Do Now: #s 1 and 2

Name: _____ Date: _____

AP Calc AB - Applications of Derivatives

1. State an equation of each horizontal tangent line to the curve $y = x^4 - 3x^2 + 2$, if any exist.

$m = 0$

$y' = 4x^3 - 6x$

$4x^3 - 6x = 0$

$2x(2x^2 - 3) = 0$

$x = 0 \quad | \quad x = \pm\sqrt{\frac{3}{2}}$

at $x = 0$

$y = 0^4 - 3(0)^2 + 2 = 2$

at $x = \pm\sqrt{\frac{3}{2}}$

$y = \left(\pm\sqrt{\frac{3}{2}}\right)^4 - 3\left(\pm\sqrt{\frac{3}{2}}\right)^2 + 2$

$\left(\sqrt{\frac{3}{2}}\right)^2\left(\sqrt{\frac{3}{2}}\right)^2$

$\frac{9}{4} + 3\left(\frac{3}{2}\right)$

$y = \frac{9}{4} - \frac{9}{2} + 2 = \frac{9 - 18 + 8}{4} = \frac{-1}{4}$

$y = 2$

$y = -\frac{1}{4}$

2. State an equation of (A) a tangent line and (B) a normal line to the curve to the curve $y = x^3 - 2x$ at the point on the curve where $x = 2$.

$y' = 3x^2 - 2$

$y'(2) = 3(2)^2 - 2 = 10$

$y(2) = 2^3 - 2(2) = 4$

A) $y - 4 = 10(x - 2)$

B) $y - 4 = -\frac{1}{10}(x - 2)$

3. A line is drawn tangent to the curve $y = x^3 - x$ at the point $(-1, 0)$. Where else does this line intersect the curve?

Plan: ① need eq. of the tan line

② set eq. of the tan = eq. of the curve

$y' = 3x^2 - 1$

$y'(-1) = 3(-1)^2 - 1 = 2$

$y - 0 = 2(x + 1)$

$y = 2(x + 1)$

$y = 2x + 2$

we know one solution bc it is $x = -1$

$2x + 2 = x^3 - x$

$0 = x^3 - 3x - 2$

$0 = (x + 1)(x^2 - x - 2)$

$0 = (x + 1)(x - 2)(x + 1)$

$0 = (x + 1)^2(x - 2)$

$x = 2$

	1	0	-3	-2
-1		-1	1	2
	1	-1	-2	0

if $x = 2$

$y = 2^3 - 2 = 6$

$(2, 6)$

4. Find the constant, c, if the curve $y = x^2 + c$ is tangent to the line $y = x$.

$m = 1$

Plan: we need to find (x, y)

tangent line to the curve at (x, y)

$y' = 2x$

$1 = 2x$

$\frac{1}{2} = x$

$\left(\frac{1}{2}, \frac{1}{2}\right)$ point of tangency

$\frac{1}{2} = \left(\frac{1}{2}\right)^2 + c$

$\frac{1}{2} = \frac{1}{4} + c$

$\frac{1}{4} = c$

5. Find the values of the constants $a, b,$ and c if the curve $y = ax^2 + bx + c$ is to pass through the point $(1,2)$ and is tangent to the line $y = x$ at the origin.

$m = 1$

$y' = 2ax + b$

$1 = 2ax + b$ @ $(0,0)$

$1 = 2a(0) + b$

$1 = b$

$(1,2)$ is on curve

$2 = a(1)^2 + b(1) + c$

$2 = a + b + c$

$2 = a + 1 + c$

$1 = a + c$

$1 = a + 0$

$a = 1$

$(0,0)$ is also on the curve

$0 = a(0)^2 + b(0) + c$

$0 = c$

6. Find the values of the constants $a, b,$ and c so that the curves $y = x^2 + ax + b$ and $y = cx - x^2$ will be tangent to each other at the point $(1,0)$ ← on both curves

slopes (derivatives) are equal at $(1,0)$

$y' = 2x + a$ $\quad y' = c - 2x$

$2x + a = c - 2x$ @ $(1,0)$

$2(1) + a = c - 2(1)$

$2 + a = c - 2$

$4 + a = c$

$0 = 1^2 + a(1) + b$

$0 = 1 + a + b$

$-1 = a + b$

$0 = c(1) - (1)^2$

$0 = c - 1$

$1 = c$

$4 + a = 1$

$a = -3$

$-1 = a + b$

$-1 = -3 + b$

$b = 2$