Name: ____ Date: __________

AP Calculus AB Ms. Loughran

Do Now:

1. Given $y = (1 + \sin^3(x^5))^{12}$, find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 12(1+\sin^3(x^5))^{11} \cdot (3\sin^2(x^5)) \cdot (\cos(x^5)) \cdot 5x^4$$

$$\frac{dy}{dx} = 180x^4(1+\sin^3(x^5))^{11} \cdot (\sin^2(x^5)\cos(x^5))$$

2. Given $f(x) = e^x - x$, find $f'(\ln 3)$.

$$f'(x) = e^x - 1$$

$$f'(\ln 3) = e^{\ln 3} - 1 = 3 - 1 = 2$$

Name: ______________________________ Date: ________________

AP Calc AB: Motion Introduction Ms. Loughran

A **position equation** is: an equation that mathematically models something in real life, specifically it gives the position of an object at specified times

Notation:

$x(t)$ or $s(t)$

By plugging in values of t, you can determine where the object is at that moment in time

$x(0)$ or $s(0)$ is often called initial position

Remember that the derivative describes the rate of change of a function. Therefore $s'(t)$ represents the velocity, $v(t)$ of the object in question at any given instant. Velocity combines an object's speed with its direction. Therefore velocity can be negative but speed can not. Speed is the absolute value of velocity.

(In a vertical motion problem, negative velocity means the object is down.
In a horizontal motion problem, negative velocity means the object is moving to the left.)

If you want an object's **average velocity, (average rate of change)** this value comes from: change in position over change in time

Acceleration is the rate of change of velocity. It is the second derivative of the position equation. The derivative of acceleration, rate of change of acceleration, is called the "jerk."

Recap:

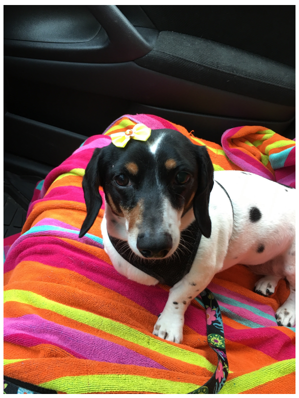
$s(t)$ is position

$s'(t) = v(t)$ is velocity

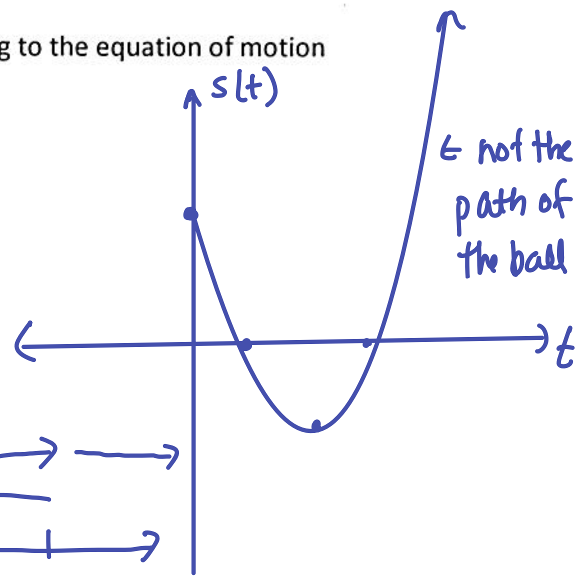
$s''(t) = a(t)$ or $v'(t)$ is acceleration

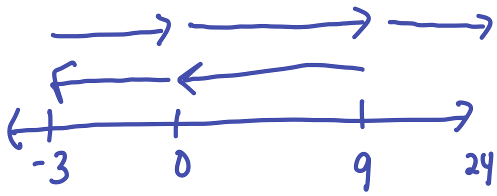
$s'''(t) = a'(t)$ or $v''(t)$ is jerk

1. My dog, Molly, is pushing a ball along a straight wall according to the equation of motion
$s(t)=3(t^2-4t+3)=s(t-1)(t-3)$
$s(t) = 3t^2 - 12t + 9$, where s is in feet and t is in seconds.



