Name: ______________________________________ Date: ________
AP Calc AB Ms. Loughran

Do Now:

1. Find $\frac{d}{dx}\sec^{3}(2x+1)$

$3\sec^{2}(2x+1)\cdot\tan(2x+1)\sec(2x+1)\cdot 2$

$6\sec^{3}(2x+1)\tan(2x+1)$

Homework 10-05

Name: ______________________ Date: __________
AP Calc: Trig Derivatives Homework Ms. Loughran

In 1- 5, find $f'(x)$.

1. $f(x) = 2\cos x - 3\sin x$

$f'(x) = -2\sin x - 3\cos x$

2. $f(x) = \frac{\sin x}{x}$

$f'(x) = \frac{x\cos x - \sin x}{x^2}$

3. $f(x) = x^3 \sin x - 5\cos x$

$f'(x) = x^3\cos x + 3x^2\sin x + 5\sin x$

4. $f(x) = \sec x - \sqrt{2}\tan x$

$f'(x) = \sec x \tan x - \sqrt{2}\sec^2 x$

5. $f(x) = \sin^2 x + \cos^2 x = 1$ or $2\sin x \cos x + 2\cos x(-\sin x)$

$f'(x) = 0$ $2\sin x\cos x - 2\cos x \sin x = 0$

6. If $y = x\cos x$, find $\frac{d^2y}{dx^2}$.

$y' = -x\sin x + \cos x$

$y'' = -x\cos x - \sin x - \sin x$

$y'' = -x\cos x - 2\sin x$

7. Find all points in the interval $[-2\pi, 2\pi]$ at which the graph of f has a horizontal tangent line.

(a) $f(x) = x + \cos x$

$f'(x) = 1 - \sin x$

$1 - \sin x = 0$

$1 = \sin x$

$\pi/2, -\frac{3\pi}{2}$

$(\pi/2, \pi/2)$

$(-3\pi/2, -3\pi/2)$

(b) $y = \tan x$

$y' = \sec^2 x$

$\sec^2 x = 0$

$\sec x = 0$

$\sec x = 0$ where $\cos x$ is undefined

$\cos x$ is never undefined so none

8. If $f(x) = x^{-3} + \frac{1}{x^7}$, find $f'(x)$. x^{-7}

$f'(x) = \frac{-3}{x^4} - \frac{7}{x^8}$

9 on next page

p. 168 – 169 #s 43, 60, 63 and Quick Quiz # 1

43. $f''(x) = x^{-\frac{1}{3}}$ $\boxed{B, C, D}$

A) $f(x) = \frac{3}{2}x^{2/3} - 3$

$f'(x) = x^{-\frac{1}{3}}$

$f''(x) = -\frac{1}{3}x^{-\frac{4}{3}}$

(B) $f(x) = \frac{9}{10}x^{5/3} - 7$

$f'(x) = \frac{5}{3} \cdot \frac{9}{10} x^{2/3} = \frac{3}{2}x^{2/3}$

$f''(x) = x^{-\frac{1}{3}}$

(C) $f'''(x) = -\frac{1}{3}x^{-\frac{4}{3}}$

(D) $f'(x) = \frac{3}{2}x^{2/3} + 6$

$f''(x) = x^{-\frac{1}{3}}$

(60) $y = \sqrt[3]{x} = x^{\frac{1}{3}}$

$y' = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$ True

(63) $y = x^{3/4}$

$y' = 3/4\, x^{-\frac{1}{4}} = \frac{3}{4x^{\frac{1}{4}}}$ E

(9) $f(x) = (x-1)(x-5)^3$

$f'(x) = 3(x-1)(x-5)^2 + (1)(x-5)^3$

$f'(x) = (x-5)^2\left[3\underbrace{(x-1)}^{3x-3} + x-5\right] = (x-5)^2(4x-8)$

$f'(x) = 0$

$(x-5)^2(4x-8) = 0$

$x = 5,\ x = 2$

QQ1

(1) $y = \sin^4(3x)$

$y' = 4\sin^3(3x) \cdot \cos(3x) \cdot 3$

$y' = 12\sin^3(3x) \cdot \cos(3x)$ (B)

From the Practice Multiple Choice for Friday’s exam packet:

7) If $f(x) = \csc x \tan x$, then $f'(x) =$

$f'(x) = -\csc x \cot x \tan x + \csc x \cdot \sec^2 x$

A) $\sec x \tan x$ $\quad = -\csc x \cdot \frac{1}{\cancel{\tan x}} \cdot \cancel{\tan x} + \csc x \cdot \sec^2 x$ $\qquad$ B) $-\dfrac{1}{\sin x}$ $\qquad$ C) $-\sec x \tan x$

