Name: ____ Date: ____________

AP Calc: Quotient Rule Ms. Loughran

Do Now

1. Find the values of a, b, and c where the curves $y = x^2 + ax + b$ and $y = cx + x^2$ have a common tangent line at $(-1,0)$.

$y' = 2x + a \qquad y' = c + 2x$

$2x + a = c + 2x$ @ $(-1,0)$

$2(-1) + a = c + 2(-1)$

$-2 + a = c - 2$

$a = c$

(-1,0) on both curves

$0 = (-1)^2 + a(-1) + b$

$0 = 1 - a + b$

$a = b + 1$

$0 = c(-1) + (-1)^2$

$0 = -c + 1$

$c = 1$

$a = 1$

$a = b + 1$

$1 = b + 1$

$b = 0$

Quotient Rule:

$$\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2}$$

1. $\frac{d}{dx}\left(\frac{1}{g(x)}\right) = \frac{g(x) \cdot 0 - 1 \cdot g'(x)}{(g(x))^2} = \frac{-g'(x)}{g^2(x)}$

Reciprocal Rule: $-\frac{\text{derivative of denominator}}{\text{denominator squared}}$

$$y = \frac{f}{g}$$

$$[yg]' = f'$$

$$yg' + y'g = f'$$

$$\frac{y'g}{g} = \frac{f' - yg'}{g}$$

$$y' = \frac{f' - yg'}{g}$$

$$y' = \frac{gf' - \frac{f}{\cancel{g}} g' \cancel{g}}{gg}$$

$$y' = \frac{gf' - fg'}{g^2}$$

"Low DHigh - High DLow" / Low²

Find y' for each of the following.

2. $y=\dfrac{x^2-1}{x^2+1}$

$$y'=\frac{(x^2+1)(2x)-2x(x^2-1)}{(x^2+1)^2}$$

$$y'=\frac{2x(\cancel{x^2}+1-(\cancel{x^2}-1))}{(x^2+1)^2}=\frac{2x(2)}{(x^2+1)^2}=\frac{4x}{(x^2+1)^2}$$

3. $y=\dfrac{2x+5}{3x-2}$

$$y'=\frac{(3x-2)2-(2x+5)3}{(3x-2)^2}$$

$$y'=\frac{6x-4-6x-15}{(3x-2)^2}=\frac{-19}{(3x-2)^2}$$

4. $y=\dfrac{x^2+5x-1}{x^2}=\dfrac{x^2}{x^2}+\dfrac{5\cancel{x}}{x^{\cancel{2}}}-\dfrac{1}{x^2}$

Could use QR or easier rewrite

$$y=1+5x^{-1}-x^{-2}$$

$$y'=-5x^{-2}+2x^{-3}=\frac{-5}{x^2}+\frac{2}{x^3}$$

5. $y=\dfrac{1}{x}=x^{-1}$

$$y'=-x^{-2}=-\frac{1}{x^2}$$

OR Recip. Rule

$$y'=\frac{-1}{x^2}$$

or QR

6. $y=\dfrac{(x-1)(x^2+x+1)}{x^3}$ $= \dfrac{x^3-1}{x^3}$

$y=\dfrac{x^3}{x^3}-\dfrac{1}{x^3}$

$y= 1-x^{-3}$

$y' = 3x^{-4} = \dfrac{3}{x^4}$

7. $y=\dfrac{1}{x^2-4x+5}$

$y' = \dfrac{-(2x-4)}{(x^2-4x+5)^2}$

also note

$y=\dfrac{5}{(x^2-4x+5)^2}$

$y = 5 \cdot \dfrac{1}{(x^2-4x+5)^2}$

$y' = \dfrac{-5(2x-4)}{(x^2-4x+5)^2}$

WHEN NOT TO USE THE QUOTIENT RULE

Not every quotient needs to be differentiated by the Quotient Rule.

When a quotient can be considered as a product of a constant times a function of x, the Constant Multiple Rule is more convenient than the Quotient Rule.

