Name: ____ Date: __________

AP Calculus Ms. Loughran

Do Now:

1. Find $\frac{dy}{dx}$ if $y = \tan(4x)$.

$$\frac{dy}{dx} = \sec^2(4x) \cdot 4$$

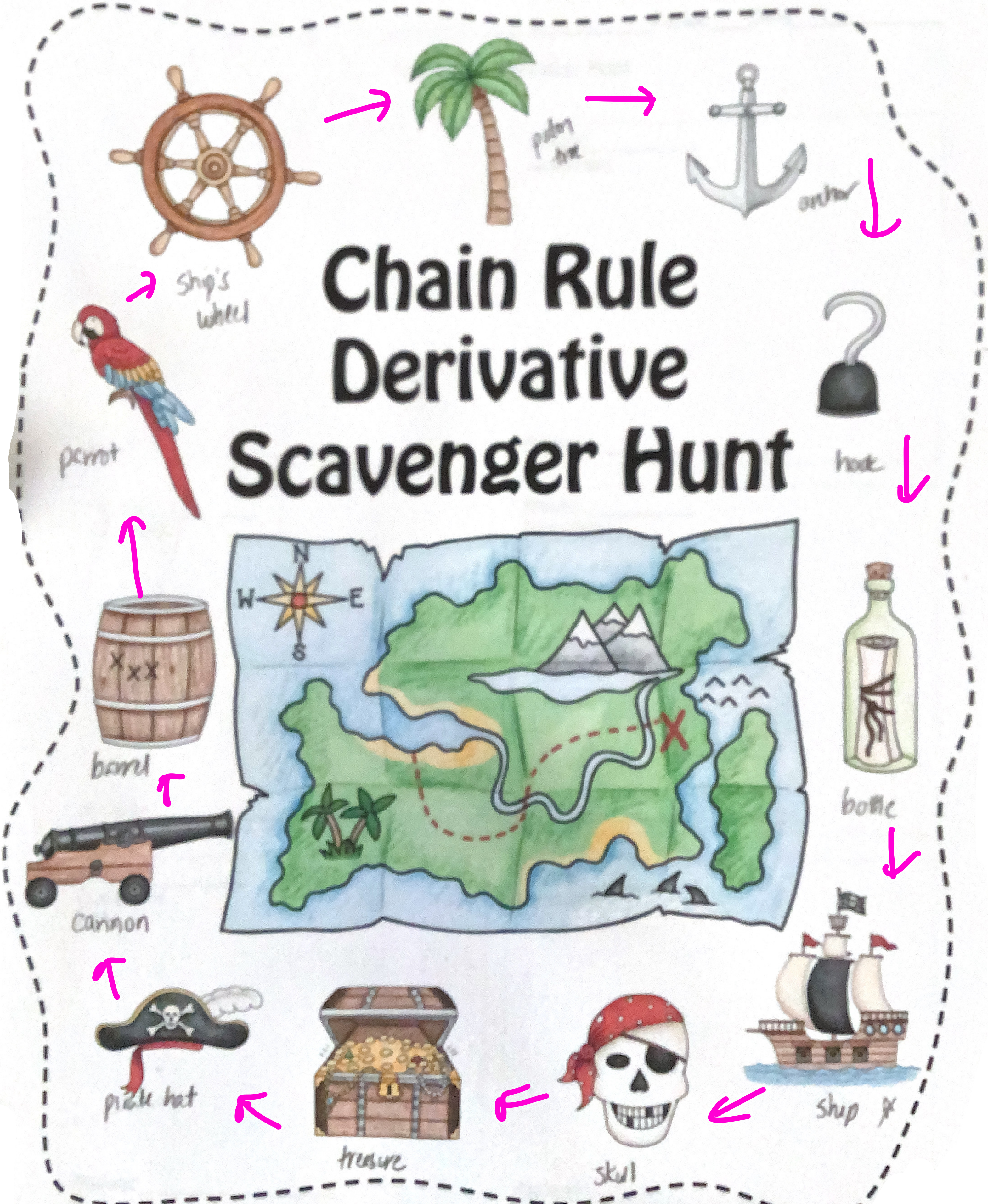
$$\frac{dy}{dx} = 4\sec^2(4x)$$

2. Find $\frac{dy}{dx}$ if $y = \cos^2(x^3 + x^2)$.

$$\frac{dy}{dx} = 2\cos(x^3+x^2) \cdot (-\sin(x^3+x^2))(3x^2+2x)$$

$$\frac{dy}{dx} = (-6x^2-4x)\cos(x^3+x^2)\sin(x^3+x^2)$$

Chain Rule Derivative Scavenger Hunt



Homework 10-10

Name: Key

AP Calculus AB: More Trig Derivatives Homework

Date:

Ms. Loughran

In 1 – 12, find $\frac{dy}{dx}$.

