Name: _____ Date: _____

AP Calculus: Product Rule Ms. Loughran

Do Now:

1. Given $f(x)=(x^2+1)(x^3+3)$. Find $f'(x)$.

$f(x) = x^5 + x^3 + 3x^2 + 3$

$f'(x) = 5x^4 + 3x^2 + 6x$

$f'(x) = 2x(x^3+3) + 3x^2(x^2+1)$

$f'(x) = 2x^4 + 6x + 3x^4 + 3x^2$

$h(x) = x^{18}$ $\quad h'(x) = 18x^{17}$

$f(x) \cdot g(x)$

$h(x) = x^{11} \cdot x^7$

$11x^{10} \cdot x^7 + 7x^6 \cdot x^{11}$

$f'g + g'f$

Product Rule: $(fg)' = f'g + fg'$

Examples

Find $f'(x)$ for each of the following.

1. $f(x)=(x^2+3)(x^3+2x-1)$

$f'(x) = 2x(x^3+2x-1) + (3x^2+2)(x^2+3)$

$f'(x) = 2x^4 + 4x^2 - 2x + 3x^4 + 9x^2 + 2x^2 + 6$

$f'(x) = 5x^4 + 15x^2 - 2x + 6$

2. $f(x)=(x+1)(2x-3)$

$f'(x) = 1(2x-3) + 2(x+1)$

$f'(x) = 2x-3+2x+2$

$f'(x) = 4x-1$

3. $f(x)=(x^2-2x+1)(x^3-1)$

$f'(x) = (2x-2)(x^3-1) + 3x^2(x^2-2x+1)$

$f'(x) = 2x^4-2x-2x^3+2+3x^4-6x^3+3x^2$

$f'(x) = 5x^4-7x^3+3x^2-2x+2$

4. $f(x)=(2x^2-1)^2 = (2x^2-1)(2x^2-1)$

$f'(x) = 4x(2x^2-1) + 4(x)(2x^2-1)$

$f'(x) = 8x^3-4x+8x^3-4x$

$f'(x) = 16x^3-8x$

5. $f(x)=(x^3+x+1)(x^4+x^2+1)$

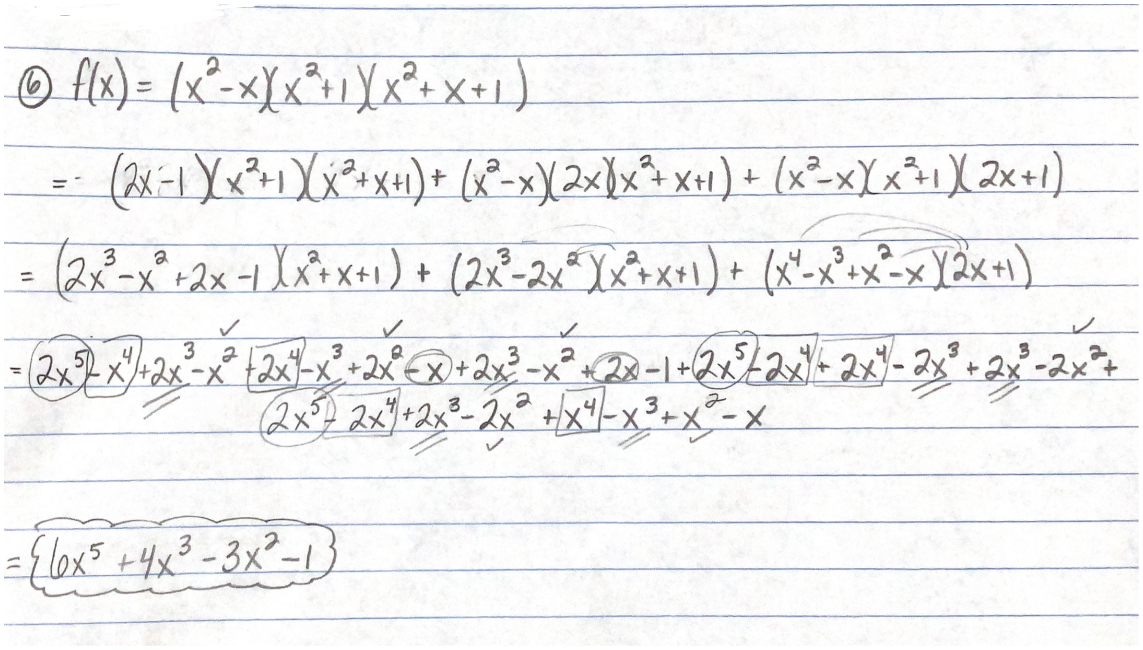
$f'(x) = (3x^2+1)(x^4+x^2+1) + (4x^3+2x)(x^3+x+1)$

