Name: ___________________________________ Date: _____________

AP Calc: Chain Rule

Ms. Loughran

Do Now

1. Find an equation for the normal line to the graph of $y = \dfrac{3x+5}{x-1}$ at $x = 3$.

$$y' = \frac{3(x-1) - (3x+5)(1)}{(x-1)^2}$$

$$y'(3) = \frac{3(2) - (14)}{(3-1)^2} = -2$$

$$y(3) = \frac{3(3)+5}{3-1} = 7$$

$$y - 7 = \frac{1}{2}(x-3)$$

2. $\dfrac{dy}{dx}\left[\dfrac{6}{x^2+2x+1}\right] = 6\left[\dfrac{1}{x^2+2x+1}\right]'$

$$6\left(\frac{-(2x+2)}{(x^2+2x+1)^2}\right)$$

$$\frac{-12x-12}{(x^2+2x+1)^2} = \frac{-12(x+1)}{((x+1)^2)^2}$$

$$\frac{-12\cancel{(x+1)}}{(x+1)^{\cancel{4}\,3}} = \frac{-12}{(x+1)^3}$$

3. Given $y = (5x^3+2)^2$, find $\dfrac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = 2(5x^3+2)(15x^2)$$

$$y = (5x^3+2)(5x^3+2)$$

$$\frac{dy}{dx} = (5x^3+2)(15x^2) + (5x^3+2)(15x^2)$$

$$\frac{dy}{dx} = 75x^5 + 30x^2 + 75x^5 + 30x^2$$

$$\frac{dy}{dx} = 150x^5 + 60x^2$$

$$\frac{d^2y}{dx^2} = 750x^4 + 120x$$

Chain Rule:

$$(f(g))' = f'(g) \cdot g'$$

$$(f(g(h)))' = f'(g(h)) \cdot g'(h) \cdot h'$$

1. If $y = (3x^2 + 5x)^7$, find y'.

$$y' = 7(3x^2 + 5x)^6 \cdot (6x + 5)$$

$$y' = (42x + 35)(3x^2 + 5x)^6$$

2. If $y = \sqrt{13x^2 - 5x + 8}$, find y'.

$$y = (13x^2 - 5x + 8)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(13x^2 - 5x + 8)^{-\frac{1}{2}} \cdot (26x - 5)$$