1. $y=\tan(4x^2)$ $\quad y' = \sec^2(4x^2)\cdot 8x = 8x\sec^2(4x^2)$

2. $y=\cos^2(3\sqrt{x})$ $\quad y' = \cancel{2}\cos(3\sqrt{x})\cdot -\sin(3\sqrt{x})\cdot \frac{3}{\cancel{2}}x^{-\frac{1}{2}} = \frac{-3\cos(3\sqrt{x})\sin(3\sqrt{x})}{\sqrt{x}}$

3. $y=3\cot^4 x$ $\quad y' = 3\cdot 4\cot^3 x\cdot -\csc^2 x = -12\cot^3 x\csc^2 x$

4. $y=4\cos^5 x$ $\quad y' = 20\cos^4 x\cdot(-\sin x) = -20\cos^4 x\sin x$

5. $y=\csc(x^3)$ $\quad y' = -\csc(x^3)\cot(x^3)\cdot 3x^2 = -3x^2\csc(x^3)\cot(x^3)$

6. $y=\sin\left(\frac{1}{x^2}\right)$ $\quad y' = \cos\left(\frac{1}{x^2}\right)\cdot -2x^{-3} = \frac{-2\cos\left(\frac{1}{x^2}\right)}{x^3}$

$y=\sin(x^{-2})$

7. $y=\tan^4(x^3)$ $\quad y' = 4\tan^3(x^3)\cdot\sec^2(x^3)\cdot 3x^2 = 12x^2\tan^3(x^3)\sec^2(x^3)$

8. $y=2\sec^2(x^7)$ $\quad y' = 4\sec(x^7)\cdot\sec(x^7)\tan(x^7)\cdot 7x^6 = 28x^6\sec^2(x^7)\tan(x^7)$

9. $y=\cos^3\left(\frac{x}{x+1}\right)$ $\quad y' = 3\cos^2\left(\frac{x}{x+1}\right)\cdot -\sin\left(\frac{x}{x+1}\right)\cdot\frac{\cancel{x}+1-\cancel{x}}{(x+1)^2} = \frac{-3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)}{(x+1)^2}$

10. $y=\sqrt{\cos(5x)}$ $\quad y' = \frac{1}{2}(\cos(5x))^{-\frac{1}{2}}\cdot -\sin(5x)\cdot 5 = \frac{-5\sin(5x)}{2\sqrt{\cos(5x)}}$

11. $y=x^2\sqrt{5-x^2}$ $\quad y' = x^2\cdot\frac{1}{\cancel{2}}(5-x^2)^{-\frac{1}{2}}\cdot -\cancel{2}x + 2x\sqrt{5-x^2}$

$y=x^2\cdot(5-x^2)^{\frac{1}{2}}$

$$\frac{-x^3}{\sqrt{5-x^2}} + \frac{2x\sqrt{5-x^2}}{1}\cdot\frac{\sqrt{5-x^2}}{\sqrt{5-x^2}}$$

$$\frac{-x^3+2x(5-x^2)}{\sqrt{5-x^2}}$$

12. $y=\frac{(2x+3)^3}{(4x^2-1)^8}$

on next page

13. Given $y=x\cos(5x)-\sin^2 x$, find $\frac{d^2y}{dx^2}$.

14. Find an equation for the tangent line to the graph of $y=x\cos 3x$ at $x=\pi$.

(12) $y = \frac{(2x+3)^3}{(4x^2-1)^8} = \frac{(4x^2-1)^8 \cdot 3(2x+3)^2 \cdot 2 - (2x+3)^3 \cdot 8(4x^2-1)^7 \cdot 8x}{((4x^2-1)^8)^2}$

$$\frac{6(4x^2-1)^{\cancel{8}}(2x+3)^2 - 64x(2x+3)^3 \cancel{(4x^2-1)^7}}{(4x^2-1)^{\cancel{16}\ 9}}$$

$-52x^2-96x-3$

$12x^2-3-64x^2-96x$

$$\frac{2\cancel{(4x^2-1)^7}(2x+3)^2\left[3(4x^2-1)-32x(2x+3)\right]}{(4x^2-1)^{\cancel{16}\ 9}}$$