In other cases, using the Power Rule with negative exponents is preferable to using the Quotient Rule.

DIRECTIONS: Complete the following table WITHOUT USING THE QUOTIENT RULE.
[* Note that two characteristics of a "simplified final form" are the absence of negative exponents and the combining of like terms.]

	FUNCTION	REWRITE	DERIVATIVE	* SIMPLIFY
1.	$y = \frac{x^2 + 2x}{x}$	$y = x + 2$	$y' = 1$	$y' = 1$
2.	$y = \frac{4x^{3/2}}{x}$	$y = 4x^{\frac{1}{2}}$	$y' = 2x^{-\frac{1}{2}}$	$y' = \frac{2}{x^{\frac{1}{2}}}$ or $\frac{2}{\sqrt{x}}$
3.	$y = \frac{7}{3x^3}$	$y = \frac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = \frac{-7}{x^4}$
4.	$y = \frac{4}{5x^2}$	$y = \frac{4}{5}x^{-2}$	$y' = \frac{-8}{5}x^{-3}$	$y' = \frac{-8}{5x^3}$
5.	$y = \frac{x^2 + 3x}{6}$	$y = \frac{1}{6}x^2 + \frac{1}{2}x$	$y' = \frac{1}{3}x + \frac{1}{2}$	$y' = \frac{1}{3}x + \frac{1}{2}$
6.	$y = \frac{3x^2 - 5}{7}$	$y = \frac{3}{7}x^2 - \frac{5}{7}$	$y' = \frac{6}{7}x$	$\longrightarrow$
7.	$y = \frac{x^2 - 4}{x + 2}$	$y = x - 2$	$y' = 1$	$\longrightarrow$
8.	$y = \frac{-3(3x - 2x^2)}{7x}$	$y = \frac{-9}{7} + \frac{6}{7}x$	$y' = \frac{6}{7}$	$\longrightarrow$

* remember when you divide and the bases are the same, subtract exponents

$(x-2)(x+2)$

$-9x + 6x^2$

$y = \frac{-9\cancel{x}}{7\cancel{x}} + \frac{6x^{\cancel{2}}}{7\cancel{x}}$

Homework 10-02

Homework

For 1-3, find $\frac{dy}{dx}$.

1. $f(x)=(3x^2+6)\left(2x-\frac{1}{4}\right)$ $\quad f'(x)=(3x^2+6)(2)+(2x-\frac{1}{4})(6x)$
$= 6x^2+12+12x^2-\frac{3}{2}x = 18x^2-\frac{3}{2}x+12$

2. $f(x)=(2-x-3x^3)(7+x^5)$ $\quad f'(x)=(2-x-3x^3)(5x^4)+(7+x^5)(-1-9x^2)$
$=-24x^7-6x^5+10x^4-63x^2-7$

3. $f(x)=(3x^2+1)^2$ $\quad f'(x)=6x(3x^2+1)+6x(3x^2+1)$
$18x^3+6x+18x^3+6x = 36x^3+12x$

4. Find $\frac{d^2y}{dx^2}$ if $y=(5x^2-3)(7x^3+x)$ $\quad y'=(5x^2-3)(21x^2+1)+(7x^3+x)(10x)$
$= 105x^4-63x^2+5x^2-3+70x^4+10x^2$
$y'=175x^4-48x^2-3$
$y''=700x^3-96x$

5. Find $\left.\frac{d^2y}{dx^2}\right|_{x=1}$, where $y=6x^5-4x^2$
$y'=30x^4-8x$
$y''=120x^3-8\big|_{x=1}$ $\quad 120(1)^3-8=112$

6. Find the coordinates of all points on the graph of $y=1-x^2$ at which the tangent line passes through the point $(2,0)$.