$s(0) = 9$
$s(1) = 0$
$s(2) = -3$
$s(3) = 0$
$s(4) = 9$
$s(5) = 24.$





(a) Find the average velocity on the interval [1,3].

$$\frac{s(3)-s(1)}{3-1} = \frac{0-0}{2} = 0 \text{ ft/s}$$

(b) Find the instantaneous velocity at t = 1 and at t = 3.

$v(t) = s'(t)$
$v(t) = 6t - 12$

$v(1) = 6(1) - 12 = -6$ ft/s
$v(3) = 6(3) - 12 = 6$ ft/s

(c) When is the particle at rest?

$v(t) = 0$

$6t - 12 = 0$

$t = 2$ sec.

2. Laura drops a stone off a cliff from a height of 256 feet. (When an object falls from rest, its equation of motion is $s = -16t^2$, with s in feet and t in seconds.)

(a) Find the velocity of the stone when t = 1 and t = 2.

$v(t) = -32t$

$v(1) = -32(1) = -32$ ft/s

$v(2) = -32(2) = -64$ ft/s

(b) How long does it take for the stone to reach the ground?

$-16t^2 = -256$

$t^2 = 16$

$t = \cancel{\pm}4$

$t = 4$ sec.

(c) Find the velocity of the stone when it reaches the ground. (impact velocity)

$v(4) = -32(4) = -128$ ft/se

Homework 10-17

* This is the key to the entire chain rule charts packet. I highlighted the questions that were hw last night *

Topic: Chain Rule Charts

$h(0) = f(g(0)) = f(2) = 0$
$h(1) = f(g(1)) = f(0) = 1$
$h(2) = f(g(2)) = f(3) = 2$
$h(3) = f(g(3)) = f(1) = 3$

) $h(x) = f(g(x))$
$h'(x) = g'(x) \cdot f'(g(x))$

So $h'(0) = g'(0) \cdot f'(g(0))$
$-5 \cdot f'(2)$
$-5 \cdot 2 = -10$

$h'(1) = g'(1) \cdot f'(g(1))$
$= 1 \cdot f'(0)$
$= 1 \cdot 5 = 5$

$h'(2) = g'(2) \cdot f'(g(2))$
$= 1 \cdot f'(3)$
$= 1 \cdot 4 = 4$

$h'(3) = g'(3) \cdot f'(g(3))$
$= -6 \cdot f'(1)$
$= -6 \cdot (-2) = 12$

$k(x) = g(f(x))$

$k'(x) = f'(x) \cdot g'(f(x))$

$k(0) = g(f(0)) = g(1) = 0$
$k(1) = g(f(1)) = g(3) = 1$
$k(2) = g(f(2)) = g(0) = 2$
$k(3) = g(f(3)) = g(2) = 3$

$k'(0) = f'(0) \cdot g'(f(0))$
$= 5 \cdot g'(1)$
$= 5 \cdot 1 = 5$

$k'(1) = f'(1) \cdot g'(f(1))$
$= -2 \cdot g'(3)$
$= -2 \cdot -6 = 12$

$k'(2) = f'(2) \cdot g'(f(2))$
$= 2 \cdot g'(0)$
$= 2 \cdot -5 = -10$

$k'(3) = f'(3) \cdot g'(f(3))$
$= 4 \cdot g'(2)$
$= 4 \cdot 1 = 4$

(2) at $x=2$

(a) $[f(x)+g(x)]' = f'(x) + g'(x)\Big|_{x=2} = f'(2) + g'(2) =$
$2 + 1 = 3$

(b) $[f(x) \cdot g(x)]' = f'(x)g(x) + g'(x)f(x) = f'(2)g(2) + g'(2)f(2) = 2 \cdot 3 + 1 \cdot 0$
$= 6$

(c) $\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} = \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} = \frac{3 \cdot 2 - 0 \cdot 1}{3^2}$
$= 6/9 = 2/3$

(d) $[f(g(x))]' = g'(x) \cdot f'(g(x))$

$h'(x) = h'(2) = 4$

3) (a) $(f+g)'(2) = f'(2) + g'(2) = 9 + 3 = 12$

(b) $\left(\frac{f}{g}\right)'(1) = \frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2} = \frac{2\cdot 4 - 2\cdot 5}{2^2} = \frac{8-10}{4} = \frac{-2}{4} = \frac{-1}{2}$

