① Find the set of x values at which the graph of $y = x^4 - 6x^2 + 4$ where the tangent lines are horizontal.

$$y' = 4x^3 - 12x$$

$$4x^3 - 12x = 0$$

$$4x(x^2 - 3) = 0$$

$x = 0$ | $x = \pm\sqrt{3}$

② Find $\left.\frac{d^3y}{dx^3}\right|_{x=2}$ given $y = \frac{5}{x} + 1 = 5x^{-1} + 1$

$$\frac{dy}{dx} = -5x^{-2}$$

$$\frac{d^2y}{dx^2} = 10x^{-3}$$

$$\frac{d^3y}{dx^3} = -30x^{-4}$$

$$\left.\frac{d^3y}{dx^3}\right|_{x=2} = \frac{-30}{2^4} = \frac{-30}{16} = \frac{-15}{8}$$

③ Find an equation of the normal line to the graph of $f(x)=\frac{x}{2x-3}$ at $x=1$.

$f(1)=\frac{1}{2(1)-3}=\frac{1}{-1}=-1$ $(1,-1)$

$$f'(x)=\frac{(2x-3)(1)-x(2)}{(2x-3)^2}$$

$f'(1)=\frac{-1-2}{(-1)^2}=-3$ $m_{norm}=\frac{1}{3}$

$y+1=\frac{1}{3}(x-1)$

④ If $f(2)=7$ and $f'(2)=3$, write an equation of the tangent line to $y=f(x)$ at $x=2$.

$$y-7=3(x-2)$$

⑤ If $f(x) = (x-4)(x^2+3x)^3$, find $f'(x)$

$$f'(x) = 3(x-4)(x^2+3x)^2(2x+3) + (1)(x^2+3x)^3$$

$$f'(x) = (x^2+3x)^2\left(3\overset{2x^2-5x-12}{\underbrace{(x-4)(2x+3)}} + x^2+3x\right)$$

$$f'(x) = (x^2+3x)^2\left(6x^2-15x-36+x^2+3x\right)$$

$$f'(x) = (x^2+3x)^2\left(7x^2-12x-36\right)$$

⑥ If the line tangent to the graph of f at the point $(2,5)$ passes through $(3,7)$, find the value of $f'(2)$.

↑ slope of tangent line at $x=2$

$$m = \frac{7-5}{3-2} = \frac{2}{1} = 2$$

$$f'(2) = 2$$

⑦ If $y = \frac{2}{x^2 \sqrt[4]{x}}$, find y'.

$$y = \frac{2}{x^2 \cdot x^{\frac{1}{4}}} = \frac{2}{x^{\frac{9}{4}}} = 2x^{-\frac{9}{4}}$$

$$y' = -\frac{18}{4} x^{-\frac{13}{4}} = \frac{-9}{2\sqrt[4]{x^{13}}} = \frac{-9}{2\sqrt[4]{x^{12}}\sqrt[4]{x}}$$

$$\frac{-9}{2x^3\sqrt[4]{x}}$$

⑧ Find an equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point $(1,-1)$

$$f'(x) = 3x(1-2x)^2 \cdot (-2) + (1)(1-2x)^3$$

$$f'(1) = 3(1)(1-2(1))^2 \cdot (-2) + 1(1-2(1))^3$$

$$f'(1) = -6 + (-1) = -7$$

$$y+1 = -7(x-1)$$

⑨ If $y = \cos x + \tan x$, find y''.

$$y' = -\sin x + \sec^2 x$$

$$y'' = -\cos x + 2\sec x \cdot \sec x \tan x$$

$$y'' = -\cos x + 2\sec^2 x \tan x$$

$(5x-6)^{\frac{1}{2}}$

⑩ If $f(x) = x^2\sqrt{5x-6}$, find $f'(x)$

$$f'(x) = \frac{1}{2}x^2(5x-6)^{-\frac{1}{2}} \cdot 5 + 2x\sqrt{5x-6}$$

$$f'(x) = \frac{5x^2}{2\sqrt{5x-6}} + \left(\frac{2x\sqrt{5x-6}}{1}\right)\left(\frac{2\sqrt{5x-6}}{2\sqrt{5x-6}}\right)$$

$20x^2 - 24x$

$$f'(x) = \frac{5x^2 + 4x(5x-6)}{2\sqrt{5x-6}} = \frac{25x^2 - 24x}{2\sqrt{5x-6}}$$

(11) Find the fourth derivative of $f(x) = (2x-3)^4$.

$$f'(x) = 4(2x-3)^3 \cdot 2 = 8(2x-3)^3$$

$$f''(x) = 24(2x-3)^2 \cdot 2 = 48(2x-3)^2$$

$$f'''(x) = 96(2x-3) \cdot 2 = 192(2x-3)$$

$$f^{iv}(x) = 192 \cdot 2 = 384$$

(12) Find an equation of the line normal to the graph of $y = \sqrt{3x^2+2x}$ at $(2,4)$.

$$y' = \frac{1}{2}(3x^2+2x)^{-\frac{1}{2}} \cdot (6x+2)$$

$$y'(2) = \frac{1}{2}\left(3(2)^2+2(2)\right)^{-\frac{1}{2}} \cdot (6(2)+2)$$

$$y'(2) = \frac{1}{2}\left(\frac{1}{4}\right) \cdot 14 = \frac{14}{8} = \frac{7}{4}$$ ← slope of tan line at $x=2$

$m_{normal} = -\frac{4}{7}$

$$7\left(y-4 = \frac{-4}{7}(x-2)\right)$$

$$7y - 28 = -4(x-2)$$

$$7y - 28 = -4x + 8$$

$$4x + 7y = 36$$

(13) For what non negative value of b is the line given by $y=-\frac{1}{3}x+b$ normal to the curve $y=x^3$.

$m_{normal} = -\frac{1}{3}$

$m_{tan} = 3$

$(1,1)$

$1 = -\frac{1}{3}(1) + b$

$\frac{4}{3} = b$

$y' = 3x^2$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$(1, 1^3) \to (1,1)$

$(-1, (-1)^3) \quad (-1,-1)$

(14) Differentiate:

$$y = -4\csc^3(2x^2+4x)$$

$y' = -12\csc^2(2x^2+4x)\left(-\cot(2x^2+4x)\right)\left(\csc(2x^2+4x)\right)\cdot(4x+4)$

$y' = 12(4x+4)\csc^3(2x^2+4x)\cot(2x^2+4x)$

$y' = (48x+48)\csc^3(2x^2+4x)\cot(2x^2+4x)$