Do Now

① Given $f(x) = 3x^2 - 2x + 1$, find $f'(x)$.

$f'(x) = 6x - 2$

② Given $f(x) = x^3 + 2x^2 + 3x - 1$, find $f'(x)$.

$f'(x) = 3x^2 + 4x + 3$

③ Given $f(x) = 2x$, find $f'(x) = 2$

a linear function, $y = mx + b$, so $f'(x) = m$

④ Given $f(x) = 5$, find $f'(x) = 0$

horizontal line hence the slope, $f'(x)$, $= 0$

From your work the past couple of days, you have found:

if $f(x) = x^2 + 4x + 9$, then $f'(x) = 2x + 4$

if $f(x) = x^2 - 5x + 1$, then $f'(x) = 2x - 5$

if $f(x) = 3x^2 - 4$, then $f'(x) = 6x$

if $f(x) = 2x^2 - 5x^3$, then $f'(x) = 4x - 15x^2$

if $f(x) = \frac{1}{x}$, then $f'(x) = -\frac{1}{x^2}$

$f(x) = x^{-1}$ $\qquad$ $f'(x) = -x^{-2}$

See a shortcut?

Homework 09-28

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

4. Given $f(x) = 4x^3 - 5x$
   (a) Find $f'(x)$. $= 12x^2 - 5$
   (b) Find $f'(-1)$, $f'(0)$, and $f'(2)$.
   (c) Write an equation of the tangent line to $f(x)$ at $x = -1$.

b) $f'(-1) = 7$
$f'(0) = -5$
$f'(2) = 43$

c) $f(-1) = 1$
$y - 1 = 7(x+1)$

5. Given $f(x) = \sqrt{8x}$
   (a) Find $f'(x)$.
   (b) Find $f'(2)$.
   (c) Write an equation of the tangent line of $f(x)$ at $x = 2$.

a) $f'(x) = \dfrac{4}{\sqrt{8x}}$ or $\dfrac{8}{2\sqrt{8x}}$

b) $f'(2) = 1$

c) $f(2) = 4$
$y - 4 = 1(x - 2)$

Homework 09-29

Quick Review 3.1 *(For help, go to Sections 2.1 and 2.4.)*

In Exercises 1–4, evaluate the indicated limit algebraically.

1. $\lim_{h\to 0} \dfrac{(2+h)^2-4}{h}$

2. $\lim_{x\to 2^+} \dfrac{x+3}{2}$

3. $\lim_{y\to 0^-} \dfrac{|y|}{y}$

4. $\lim_{x\to 4} \dfrac{2x-8}{\sqrt{x}-2}$

5. Find the slope of the line tangent to the parabola $y = x^2 + 1$ at its vertex.

6. By considering the graph of $f(x) = x^3 - 3x^2 + 2$, find the intervals on which f is increasing.

In Exercises 7–10, let

$$f(x) = \begin{cases} x+2, & x \le 1 \\ (x-1)^2, & x > 1. \end{cases}$$

7. Find $\lim_{x\to 1^+} f(x)$ 0 and $\lim_{x\to 1^-} f(x)$. 3

8. Find $\boxed{\lim_{h\to 0^+} f(1+h).}$ $= f(1)$

9. Does $\lim_{x\to 1} f(x)$ exist? Explain. No $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

10. Is f continuous? Explain. f is not continuous at 1 bc $\lim_{x\to 1} f(x)$ dne

$\lim_{h\to 0} f(1+h) = f(1+0) = f(1) = 1+2 = 3$

① $\lim_{h\to 0} \dfrac{((2+h)-2)(2+h+2)}{h}$

$\lim_{h\to 0} \dfrac{\cancel{h}(h+4)}{\cancel{h}} = 4$

② $\lim_{x\to 2^+} \dfrac{x+3}{2} = \dfrac{2+3}{2} = \dfrac{5}{2}$

③ $\lim_{y\to 0^-} \dfrac{|y|}{y} = -1$

④ $\lim_{x\to 4} \dfrac{\overset{2(x-4)}{2x-8}}{\sqrt{x}-2}$

$\lim_{x\to 4} \dfrac{2\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)}{\cancel{\sqrt{x}-2}} = 2(4) = 8$

Name:______________________________ Date:__________

AP Calc AB: Derivatives Ms. Loughran

Notation

There are many ways to denote the derivative of a function $y = f(x)$. Besides $f'(x)$, the most common notations are these:

y'	"y prime"	Nice and brief, but does not name the independent variable.
$\dfrac{dy}{dx}$	"dy dx" or "the derivative of y with respect to x"	Names both variables and uses d for derivative.
$\dfrac{df}{dx}$	"df dx" or "the derivative of f with respect to x"	Emphasizes the function's name.
$\dfrac{d}{dx}f(x)$	"d dx of f at x" or "the derivative of f at x"	Emphasizes the idea that differentiation is an operation performed on f.

RULE 1 Derivative of a Constant Function

If f is the function with the constant value c, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0.$$

RULE 2 Power Rule for Positive Integer Powers of x

If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

If n is a negative integer and $x \neq 0$, then

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

RULE 3 The Constant Multiple Rule

If u is a differentiable function of x and c is a constant, then

$$\frac{d}{dx}(cu) = c\frac{du}{dx}.$$

RULE 4 The Sum and Difference Rule

If u and v are differentiable functions of x, then their sum and difference are differentiable at every point where u and v are differentiable. At such points,

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}.$$

In function notation,

$$(f + g)' = f' + g' \qquad (f - g)' = f' - g'$$

Examples:

1. Find $\frac{dp}{dt}$ if $p = t^3 + 6t^2 - \frac{5}{3}t + 16$

$$\frac{dp}{dt} = 3t^2 + 12t - \frac{5}{3}$$

2. Find $\frac{dy}{dx}$ if $y = -x^2 + 3$

$$\frac{dy}{dx} = -2x$$

3. Find $\frac{dy}{dx}$ if $y = \frac{x^3}{3} - 3$

$$y = \frac{1}{3}x^3 - 3$$

$$\frac{dy}{dx} = x^2$$

4. Find $\frac{dy}{dx}$ if $y = x^2 + x + 1$

$$\frac{dy}{dx} = 2x + 1$$

5. Find $\frac{dy}{dx}$ if $y = x^4 - 7x^3 + 2x^2 + 15$

$$\frac{dy}{dx} = 4x^3 - 21x^2 + 4x$$

6. Find $\frac{dy}{dx}$ if $y = 4x^{-2} - 8x + 1$

$$\frac{dy}{dx} = -8x^{-3} - 8 = \frac{-8}{x^3} - 8$$

7. Find $\frac{dy}{dx}$ if $y = \frac{x^{-4}}{4} - \frac{x^{-3}}{3} + \frac{x^{-2}}{2} - x^{-1} + 3$

$$y = \frac{1}{4}x^{-4} - \frac{1}{3}x^{-3} + \frac{1}{2}x^{-2} - x^{-1} + 3$$

$$\frac{dy}{dx} = -x^{-5} + x^{-4} - x^{-3} + x^{-2} = -\frac{1}{x^5} + \frac{1}{x^4} - \frac{1}{x^3} + \frac{1}{x^2}$$

8. Use the definition of derivative to show that $\frac{d}{dx}(x) = 1$.

$$\lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x}}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$$

⑨ Find $\frac{dy}{dx}$ if $y = \sqrt{x}$

$y = x^{\frac{1}{2}}$ $\qquad \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \qquad = \frac{1}{2\sqrt{x}}$