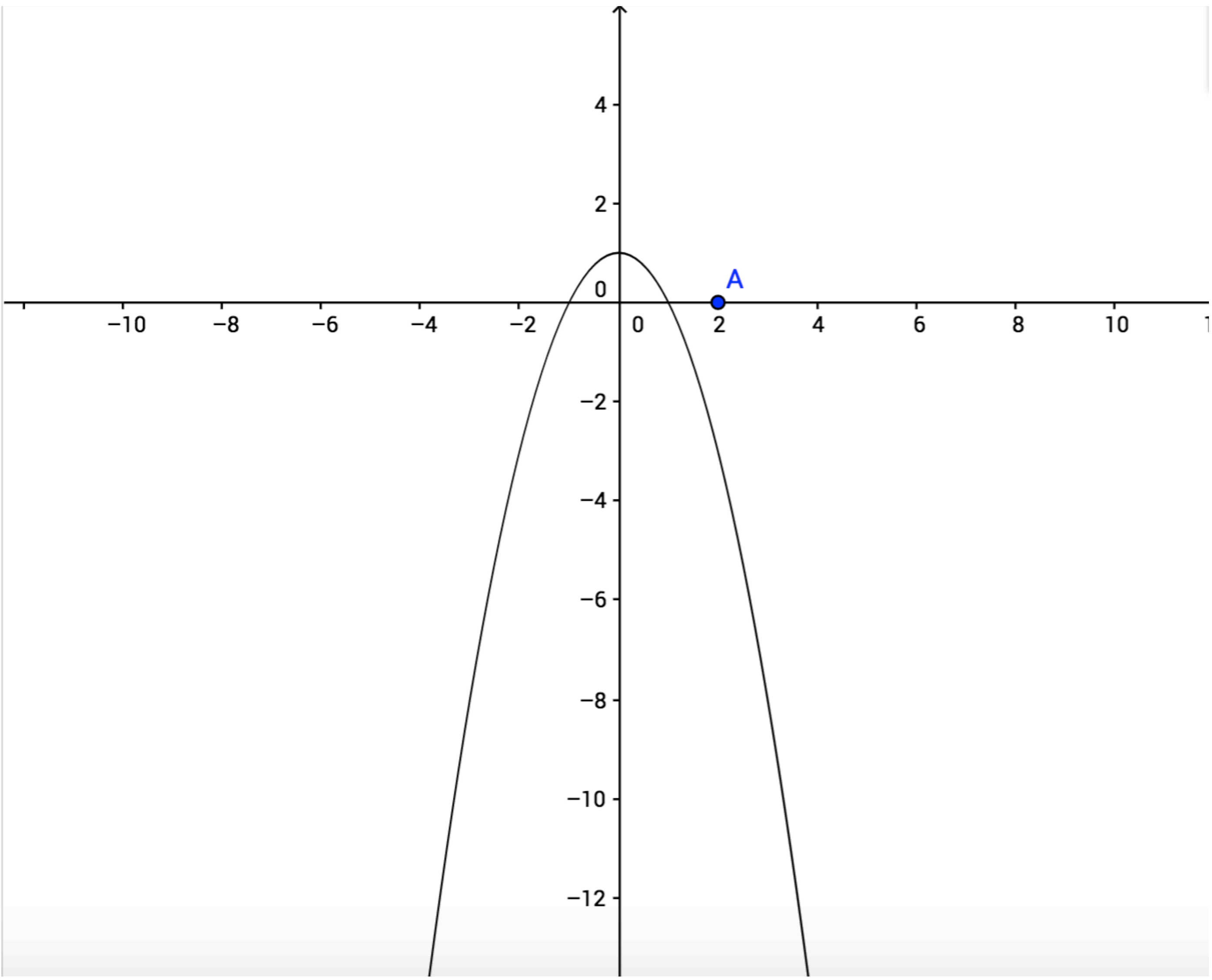
$(2,0)$
$(x, 1-x^2)$

$m_{tan}=\frac{0-(1-x^2)}{2-x}$

$-2x=\frac{-1+x^2}{2-x}$

$-4x+2x^2=-1+x^2$
$x^2-4x+1=0$
$x=\frac{4\pm\sqrt{16-4(1)(1)}}{2(1)}=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$