$$y' = \frac{26x - 5}{2\sqrt{13x^2 - 5x + 8}}$$

3. $y = (x^3 + 3)^5$

$$y' = 5(x^3 + 3)^4 \cdot (3x^2)$$

$$y' = 15x^2(x^3 + 3)^4$$

4. $y=(-5x^3-3)^3$

$y' = 3(-5x^3-3)^2 \cdot (-15x^2)$

$y' = -45x^2(-5x^3-3)^2$

5. $y=(-3x^5+1)^3$

$y' = 3(-3x^5+1)^2 \cdot (-15x^4)$

$y' = -45x^4(-3x^5+1)^2$

6. $y=(5x^2+3)^4$

$y' = 4(5x^2+3)^3 \cdot 10x$

$y' = 40x(5x^2+3)^3$

7. $f(x)=\sqrt[4]{-3x^4-2}$ $=\left(-3x^4-2\right)^{\frac{1}{4}}$

$f'(x)=\frac{1}{4}\left(-3x^4-2\right)^{-\frac{3}{4}}\cdot -12x^3$

$f'(x)=\dfrac{-3x^3}{\sqrt[4]{\left(-3x^4-2\right)^3}}$

8. $f(x)=\sqrt{-2x^2+1}$ $=\left(-2x^2+1\right)^{\frac{1}{2}}$

$f'(x)=\frac{1}{2}\left(-2x^2+1\right)^{-\frac{1}{2}}\cdot(-4x)$

$f'(x)=\dfrac{-2x}{\sqrt{-2x^2+1}}$

9. $y=\sqrt[3]{-2x^4+5}$ $=\left(-2x^4+5\right)^{\frac{1}{3}}$

$y'=\frac{1}{3}\left(-2x^4+5\right)^{-2/3}\cdot -8x^3$

$y'=\dfrac{-8x^3}{3\sqrt[3]{\left(-2x^4+5\right)^2}}$

10. $y=(-x^4-3)^{-2}$

$y'=-2\left(-x^4-3\right)^{-3}\cdot -4x^3$

$y'=\dfrac{8x^3}{\left(-x^4-3\right)^3}$

Homework 10-03

Name: Key Date: ______
AP Calc: Quotient Rule Homework Ms. Loughran

$\rightsquigarrow f'(x) = 2(3x^2+1)\cdot 6x$

1. If $f(x)=(3x^2+1)^2$, find $f'(x)$.

$f'(x) = 6x(3x^2+1) + 6x(3x^2+1) = 12x(3x^2+1)$

2. If $y=\dfrac{1}{5x-3}$, find $y'(1)$.

$y' = \dfrac{-5}{(5x-3)^2}$ $y'(1) = \dfrac{-5}{(5(1)-3)^2} = \dfrac{-5}{4}$

3. If $x=\dfrac{3t}{2t+1}$, find $\dfrac{dx}{dt}$.

$\dfrac{dx}{dt} = \dfrac{3(2t+1)-2(3t)}{(2t+1)^2} = \dfrac{6t+3-6t}{(2t+1)^2} = \dfrac{3}{(2t+1)^2}$

4. If $y=\dfrac{2x-1}{x+3}$, find $\left.\dfrac{dy}{dx}\right|_{x=1}$

$y' = \left.\dfrac{2(x+3)-(2x-1)}{(x+3)^2}\right|_{x=1}$ $\dfrac{2(4)-1}{(1+3)^2} = \dfrac{7}{16}$

5. If $y=(2x^7-x^2)\left(\dfrac{x-1}{x+1}\right)$, find $\left.\dfrac{dy}{dx}\right|_{x=1}$

6. If $y=\dfrac{x+1}{x}$, find $\dfrac{d^2y}{dx^2}$.

$y = 1 + x^{-1}$ $y' = -x^{-2}$ $y'' = 2x^{-3} = \dfrac{2}{x^3}$

Also from your textbook, p.126 #s 55-58 and Quick Quiz #s 2-4.

(5) $y' = (2x^7-x^2)\left(\dfrac{x-1}{x+1}\right)' + \left(\dfrac{x-1}{x+1}\right)(14x^6-2x)$

$y' = (2x^7-x^2)\left[\dfrac{(x+1)(1)-1(x-1)}{(x+1)^2}\right] + \dfrac{(14x^6-2x)(x-1)}{x+1}$

$x+1-x+1$

$\left.\dfrac{(2x^7-x^2)(2)}{(x+1)^2} + \dfrac{(14x^6-2x)(x-1)}{x+1}\right|_{x=1}$

$\dfrac{(2-1)(2)}{2^2} + \dfrac{(14-2)(0)}{2} = \dfrac{2}{4} = \dfrac{1}{2}$

p.126

#55 $y = uv$

$y' = uv' + vu'$

$y'(1) = u(1)\,v'(1) + v(1)\,u'(1)$

$2 \cdot 1 + (-1)(3) = 2 - 3 = -1$ (B).

56 $f(x) = x - \frac{1}{x} = x - x^{-1}$

$f'(x) = 1 + x^{-2}$ $\quad f''(x) = -2x^{-3}$ $\quad$ D.

57 $\frac{d}{dx}\left(\frac{x+1}{x-1}\right) = \frac{\overbrace{(x-1)(1) - (x+1)(1)}^{x-1-x-1}}{(x-1)^2} = \frac{-2}{(x-1)^2}$ E.

58 $f(x) = (x^2-1)(x^2+1)$ $\quad$ or $x^4 - 1$ $\quad$ $4x^3 = 0$

$f'(x) = \overbrace{(x^2-1)(2x)}^{2x^3-2x} + \overbrace{2x(x^2+1)}^{+2x^3+2x}$ $\quad$ $x = 0$

$f'(x) = 2x\left[x^2 \cancel{-1} + x^2 \cancel{+1}\right] = 2x\left[2x^2\right] = 4x^3$

(B).

