Name:_________________________________ Date:__________

AP Calculus AB Trig Derivatives Ms. Loughran

Do Now:

1. Find an equation for the normal line to the graph of $y=\dfrac{x+3}{x-3}$ at $x=4$.

$y(4)=\dfrac{4+3}{4-3}=7$

$y'=\dfrac{(x-3)-(x+3)}{(x-3)^2}=\dfrac{-6}{(x-3)^2}$

$y'(4)=\dfrac{-6}{(4-3)^2}=-6$

$y-7=\frac{1}{6}(x-4)$

2. If $y=\left(\dfrac{8x-x^6}{x^3}\right)^{-\frac{4}{5}}$, find $\dfrac{dy}{dx}$.

$y=\left(8x^{-2}-x^3\right)^{-\frac{4}{5}}$

$\dfrac{dy}{dx}=-\frac{4}{5}\left(8x^{-2}-x^3\right)^{-\frac{9}{5}}\cdot\left(-16x^{-3}-3x^2\right)$

$\dfrac{-4\left(-16x^{-3}-3x^2\right)}{5\left(8x^{-2}-x^3\right)^{\frac{9}{5}}}$

$\dfrac{d}{dx}[sinx]=$ $\cos x$

$\dfrac{d}{dx}[\cos x]=$ $-\sin x$

1. $\frac{d}{dx}[tanx] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$

$\cos^2 x + \sin^2 x$

$\frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$

$\frac{1}{\cos^2 x}$

$\sec^2 x$

2. $\frac{d}{dx}[cscx] = \frac{d}{dx}\left[\frac{1}{\sin x}\right]$

$\frac{-\cos x}{\sin^2 x}$

$-\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$

$-\cot x \csc x$

3. $\frac{d}{dx}[secx] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{-(-\sin x)}{\cos^2 x}$

$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$\tan x \sec x$

4. $\frac{d}{dx}[cotx] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right]$

$-(\sin^2 x + \cos^2 x)$

$-\sin^2 x - \cos^2 x$

$\frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x}$

$\frac{-1}{\sin^2 x}$

$-\csc^2 x$

Examples: Find $\frac{dy}{dx}$.

1. $y = x^2 sinx$

$\frac{dy}{dx} = 2x\sin x + x^2\cos x$

2. $y = \frac{cos^2 x}{3x}$

3. $y = x + cotx$

4. $y = \frac{secx}{x}$

5. $y = x^2 tanx$

6. $y = xsinx + cosx$

7. $y = sin^2(\sqrt{x})$

8. $y = \tan^3(9x^2)$

2. $y = \frac{cos^2 x}{3x}$

$\leftarrow$ Double $\measuredangle$ formula

$$\frac{dy}{dx} = \frac{3x \cdot 2\cos x(-\sin x) - \cos^2 x \cdot 3}{(3x)^2}$$

$-3x \quad 2\cos x \sin x$

$$\frac{dy}{dx} = \frac{-3x \sin 2x - 3\cos^2 x}{9x^2}$$

$$\frac{dy}{dx} = \frac{-x \sin 2x - \cos^2 x}{3x^2}$$

3. $y = x + cotx$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

$$\frac{dy}{dx} = 1 - \csc^2 x$$

$$\frac{dy}{dx} = -\cot^2 x$$

4. $y = \frac{secx}{x}$

$$\frac{dy}{dx} = \frac{x \tan x \sec x - \sec x}{x^2}$$

5. $y = x^2 tanx$

$$\frac{dy}{dx} = x^2 \sec^2 x + 2x \tan x$$

6. $y = xsinx + cosx$

$$\frac{dy}{dx} = (1)\cancel{\sin x} + x\cos x \cancel{- \sin x}$$

$$\frac{dy}{dx} = x\cos x$$

7. $y = \sin^2(\sqrt{x})$

$$\frac{dy}{dx} = \cancel{2}\sin\sqrt{x} \cdot \cos\sqrt{x} \cdot \frac{1}{\cancel{2}}x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{\sin\sqrt{x}\,\cos\sqrt{x}}{\sqrt{x}}$$

8. $y = \tan^3(9x^2)$

$$\frac{dy}{dx} = 3\tan^2(9x^2) \cdot \sec^2(9x^2) \cdot 18x$$

$$\frac{dy}{dx} = 54x\tan^2(9x^2)\sec^2(9x^2)$$

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AP Calculus: Derivatives of Sine and Cosine

Ms. Loughran

Answer each of the following questions using the formal definition of a derivative.

1. If $f(x) = \sin x$, find $f'(x)$.

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$\lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$\lim_{h \to 0} \frac{\sin x \cos h - \sin x + \cos x \sin h}{h}$$

$$\lim_{h \to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$\lim_{h \to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h \to 0} \frac{\cos x \sin h}{h}$$

$$\lim_{h \to 0} \sin x \cdot \frac{(\cos h - 1)}{h} + \lim_{h \to 0} \frac{\sin h}{h} \cdot \cos x$$

$$\sin x \cdot 0 \qquad 1 \cdot \cos x$$

$$0 + \cos x = \cos x$$

2. If $f(x) = \cos x$, find $f'(x)$.

$$\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$\lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$\lim_{h \to 0} \frac{\cos x \cos h - \cos x - \sin x \sin h}{h}$$

$$\lim_{h \to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$$

$$\lim_{h \to 0} \cos x \cdot \frac{\cos h - 1}{h} - \lim_{h \to 0} \frac{\sin x \sin h}{h}$$

$$\cos x\ (0) - \sin x\ 1$$

$$0 - \sin x = -\sin x$$

Therefore:

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

Homework 10-04

Name: ____________________ Date: ____________

AP Calc: Chain Rule Homework Ms. Loughran

In 1-6, find $f'(x)$.