(13) $y = x\cos(5x) - \sin^2 x$

$y' = x \cdot -\sin(5x) \cdot 5 + \cos(5x) - 2\sin x \cos x$

$y' = -5x\sin(5x) + \cos(5x) - 2\sin x\cos x$

$y' = -5x\sin(5x) + \cos(5x) - \sin 2x$

$y'' = -5x \cdot \cos(5x) \cdot 5 + \sin(5x) \cdot -5 - 5\sin(5x) - 2\cos(2x)$

$y'' = -25x\cos(5x) - 5\sin(5x) - 5\sin 5x - 2\cos(2x)$

$y'' = -25x\cos(5x) - 10\sin(5x) - 2\cos(2x)$

(14) $y = x\cos(3x)$

$y' = -3x\sin(3x) + \cos(3x)$

$y(\pi) = \pi\cos(3\pi) = -\pi$

$y'(\pi) = -3\pi\sin(3\pi) + \cos(3\pi)$

$= -3\pi(0) + -1 = -1$

$y + \pi = -1(x-\pi)$

$y + \pi = -x + \pi$

$y = -x$

Some Practice Key

Set A

① $y = \frac{x+3}{x^2+1}$ at $x=1$

at $x=1$, $y = \frac{1+3}{1^2+1} = \frac{4}{2} = 2$

$$y' = \frac{(x^2+1) - (x+3)(2x)}{(x^2+1)^2}$$

$$y'\Big|_{x=1} = \frac{2-(4)(2)}{4} = \frac{2-8}{4} = \frac{-6}{4} = \frac{-3}{2}$$

$$y - 2 = -3/2(x-1)$$

$$y - 2 = -3/2x + 3/2$$

$$y = -3/2x + 7/2 \quad (E)$$

② $f(x) = \frac{x^2-2}{x+1}$

$$f'(x) = \frac{(x+1)(2x) - (x^2-2)}{(x+1)^2}$$

$$f'(3) = \frac{4(6) - 7}{16} = \frac{17}{16} \quad (E)$$

③ $\lim_{x\to 1} \frac{\ln x}{3x}$

$$\frac{\ln 1}{3(1)} = \frac{0}{3} = 0 \quad (A)$$

④ $f(x) = x\sqrt{4x-1} = x(4x-1)^{\frac{1}{2}}$

$$f'(x) = 1(4x-1)^{\frac{1}{2}} + \tfrac{1}{2}(4x-1)^{-\frac{1}{2}}(4)x$$

$$f'(x) = \frac{\sqrt{4x-1}}{1} + \frac{2x}{\sqrt{4x-1}}$$

$$\frac{4x-1+2x}{\sqrt{4x-1}} = \frac{6x-1}{\sqrt{4x-1}}$$

(A)

(5) $y = -\sqrt{x+4} = -(x+4)^{\frac{1}{2}}$

$y' = -\frac{1}{2}(x+4)^{-\frac{1}{2}}(1) = -\frac{1}{2\sqrt{x+4}}\Big|_{x=0}$ $\quad -\frac{1}{4} = m_{tan}$

$m_{normal} = 4$ (E)

(6) $4x - 8y = 5$

$4x - 5 = 8y$

$\frac{1}{2}x - \frac{5}{8} = y$

$m = \frac{1}{2}$

$y = \frac{1}{2}x^2 - \frac{3}{2}$

$y' = x$

$x = \frac{1}{2}$

(B)

$y = \frac{1}{2}\left(\frac{1}{2}\right)^2 - \frac{3}{2}$

$y = \frac{1}{8} - \frac{3}{2} = \frac{1}{8} - \frac{12}{8} = -\frac{11}{8}$

(7) $f(x) = -x^5 + x + \frac{1}{x^2}$, $f'(-1)$

$f(x) = -x^5 + x + x^{-2}$

$f'(x) = -5x^4 + 1 - 2x^{-3}$

$f'(-1) = -5(-1)^4 + 1 - 2(-1)^{-3}$

$= -5 + 1 + 2$

$= -2$ (C)

* (8) $\lim_{h \to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$

$f(x) = 8x^8$ $\quad x = \frac{1}{2}$

$f'(x) = 64x^7$

$f'\left(\frac{1}{2}\right) = 64\left(\frac{1}{2}\right)^7$

$\frac{64}{128} = \frac{1}{2}$ (B)