Quick Quiz

(1,2) (-1,1)

(2) $m_{\text{normal}} = \frac{2-1}{1-(-1)} = \frac{1}{2}$ $\quad$ $f'(1)$ = slope of tan line @ $x=1$

$f'(1) = -2$ (A).

(3) $y = \frac{4x-3}{2x+1}$

$y' = \frac{4(2x+1) - 2(4x-3)}{(2x+1)^2}$ $\quad$ ($8x+4-8x+6$) $= \frac{10}{(2x+1)^2}$ (C).

(4) $f(x) = x^4 - 4x^2$

$f'(x) = 4x^3 - 8x$

(a) $4x^3 - 8x = 0$

$4x(x^2-2) = 0$

$x = 0 \quad x = \pm\sqrt{2}$

(0,0) $\quad$ $4 - 8$

$(\sqrt{2}, (\sqrt{2})^4 - 4(\sqrt{2})^2)$

$(\sqrt{2}, -4)$ $\quad$ $(-\sqrt{2}, -4)$

(b) $f(1) = 1 - 4 = -3$

$f'(1) = -4$

$y + 3 = -4(x-1)$

$y = -4x + 1$

(c) $y + 3 = \frac{1}{4}(x-1)$

$y = \frac{1}{4}x - \frac{13}{4}$

55. Multiple Choice Let $y = uv$ be the product of the functions u and v. Find $y'(1)$ if $u(1) = 2$, $u'(1) = 3$, $v(1) = -1$, and $v'(1) = 1$.

(A) -4 (B) -1 (C) 1 (D) 4 (E) 7

$y' = uv' + u'v$

$y'(1) = u(1)v'(1) + u'(1)v(1)$

$2 \cdot 1 + 3 \cdot (-1)$

$2 + -3$

56. Multiple Choice Let $f(x) = x - \frac{1}{x}$. Find $f''(x)$.

(A) $1 + \frac{1}{x^2}$ (B) $1 - \frac{1}{x^2}$ (C) $\frac{2}{x^3}$

(D) $-\frac{2}{x^3}$ (E) does not exist

57. Multiple Choice Which of the following is $\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$?

(A) $\frac{2}{(x-1)^2}$ (B) 0 (C) $-\frac{x^2+1}{x^2}$

(D) $2x - \frac{1}{x^2} - 1$ (E) $-\frac{2}{(x-1)^2}$

58. Multiple Choice Assume $f(x) = (x^2 - 1)(x^2 + 1)$. Which of the following gives the number of horizontal tangents of f?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Extending the Ideas

59. *Leibniz's Proof of the Product Rule* Here's how Leibniz explained the Product Rule in a letter to his colleague John Wallis:

It is useful to consider quantities infinitely small such that when their ratio is sought, they may not be considered zero, but which are rejected as often as they occur with quantities incompara[bly] greater. Thus if we have $x + dx$, dx is rejected. Similarly we cannot have xdx and $dxdx$ standing together, as xdx is incom[parably] greater than $dxdx$. Hence if we are to differentiate uv, we write

$$\begin{aligned} d(uv) &= (u + du)(v + dv) - uv \\ &= uv + vdu + udv + dudv - uv \\ &= vdu + udv. \end{aligned}$$

Answer the following questions about Leibniz's proof.

(a) What does Leibniz mean by a quantity being "rejected"?

(b) What happened to $dudv$ in the last step of Leibniz's proo[f]?

(c) Divide both sides of Leibniz's formula

$$d(uv) = vdu + udv$$

by the differential dx. What formula results?

(d) Why would the critics of Leibniz's time have objected to dividing both sides of the equation by dx?

(e) Leibniz had a similar simple (but not-so-clean) proof of t[he] Quotient Rule. Can you reconstruct it?