D) $\dfrac{1}{\sin^2 x \cos x}$ $\qquad$ E) $-\csc x \cot x$

$= -\csc x + \csc x \sec^2 x$

$= \csc x(-1 + \sec^2 x)$

$\cdot \tan^2 x = \frac{1}{\sin x} \cdot \frac{\sin^2 x}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

11) If $f(x) = \sin x + \cos x$, then $f''(x) =$ $-\sin x - \cos x$

A) $\sin x - \cos x$ $\qquad$ B) $2 \sin x \cos x$ $\qquad$ C) $-(\cos x + \sin x)$

D) $\sin x + \cos x$ $\qquad$ E) $\cos x - \sin x$

$f'(x) = \cos x - \sin x$

12) If $y = \tan x$, then $\dfrac{d^2y}{dx^2}$ at $x = \dfrac{\pi}{4}$ is

$y' = \sec^2 x$

$y'' = 2\sec x \cdot \sec x \tan x$

$y''\left(\frac{\pi}{4}\right) = 2\left(\frac{2}{\sqrt{2}}\right)\left(\frac{2}{\sqrt{2}}\right)(1) = \frac{8}{2}$

A) 4 $\qquad$ B) $2\sqrt{2}$ $\qquad$ C) $4\sqrt{2}$ $\qquad$ D) 2 $\qquad$ E) $\sqrt{2}$

13) If $y = x \cos x$, then $\dfrac{d^2y}{dx^2}$ when $x = \dfrac{\pi}{2}$ is

$y' = -x \sin x + \cos x$

$y'' = -x \cos x - \sin x - \sin x$

$y''\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}(0) - 1 - 1 = -2$

A) 0 $\qquad$ B) 2 $\qquad$ C) -2 $\qquad$ D) π $\qquad$ E) $-\pi$

16) An equation of the line tangent to the graph of $y = \sec x - \tan x$ at the point (0,1) is

A) $y = x + 1$ $\qquad$ B) $y = -x$ $\qquad$ C) $y = -x + 1$

D) $y = 1$ $\qquad$ E) $y = -2x + 1$

$y' = \sec x \tan x - \sec^2 x$ $\qquad$ $y'(0) = (1)(1) - 1^2 = 1 - 1 = -1$

$y - 1 = -1(x - 0)$

$y - 1 = -x$

$y = -x + 1$

17) What is the y-intercept of the line tangent to the graph of the function $f(x) = x^2 + \dfrac{1}{x}$ at $x = 2$?

$f(2) = 2^2 + \frac{1}{2} = \frac{9}{2}$

A) $\dfrac{15}{4}$ $\qquad$ B) 3 $\qquad$ C) $-\dfrac{15}{4}$ $\qquad$ D) -3 $\qquad$ E) $\dfrac{9}{2}$

$f'(x) = 2x - x^{-2}$ $\qquad$ $f'(2) = 2(2) - 2^{-2} = 4 - \frac{1}{4} = \frac{15}{4}$

$y - \frac{9}{2} = \frac{15}{4}(x - 2)$

$4y - 18 = 15x - 30$

$4y = 15x - 12$

$y = \frac{15}{4}x - 3$

18) At what point on the graph of $f(x) = 2x - \dfrac{1}{x}$ is the tangent line parallel to the line $2y - 12x = 7$?

A) $\left(2, \frac{7}{2}\right)$ $\qquad$ B) $\left(-\frac{1}{2}, 2\right)$ $\qquad$ C) (1,1) $\qquad$ D) $\left(-2, \frac{7}{2}\right)$ $\qquad$ E) $\left(\frac{1}{2}, -1\right)$

$2y - 12x = 7$

$2y = 12x + 7$

$y = 6x + \frac{7}{2}$ $\qquad$ $m = 6$

$f'(x) = 2 + x^{-2}$

$2 + \frac{1}{x^2} = 6$

$\frac{1}{x^2} = 4$

$4x^2 = 1$

$x^2 = \frac{1}{4}$ $\quad x = \pm\frac{1}{2}$

$\left(\frac{1}{2}, -1\right)$

---

20) What is the instantaneous rate of change of $f(x) = \dfrac{2x}{x^2 + 1}$ at $x = -1$?

derivative

A) $\dfrac{1}{2}$ $\qquad$ B) 1 $\qquad$ C) -1 $\qquad$ D) $-\dfrac{1}{2}$ $\qquad$ E) 0

$$f'(x) = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2}$$

$$f'(-1) = \frac{2((-1)^2+1) - 2(-1)(2(-1))}{((-1)^2+1)^2} = \frac{4-4}{4} = \frac{0}{4} = 0$$