$y-0=-2x(x-2)$
$y=-2x^2+4x$ eq. of tan line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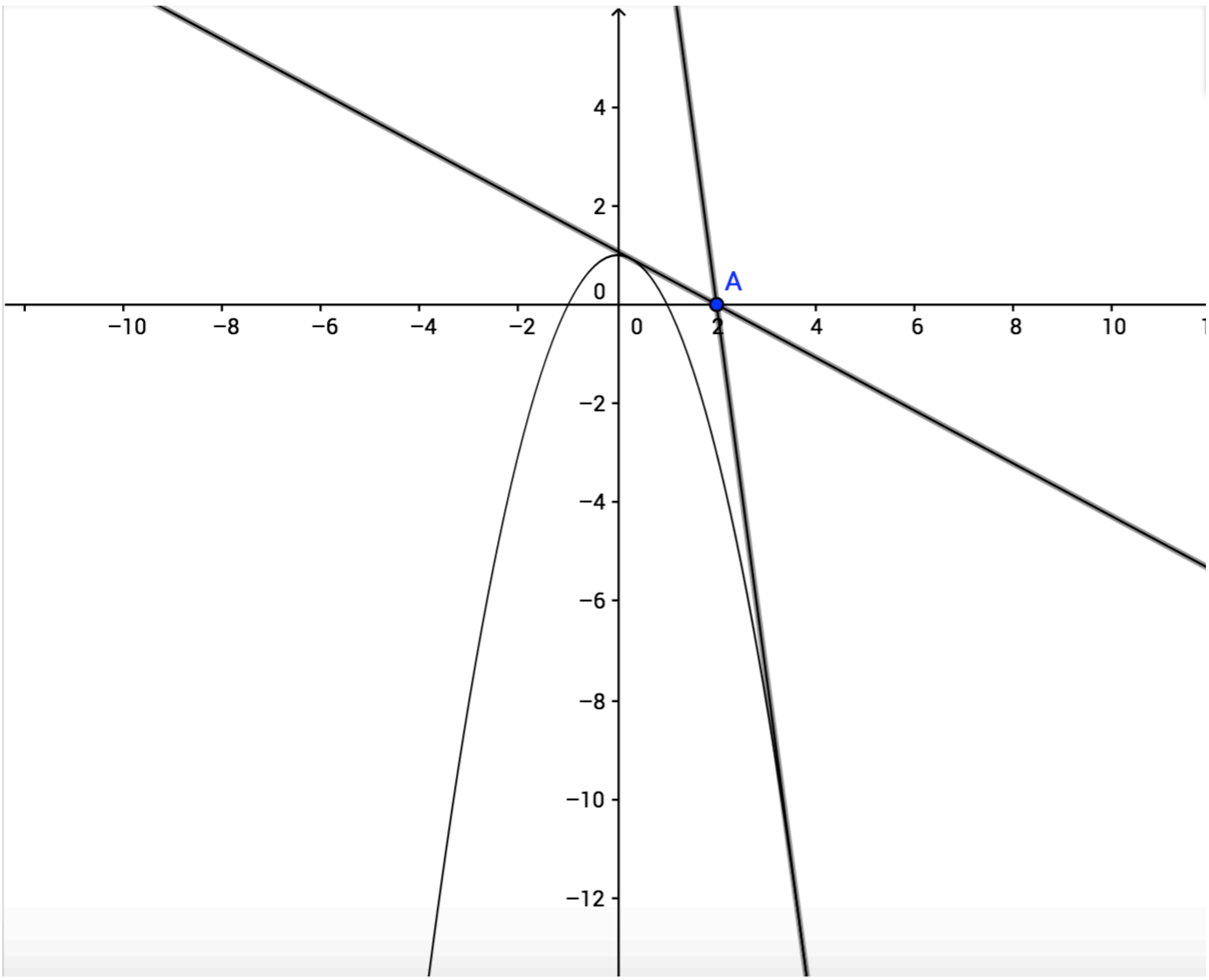

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−8
−10
−12

$0 \neq 1-(2)^2$

$y = 1 - x^2$

$(2,0)$ is NOT the pt of tangency - it's not even on curve.