(58) $f(x) = x^4 - 1$

$f'(x) = 4x^3$

$4x^3 = 0$

Quick Quiz for AP* Preparation: Sections 3.1–3.3

1. Multiple Choice Let $f(x) = |x + 1|$. Which of the following statements about f are true?

I. f is continuous at $x = -1$.

II. f is differentiable at $x = -1$.

III. f has a corner at $x = -1$.

(A) I only (B) II only (C) III only

(D) I and III only (E) I and II only

2. Multiple Choice If the line normal to the graph of f at the point $(1, 2)$ passes through the point $(-1, 1)$, then which of the following gives the value of $f'(1) = ?$

(A) -2 (B) 2 (C) $-1/2$ (D) $1/2$ (E) 3

$m_{norm} = \frac{1-2}{-1-1} = \frac{-1}{-2} = \frac{1}{2}$

slope of the tan line at 1

3. Multiple Choice Find dy/dx if $y = \frac{4x-3}{2x+1}$.

(A) $\frac{10}{(4x-3)^2}$ (B) $-\frac{10}{(4x-3)^2}$ (C) $\frac{10}{(2x+1)^2}$

(D) $-\frac{10}{(2x+1)^2}$ (E) 2

4. Free Response Let $f(x) = x^4 - 4x^2$.

(a) Find all the points where f has horizontal tangents.

(b) Find an equation of the tangent line at $x = 1$.

(c) Find an equation of the normal line at $x = 1$.

From the Practice Derivative Multiple Choice

Name: ____

AP Calculus - Multiple Choice Practice

1) If $y = \frac{1 - x}{2x + 1}$, then $\frac{dy}{dx} =$ $\frac{(2x+1)(-1)-2(1-x)}{(2x+1)^2} = \frac{\cancel{-2x}-1-2+\cancel{2x}}{(2x+1)^2}$

A) $\frac{-3}{8x + 4}$ B) $\frac{3}{(2x + 1)^2}$ C) $-\frac{4x + 1}{(2x + 1)^2}$ D) $\frac{-3}{(2x + 1)^2}$ E) $-\frac{1}{2}$

2) If $f(x) = \frac{x^2 - 7}{7 - x^2}$, then $\frac{dy}{dx} =$ 0

-1

A) 1 B) $\frac{14}{(7 - x^2)^2}$ C) $\frac{-14}{(7 - x^2)^2}$ D) -1 E) 0

3) If $f(x) = 3x^{\frac{1}{3}} - 2x^{\frac{1}{2}} + x^{-1}$, then $f'(x) =$ $x^{-2/3} - x^{-1/2} - x^{-2}$

A) $x^{-\frac{2}{3}} - x^{-\frac{1}{2}} - 1$ B) $x^{-\frac{2}{3}} - x^{-\frac{1}{2}} - \frac{1}{x^2}$ C) $x^{-\frac{1}{3}} - x^{-\frac{1}{2}} - x$

D) $x^{-\frac{2}{3}} - x^{-\frac{1}{2}} - x$ E) $x^{-\frac{2}{3}} - x^{-\frac{3}{2}} - x$

$y = x^2 \cdot x^{\frac{1}{2}} = x^{\frac{5}{2}}$

4) If $y = x^2\sqrt{x}$, then $\frac{dy}{x} =$ $\frac{5}{2}x^{3/2}$

A) $2\sqrt{x}$ B) $\frac{3}{2}x\sqrt{x}$ C) $x\sqrt{x}$ D) $2x\sqrt{x}$ E) $\frac{5}{2}x\sqrt{x}$

Questions 5 and 6 refer to the following:

Differentiate the given function.

5) $h(x) = (3 - 2x + x^3)(x^4 + 7)$

$h'(x) = 7x^6 - 10x^4 + 12x^3 + 21x^2 - 14$

$h'(x) = (3-2x+x^3)(4x^3) + (3x^2-2)(x^4+7)$

$= 4x^6 - 8x^4 + 12x^3 + 3x^6 - 2x^4 + 21x^2 - 14$