(c) $\frac{d}{dx} f(g(x))$ at $x=1$

$f'(g(x)) \cdot g'(x)\Big|_{x=1}$ $f'(g(1)) \cdot g'(1) = f'(2) \cdot 5 = 9 \cdot 5 = 45$

(d) $(g\circ f)'(1)$

$(g(f(x)))' = g'(f(x)) \cdot f'(x)\Big|_{x=1}$ $g'(f(1)) \cdot f'(1) = g'(2) \cdot 4 = 3 \cdot 4 = 12$

(e) $[f^2(x)]' = 2f(x) \cdot f'(x)\Big|_{x=2}$ $2f(2) \cdot f'(2) = 2 \cdot 5 \cdot 9 = 90$

) $[f(x^2)]'$ at $x=1$ $= f'(x^2) \cdot 2x$

$f'(1) \cdot 2(1) = 4 \cdot 2 = 8$

) $(f\circ f)'(1)$

$(f(f(x)))' = f'(f(x)) \cdot f'(x)\Big|_{x=1}$ $f'(f(1)) \cdot f'(1) = f'(2) \cdot 4 = 9 \cdot 4 = 36$

) (a) $g(x) = f(x^2)$

$g'(x) = f'(x^2) \cdot 2x$

$g'(x) = 2x f'(x^2)$

(b) $g(x) = f(\sin^2 x) + f(\cos^2 x)$

$g'(x) = f'(\sin^2 x) \cdot 2\sin x \cdot \cos x +$

$f'(\cos^2 x) \cdot -2\cos x \sin x$

$g'(x) = 2\sin x \cos x f'(\sin^2 x) - 2\sin x \cos x f'(\cos^2 x)$

(c) $g(x) = f(f(x))$

$g'(x) = f'(f(x)) \cdot f'(x)$

(d) $g(x) = f(f(f(x)))$

$g'(x) = f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x)$

Chain Rule Problems

(1) $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$

$f'(2)g(2) + g'(2)f(2)$

$1(7) + -3(3)$

$7 - 9 = -2$

(2) $\frac{d}{dx}[f^2(x)g(x)] = [f^2(x)]' g(x) + g'(x) f^2(x)$

$2f(x)f'(x)g(x) + g'(x)f^2(x)$

$2f(2)f'(2)g(2) + g'(2)f^2(2)$

$2 \cdot 3 \cdot 1 \cdot 7 + -3 \cdot (3)^2$

$42 - 27 = 15$

(3) $\frac{d}{dx}[f^3(x)] = 3f^2(x) \cdot f'(x)$

$3 \cdot f^2(2) \cdot f'(2)$

$3 \cdot (3)^2 \cdot 1 = 27$

④ $\frac{d}{dx}\left[3f(x) - g^3(x)\right]$

$3 \cdot f'(x) - 3g^2(x) \cdot g'(x)$

$3 \cdot f'(2) - 3\,g^2(2) \cdot g'(2)$

$3 \cdot 1 - 3(7)^2 \cdot -3$

$3 - 3(49)(-3) = 3 + 441 = 444$

⑤ $d/dx\left[\frac{2g(x) - f(x)}{f^2(x)}\right] = \frac{f^2(x)\left[2g(x) - f(x)\right]' - (2g(x) - f(x)) \cdot 2f(x) \cdot f'(x)}{(f^2(x))^2}$

$= \frac{f^2(x)(2g'(x) - f'(x)) - (2g(x) - f(x))2f(x) \cdot f'(x)}{f^4(x)}$

$\frac{f^2(2)(2g'(2) - f'(2)) - (2g(2) - f(2)) \cdot 2f(2) \cdot f'(2)}{f^4(2)}$

$\frac{(3)^2(2 \cdot (-3) - 1) - (2 \cdot 7 - 3) \cdot 2(3) \cdot 1}{(3)^4}$ ✓

$\frac{9(-7) - 11(6)}{81} = \frac{-63 - 66}{81} = \frac{-129}{81}$

⑥ $\frac{d}{dx}\left(f(x) / 3g(x)\right) = \frac{3g(x) \cdot f'(x) - f(x)\,3g'(x)}{(3g(x))^2}$