$f'(x) = 3x^6+3x^4+3x^2+x^4+x^2+1+4x^6+4x^4+4x^3+2x^4+2x^2+2x$

$f'(x) = 7x^6+10x^4+4x^3+6x^2+2x+1$

$(fgh)' = f'gh + fg'h + fgh'$

6. $f(x) = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$



Homework

For 1-3, find $\frac{dy}{dx}$.

1. $f(x) = (3x^2 + 6)\left(2x - \frac{1}{4}\right)$

2. $f(x) = (2 - x - 3x^3)(7 + x^5)$

3. $f(x) = (3x^2 + 1)^2$

4. Find $\frac{d^2y}{dx^2}$ if $y = (5x^2 - 3)(7x^3 + x)$

5. Find $\left.\frac{d^2y}{dx^2}\right|_{x=1}$, where $y = 6x^5 - 4x^2$

6. Find the coordinates of all points on the graph of $y = 1 - x^2$ at which the tangent line passes through the point $(2, 0)$.

Homework 09-28

Name: Key
AP Calculus: Higher Derivatives Homework

Date: _____
Ms. Loughran

(-3, 2)
$m = 5$

1. Find an equation of the tangent line to the graph of $y = f(x)$ at the point where $x = -3$ if $f(-3) = 2$ and $f'(-3) = 5$.

$y - 2 = 5(x + 3)$
$y = 5x + 17$

2. Find $\frac{d^2y}{dx^2}$

(a) $y = 7x^3 - 5x^2 + x$
$y' = 21x^2 - 10x + 1$
$y'' = 42x - 10$.

(b) $y = 12x^2 - 2x + 3$
$y' = 24x - 2$
$y'' = 24$.

3. Find y'''.

(a) $y = x^{-5} + x^5$ $y' = -5x^{-6} + 5x^4$ $y'' = 30x^{-7} + 20x^3$ $y''' = -210x^{-8} + 60x^2$

(b) $y = \frac{1}{x}$ $y = x^{-1}$ $y' = -x^{-2}$ $y'' = 2x^{-3}$ $y''' = -6x^{-4}$.

(c) $y = ax^3 + bx + c$ (a, b and c are constants)
$y' = 3ax^2 + b$ $y'' = 6ax$ $y''' = 6a$.

4. Find (a) $f'''(2)$ where $f(x) = 3x^2 - 2$ $f'(x) = 6x$ $f''(x) = 6$ $f'''(x) = 0$ $f'''(2) = 0$

(b) $\left.\frac{d^4y}{dx^4}\right|_{x=1}$, where $y = \frac{1}{x^3}$ $y = x^{-3}$ $y' = -3x^{-4}$ $y'' = 12x^{-5}$ $y''' = -60x^{-6}$ $y^{IV} = 360x^{-7}$ 360.

5. Show that $y = x^3 + 3x + 1$ satisfies $y''' + xy'' - 2y' = 0$.

6. Given that $f(-2) = 3$ and $f'(-2) = 5$, find an equation for the tangent line to the graph of $y = f(x)$ at the point where $x = -2$.

(5) $y' = 3x^2 + 3$
$y'' = 6x$
$y''' = 6$

$6 + x(6x) - 2(3x^2 + 3) = 0$
$6 + 6x^2 - 6x^2 - 6 = 0$ ✓

(6) $(-2, 3)$
$m = 5$

$y - 3 = 5(x + 2)$
$y - 3 = 5x + 10$
$y = 5x + 13$

$y = x^{-3}$
$y' = -3x^{-4}$
$y'' = 12x^{-5}$
$y''' = -60x^{-6}$
$y^{IV} = 360x^{-7}$