^5 = 0$

$y = 5(1)^3 - 3(1)^5 = 2$

$y = 5(-1)^3 - 3(-1)^5 = -5 + 3 = -2$

(12) $x = 0, \frac{21 \pm \sqrt{377}}{8}$

$y = 0^4 - 7(0)^3 + 2(0)^2 + 15$

$y = \left(\frac{21+\sqrt{377}}{8}\right)^4 - 7\left(\frac{21+\sqrt{377}}{8}\right)^3 + 2\left(\frac{21+\sqrt{377}}{8}\right)^2 + 15$

$y = \left(\frac{21-\sqrt{377}}{8}\right)^4 - 7\left(\frac{21-\sqrt{377}}{8}\right)^3 + 2\left(\frac{21-\sqrt{377}}{8}\right)^2 + 15$

39. Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ where the tangent is parallel to the x-axis.

40. Find the x- and y-intercepts of the line that is tangent to the curve $y = x^3$ at the point $(-2, -8)$.

41. Find the tangents to *Newton's serpentine*,

$$y = \frac{4x}{x^2 + 1},$$

at the origin and the point $(1, 2)$.

42. Find the tangent to the *witch of Agnesi*,

$$y = \frac{8}{4 + x^2},$$

at the point $(2, 1)$.

Standardized Test Questions

53. True or False $\frac{d}{dx}(\pi^3) = 3\pi^2$. Justify your answer.

54. True or False The graph of $f(x) = 1/x$ has no horizontal tangents. Justify your answer.

$f(x) = x^{-1}$

$f'(x) = -x^{-2} = -\frac{1}{x^2}$

$-\frac{1}{x^2} = 0$

$0 \neq 1$

(53) False

π^3 is a constant so $\frac{d}{dx}(\pi^3) = 0$

(54) True

$f'(x) = \frac{-1}{x^2}$, $\frac{-1}{x^2} \neq 0$

7. $y = x^3 - 2x^2 + x + 1$

$y' = 3x^2 - 4x + 1$

$0 = 3x^2 - 4x + 1$

$0 = (3x-1)(x-1)$

$x = \frac{1}{3}$ | $x = 1$

$y = \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \frac{1}{3} + 1$

$y = (1)^3 - 2(1)^2 + 1 + 1$

12. $y = x^4 - 7x^3 + 2x^2 + 15$

$y' = 4x^3 - 21x^2 + 4x$

$4x^3 - 21x^2 + 4x = 0$

$x(4x^2 - 21x + 4) = 0$

$x = 0$

$y = 0^4 - 7(0)^3 + 2(0)^2 + 15$

$y = 15$

$x = \frac{21 \pm \sqrt{(21)^2 - 4(4)(4)}}{2(4)}$

$y = \left(\frac{21 + \sqrt{(21)^2 - 4(4)(4)}}{2(4)}\right)^4 - 7\left(\frac{21 + \sqrt{(21)^2 - 4(4)(4)}}{2(4)}\right)^3 + 2\left(\frac{21 + \sqrt{(21)^2 - 4(4)(4)}}{2(4)}\right)^2 + 15$

do the same $\frac{21 - \sqrt{21^2 - 4(4)(4)}}{2(4)}$