$= \frac{3g(2) \cdot f'(2) - (f(2) \cdot 3g'(2))}{9g^2(2)} = \frac{3 \cdot 7 - (3 \cdot 3 \cdot -3)}{9(7)^2}$

$= \frac{21 + 27}{441} = \frac{48}{441} = \frac{16}{147}$

$\begin{array}{r} 49 \\ \underline{\times 9} \\ 441 \end{array}$

8) $h(x) = f(u(x)) = f'(u(x)) \cdot u'(x)$

$f'(u(2)) \cdot u'(2)$

$f'(3) \cdot 7$

$-2 \cdot 7 = -14$

9) $dy/dt\,(u^2(t)+1)^3 = 3(u^2(t)+1)^2 \cdot 2u(t) \cdot u'(t)$ ✓

$= 3(u^2(3)+1)^2 \cdot 2u(3) \cdot u'(3)$

$= 3((-2)^2+1)^2 \cdot 2(-2) \cdot -1$

$= 3(25)(-4)(-1) = 300$

10) $h'(x) = g'(f(x)) \cdot f'(x) \Big|_{x=\frac{1}{4}\pi}$

$g'(f(\pi/4)) \cdot f'(\pi/4)$

$g'(\sqrt{2}/2) \cdot \cos \pi/4$

$2(\sqrt{2}/2) \cdot \sqrt{2}/2 = \sqrt{2} \cdot \frac{\sqrt{2}}{2} = 2/2 = 1$

$h(\pi/4) = g(f(\pi/4))$

$g(\sqrt{2}/2) = (\sqrt{2}/2)^2 = \frac{1}{2}$

$g(x) = x^2$

$g'(x) = 2x$

$y - \frac{1}{2} = 1(x - \pi/4)$

$y = x - \pi/4 + 1/2$

$f'(x) = \frac{1}{x}$

$g'(x) = 2x$

12) $h'(x) = f'(g(x)) \cdot g'(x) = \frac{1}{g(x)} \cdot 2x$

$= \frac{1}{g(7)} \cdot 2(7)$

$\frac{1}{45} \cdot 14 = \frac{14}{45}$

p.159 #56,

(56) (a) $2f'(x)\Big|_{x=2}$ $\quad 2\cdot f'(2) = 2\cdot\frac{1}{3} = \frac{2}{3}$

(b) $f'(x)+g'(x)\Big|_{x=3}$ $\quad f'(3)+g'(3) = 2\pi+5$

(c) $f'(x)g(x)+g'(x)f(x)\Big|_{x=3}$ $\quad f'(3)g(3)+g'(3)f(3) = 2\pi(-4)+5(3)$

$-8\pi+15$

d) $\dfrac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}\Bigg|_{x=2}$ $\quad \dfrac{g(2)f'(2)-f(2)g'(2)}{(g(2))^2} = \dfrac{2(\frac{1}{3})-(8)(-3)}{(2)^2}$ $\quad \frac{24}{\frac{3}{72}}$

$= \dfrac{\frac{2}{3}+24}{4} = \dfrac{2+72}{12}$

$= \dfrac{74}{12} = \dfrac{37}{6}$

e) $(f(g(x)))'\Big|_{x=2}$

$f'(g(x))\cdot g'(x)$

$f'(\overset{2}{g(2)})\cdot g'(2) = \frac{1}{3}\cdot(-3) = -1$

(f) $\left((f(x))^{\frac{1}{2}}\right)' = \frac{1}{2}f(x)^{-\frac{1}{2}}\cdot f'(x)\Big|_{x=2}$ $\quad \frac{1}{2}f(2)^{-\frac{1}{2}}\cdot f'(2)$

$\frac{1}{2}\cdot 8^{-\frac{1}{2}}\cdot\frac{1}{3}$

$\dfrac{1}{2\sqrt{8}}\cdot\dfrac{1}{3} = \dfrac{1}{6\sqrt{8}} = \dfrac{1}{12\sqrt{2}}$

(g) $\left(\frac{1}{g^2(x)}\right)' = \left.\frac{-2g(x)g'(x)}{(g^2(x))^2}\right|_{x=3} = \frac{-2g(3)\cdot g'(3)}{g^4(3)} = \frac{-2(-4)\cdot 5}{(-4)^4}$