* need eq. of tan line to find pts of intersection

$y' = -2x$

Eq. of tan line

$m = -2x$

$(2,0)$

not finding eq. of tan line at $(2,0)$ b/c $(2,0)$ is not pt of tangency.

$y - 0 = -2x(x-2)$

$y = -2x^2 + 4x$

$1 - x^2 = -2x^2 + 4x$

$x^2 - 4x + 1 = 0$

$x^2 - 4x = -1$

$x^2 - 4x + 4 = 3$

$(x-2)^2 = 3$

$x - 2 = \pm\sqrt{3}$

$x = 2 \pm \sqrt{3}$

remind them of completing the square technique

Name: ______________________________ Date: ____________

AP Calculus: Applications of Derivatives Homework Ms. Loughran

1. $\frac{d}{dt}\left[16t^2\right] = 32t$

2. Find $V'(r)$, where $V = \pi r^3$ $V'(r) = 3\pi r^2$

3. Find $F'(2)$ given that $f(2) = -1$, $f'(2) = 4$, $g(2) = 1$, $g'(2) = -5$ and $F(x) = 5f(x) + 2g(x)$.

$F'(x) = 5f'(x) + 2g'(x)$ $F'(2) = 5f'(2) + 2g'(2)$

$5(4) + 2(-5) = 10$

4. Find y''', where $y = 5x^2 - 4x + 7$

$y' = 10x - 4$ $y'' = 10$ $y''' = 0$

5. Find a function $y = ax^2 + bx + c$ whose graph has an x-intercept of 1, a y-intercept of -2, and a tangent line with a slope of -1 at the y-intercept.

6. Find k if the curve $y = x^2 + k$ is tangent to the line $y = 2x$.

7. Find the x-coordinate of the point on the graph of $y = x^2$ where the tangent line is parallel to the secant line that cuts the curve at $x = -1$ and $x = 2$.

5 x-int of 1

(1,0)

y-nt of -2

(0,-2)

$c = -2$

$y = ax^2 + bx + c$

$y' = 2ax + b$

$2ax + b = -1$ @ y-int. @ (0,-2)

$2a(0) + b = -1$

$b = -1$

$y = ax^2 + bx + c$

$0 = a + b + c$

$0 = a + b + c$

$0 = -1 - 2 + a$

$0 = -3 + a$

$a = 3$

6 $y = x^2 + k$

$y' = 2x$

$y = 2x$

slope $= 2$

$2x = 2$

$x = 1$

(1,2)

$2 = 1^2 + k$

$1 = k$

(7) $y = x^2$

tan line ∥ to secant line
∴ slopes are =

$x = -1 \quad y = 1$
$x = 2 \quad y = 4$

need slope of secant line

$$m = \frac{4-1}{2-(-1)} = \frac{3}{3} = 1$$

$y' = 2x$

$2x = 1$

$x = \frac{1}{2}$

(5) $y = ax^2 + bx + c$

$(1,0)$
$(0,-2)$

$m = -1$ @ $(0,-2)$

$0 = a(1)^2 + b(1) + c$

$0 = a + b + c$

$y' = 2ax + b$

$-2 = a(0)^2 + b(0) + c$

$-2 = c$

$-1 = 2a(0) + b$

$-1 = b$

$0 = a + b - 2$

$2 = a + b$

$2 = a - 1$

$3 = a$

7 $y = x^2$ tan line ∥ sec line

$(-1, (-1)^2) \rightarrow (-1, 1)$

$(2, 2^2) \rightarrow (2, 4)$

$m_{sec} = \frac{1-4}{-1-2} = \frac{-3}{-3} = 1$

$y' = 2x$

$m_{tan} = 1$

$2x = 1$

$x = \frac{1}{2}$

3 $F(x) = 5f(x) + 2g(x)$

$F'(2) =$

$F'(x) = 5f'(x) + 2g'(x)$

$F'(2) = 5f'(2) + 2g'(2)$

$= 5(4) + 2(-5) = 10$