1. $f(x)=(x^3+2x)^{37}$

$f'(x)=37(x^3+2x)^{36}\cdot(3x^2+2)$

2. $f(x)=\left(x^3-\frac{7}{x}\right)^{-2}$ $\quad -7x^{-1}$

$f'(x)=-2\left(x^3-\frac{7}{x}\right)^{-3}\cdot(3x^2+7x^{-2})$

3. $f(x)=\dfrac{4}{(3x^2-2x+1)^3}$

$f'(x)=4\left[\dfrac{-3(3x^2\cancel{-2x+1)^2}\cdot(6x-2)}{(3x^2-2x+1)^{\cancel{6}4}}\right]=\dfrac{-12(6x-2)}{(3x^2-2x+1)^4}$

4. $f(x)=\sqrt{x^3-2x+5}$

$f'(x)=\frac{1}{2}(x^3-2x+5)^{-\frac{1}{2}}\cdot(3x^2-2)$ $\quad \dfrac{3x^2-2}{2\sqrt{x^3-2x+5}}$

5. $f(x)=(5x+8)^{13}(x^3+7x)^{12}$

$f'(x)=13(5x+8)^{12}\cdot5\cdot(x^3+7x)^{12}+12(x^3+7x)^{11}\cdot(3x^2+7)(5x+8)^{13}$

$\boxed{f'(x)=(5x+8)^{12}(x^3+7x)^{11}\left(65(x^3+7x)+12(5x+8)(3x^2+7)\right)}$

6. $f(x)=\left(\dfrac{x-5}{2x+1}\right)^3$

$f'(x)=3\left(\dfrac{x-5}{2x+1}\right)^2\cdot\left(\dfrac{(2x+1)-2(x-5)}{(2x+1)^2}\right)$ $\quad -2x+10$

$=3\dfrac{(x-5)^2}{(2x+1)^2}\cdot\dfrac{11}{(2x+1)^2}=\dfrac{33(x-5)^2}{(2x+1)^4}$

7. If $y=\dfrac{1+x}{1-x}$, find $\dfrac{d^2y}{dx^2}$.

$y'=\dfrac{(1-x)(1)-(-1)(1+x)}{(1-x)^2}$ $\quad 1-x+1+x$ $\quad =\dfrac{2}{(1-x)^2}$

$y''=2\left(\dfrac{-2(1-x)(-1)}{(1-x)^4}\right)$ $\quad 2(1-x)$ $\quad =\dfrac{4\cancel{(1-x)}}{(1-x)^{\cancel{4}3}}$

8. Given $f(x)=(2x+7)^6(x-2)^5$, find the values of x at which the curve $y=f(x)$ has a horizontal tangent line.

$f'(x)=(2x+7)^6\cdot5(x-2)^4+(x-2)^5\cdot6(2x+7)^5\cdot2$

$f'(x)=5(2x+7)^6(x-2)^4+12(x-2)^5(2x+7)^5$

$f'(x)=(2x+7)^5(x-2)^4\left[5(2x+7)+12(x-2)\right]$ $\quad 10x+35+12x-24$

$f'(x)=(2x+7)^5(x-2)^4\left[22x+11\right]$

$x=-7/2 \quad x=2 \quad x=-1/2$

6) $y = \frac{1}{t^2} - \frac{1}{\sqrt[3]{t}}$

$y = t^{-2} - t^{-\frac{1}{3}}$

$y' = -2t^{-3} + \frac{1}{3}t^{-2/3} = \frac{-2}{t^3} + \frac{1}{3\sqrt[3]{t^2}}$

8) If f(x) = 3x, then f′(-2) is

A) -6 B) 3 C) 6 D) -3 E) 0

$f'(x) = 3$

$f(x) = -x^{-3} + 2x^{-2} - x^{-1}$ $f'(x) = 3x^{-4} - 4x^{-3} + x^{-2}$

9) If $f(x) = -\frac{1}{x^3} + \frac{2}{x^2} - \frac{1}{x}$, then f′(-1) = $3 + 4 + 1 = 8$

A) 8 B) 6 C) 2 D) 1 E) 0

10) If $f(x) = -2x^{\frac{2}{3}}$, then f′(8) =

A) $\frac{2}{3}$ B) $-\frac{8}{3}$ C) $-\frac{1}{3}$ D) $-\frac{2}{3}$ E) $\frac{1}{3}$

$f'(x) = -\frac{4}{3}x^{-\frac{1}{3}}$

$f'(8) = -\frac{4}{3}(8)^{-\frac{1}{3}} = \frac{-4}{3} \cdot \frac{1}{2} = \frac{-2}{3}$

15) An equation of the line tangent to the graph of $f(x) = \frac{x - 1}{2x + 1}$ at the point (-1,2) is

A) $3x + y = -1$ B) $x + y = 1$ C) $3x - y = 1$

D) $3x - y = -5$ E) $x + 2y = 3$

$$f'(x) = \frac{2x+1 - (x-1)(2)}{(2x+1)^2}$$

$$f'(-1) = \frac{2(-1)+1 - (-1-1)(2)}{(2(-1)+1)^2}$$

$$f'(-1) = \frac{-1+4}{1} = 3$$

$$y-2 = 3(x+1)$$

$$y-2 = 3x+3$$

$$-5 = 3x-y$$