$\frac{40}{256} = \frac{20}{128} = \frac{10}{64}$

$= \frac{5}{32}$

(h) $\left.\left((f^2(x)+g^2(x))^{\frac{1}{2}}\right)'\right|_{x=2}$

$\frac{1}{2}(f^2(x)+g^2(x))^{-\frac{1}{2}}\cdot(2f(x)f'(x)+2g(x)g'(x))$

$\frac{1}{2}(f^2(2)+g^2(2))^{-\frac{1}{2}}\cdot(2f(2)f'(2)+2g(2)g'(2))$

$17\overline{)68}^{\,4}$

$\frac{16}{3} - 12 \qquad \frac{16}{3} - \frac{36}{3}$

$\frac{1}{2}(64+4)^{-\frac{1}{2}}\cdot(2(8)(\frac{1}{3})+2(2)(-3))$

$\frac{1}{2\sqrt{68}}\cdot\frac{-20}{3} = \frac{-20}{6\sqrt{68}} = \frac{-10}{3\sqrt{68}} = \frac{-10}{3\cdot 2\sqrt{17}}$

$= \frac{-10}{6\sqrt{17}} = \frac{-5}{3\sqrt{17}}$

(58) (a) $5f'(x)-g'(x)\Big|_{x=1} = 5f'(1)-g'(1) = 5\left(-\frac{1}{3}\right)-\left(-\frac{8}{3}\right) = \frac{-5}{3}+\frac{8}{3} = 1$

(b) $\left(f(x)g^3(x)\right)'\Big|_{x=0}$ $\quad f(x)\cdot 3g^2(x)\cdot g'(x) + g^3(x)f'(x)\Big|_{x=0}$

$f(0)\cdot 3g^2(0)\cdot g'(0) + g^3(0)f'(0)$

$1\cdot 3(1)^2\cdot \frac{1}{3} + (1)^3\cdot 5$

$1 + 5 = 6$

(c) $\left(\dfrac{f(x)}{g(x)+1}\right)'\Bigg|_{x=1}$

$$\frac{(g(x)+1)f'(x) - f(x)(g'(x))}{(g(x)+1)^2}\Bigg|_{x=1} \quad \frac{(g(1)+1)f'(1) - f(1)g'(1)}{(g(1)+1)^2}$$

$$= \frac{\overset{-3}{(-4+1)}\left(-\frac{1}{3}\right) - (3)\left(-\frac{8}{3}\right)}{(-4+1)^2}$$

$$= \frac{1+8}{9} = \frac{9}{9} = 1$$

(d) $\left(f(g(x))\right)'\Big|_{x=0}$ $\quad f'(g(x))\cdot g'(x)\Big|_{x=0}$

$f'(g(0))\cdot g'(0) = f'(1)\cdot g'(0) = -\frac{1}{3}\cdot\frac{1}{3} = \frac{-1}{9}$

(e) $\left(g(f(x))\right)'\Big|_{x=0}$ $\quad g'(f(x))\cdot f'(x)\Big|_{x=0}$ $\quad g'(f(0))\cdot f'(0) = -\frac{8}{3}\cdot 5$

$= \frac{-40}{3}$

(f) $\left[\left(g(x)+f(x)\right)^{-2}\right]'\Big|_{x=1}$

$-2\left(g(x)+f(x)\right)^{-3}\cdot\left(g'(x)+f'(x)\right)$

$-2\left(g(1)+f(1)\right)^{-3}\cdot\left(g'(1)+f'(1)\right)$

$=-2(-4+3)^{-3}\cdot\left(-8/3+-1/3\right)$

$-2(-1)^{-3}\qquad(-3)$

$-2(-1)(-3) = -6$

(g) $\left(f(x+g(x))\right)'\quad x=0$

$f'(x+g(x))\cdot(1+g'(x))$

$f'(0+g(0))\cdot(1+g'(0))$

$f'(0+1)\cdot(1+\frac{1}{3})$

$f'(1)\cdot\left(\frac{4}{3}\right)$

$-\frac{1}{3}\cdot\frac{4}{3} = -\frac{4}{9}$