$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

Name: ______________________________ Date: __________

AP Calc: Derivatives of e^u and $\ln u$ Ms. Loughran

Do Now:

Evaluate each:

1. $\lim_{h\to 0} \frac{\tan\left(\overset{x+h}{\frac{\pi}{6}+h}\right)-\tan\left(\overset{x}{\frac{\pi}{6}}\right)}{h} = f'\left(\frac{\pi}{6}\right)$ $f(x) = \tan x$ $f'(x) = \sec^2 x$

   $f'\left(\frac{\pi}{6}\right) = \sec^2\left(\frac{\pi}{6}\right) = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$

2. $\lim_{h\to 0} \frac{5\left(\frac{1}{2}+h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$ $f(x) = 5x^4$ $f'\left(\frac{1}{2}\right) = 20\left(\frac{1}{2}\right)^3 = \frac{20}{8} = \frac{5}{2}$

   $f'(x) = 20x^3$

3. $\lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$ $f(x) = \sin x$

   $f'(x) = \cos x$

Using calculator,

$\frac{d}{dx}e^x = e^x$ $\frac{d}{dx}\ln x = \frac{1}{x}$, $x > 0$

In general:

$\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$

$e^u \cdot u'$

$\frac{d}{dx}\ln u = \frac{1}{u} \cdot \frac{du}{dx}$

$\frac{1}{u} \cdot u'$

Examples.

Find $\frac{dy}{dx}$ for each of the following.

1. $y = e^{(x+x^2)}$

$\frac{dy}{dx} = e^{x+x^2} \cdot (1+2x)$

2. $y = e^{\sqrt{x}}$

$$\frac{dy}{dx} = e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

3. $y = \ln(\overset{u}{x^2})$

$$\frac{dy}{dx} = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$$

4. $y = e^{-\frac{x}{4}} \rightarrow e^{-\frac{1}{4}x}$

$$\frac{dy}{dx} = e^{-\frac{x}{4}} \cdot -\frac{1}{4} = \frac{-1}{4e^{\frac{x}{4}}}$$

5. $y = \ln\overset{u}{\left(\frac{1}{x}\right)}$

$$\frac{1}{\frac{1}{x}}$$

$$\uparrow$$

$$\frac{dy}{dx} = x \cdot -x^{-2} = x \cdot -\frac{1}{x^2} = \frac{-1}{x}$$

6. $y = x^2e^x - xe^x$

$y = e^x(x^2 - x)$

$\frac{dy}{dx} = e^x(2x-1) + e^x(x^2-x)$

$\frac{dy}{dx} = e^x(2x-1+x^2-x)$

$\frac{dy}{dx} = e^x(x^2+x-1)$

7. $y = \ln(\overset{u}{\ln x})$

$\frac{dy}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x\ln x}$

8. $y = (\ln x)^2$

$\frac{dy}{dx} = 2\ln x \cdot \frac{1}{x} = \frac{2\ln x}{x}$

9. $y = x\ln x - x$

$\frac{dy}{dx} = \cancel{x} \cdot \frac{1}{\cancel{x}} + \ln x(1) - 1$

$\frac{dy}{dx} = 1 + \ln x - 1 = \ln x$

10. $y = \ln(\overset{u}{2 - \cos x})$

$$\frac{dy}{dx} = \frac{1}{2-\cos x} \cdot -(-\sin x) = \frac{\sin x}{2-\cos x}$$

11. $y = \ln(\overset{u}{x^2 + 1})$

$$\frac{dy}{dx} = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$$

12. $y = x\overset{\text{constant}}{e^2} - e^x$

$$\frac{dy}{dx} = e^2 - e^x$$

13. $y = \ln\left(\dfrac{x^2 \sin x}{\sqrt{1+x}}\right)$

* $\ln(AB) = \ln A + \ln B$
* $\ln\left(\frac{A}{B}\right) = \ln A - \ln B$
* $\ln A^B = B \ln A$

$$y = \ln(x^2 \sin x) - \ln\sqrt{1+x}$$

$$y = \ln x^2 + \ln \sin x - \ln(1+x)^{\frac{1}{2}}$$

$$y = 2\ln x + \ln \overset{u}{\sin x} - \frac{1}{2}\ln(\overset{u}{1+x})$$

$$\frac{dy}{dx} = \frac{2}{x} + \frac{1}{\sin x} \cdot \cos x - \frac{1}{2} \cdot \frac{1}{1+x} \cdot 1$$

$$\frac{dy}{dx} = \frac{2}{x} + \frac{\cos x}{\sin x} - \frac{1}{2} \cdot \frac{1}{1+x}$$

$$\frac{dy}{dx} = \frac{2}{x} + \cot x - \frac{1}{2(1+x)}$$