Name: _____ Date: _____

AP Calculus AB Ms. Loughran

Do Now:

b

2+2+2

1. Find the value of $a+b+c$, if $f(x)=ax^2+bx+c$, $f(0)=2$, $f'(2)=10$, and $f''(10)=4$.

✓ ✓

$f(0)=2$

$2=a(0)^2+b(0)+c$

$2=c$

$f'(x)=2ax+b$

$10=2a(2)+b$

$10=4a+b$

$10=4(2)+b$

$2=b$

$f''(x)=2a$

$4=2a$

$a=2$

2. Given $g(x)=(x+1)\ln^2(x+1)$, find $g''(x)$.

$g'(x)=(x+1)\cdot 2\ln(x+1)\cdot\frac{1}{x+1}\cdot 1+\ln^2(x+1)$

$g'(x)=2\ln(x+1)+\ln^2(x+1)$

$g''(x)=\frac{2}{x+1}+2\ln(x+1)\cdot\frac{1}{x+1}=\frac{2}{x+1}+\frac{2\ln(x+1)}{x+1}=\frac{2+2\ln(x+1)}{x+1}$

VELOCITY PROBLEMS

1. **THE BALL IN THE AIR PROBLEM.** A ball is thrown upward from the ground with initial velocity v_0 = 64 ft/sec so that $s = 64t - 16t^2$, with t in seconds.

a) Find the velocity of the ball at the end of 1 second.

$v(t)=64-32t$ $v(1)=64-32(1)=32$ ft/s

b) Find the average velocity of the ball during the first 2 seconds. [Ans: 32 fps]

$\frac{s(2)-s(0)}{2-0}=32$ ft/s

c) Find the velocity of the ball at the end of 3 seconds.

$v(3)=64-32(3)=-32$ ft/s

d) How long does it take the ball to reach its highest point?

$64-32t=0$

$t=2$ s

e) Find the velocity and acceleration of the ball when it is 48 ft from ground level. (There are two answers to each part of this question.)

$64t-16t^2=48$ $t=1s,3s$

$0=16t^2-64t+48$

$0=16(t^2-4t+3)$

$0=16(t-3)(t-1)$

$v(1)=32$ ft/s

$v(3)=-32$ ft/s

$v'(t)=a(t)$

$a(t)=-32$

$a(1)=-32$ ft/s^2

$a(3)=-32$ ft/s^2

f) Find the maximum height the ball reaches?

$s(2)=64(2)-16(2)^2=64$ ft

g) Find the SPEED (which is the absolute value of velocity) of the ball at the end of 1 second and at the end of 3 seconds.

speed = |velocity|

$|v(1)|=32$ ft/s

$|v(3)|=32$ ft/s

h) After how long will the ball return to the ground?

$0=64t-16t^2$ $t=0$ 4 seconds ← symmetry of parabola

$0=16t(4-t)$ $t=4$

i) Find the ball's average vel. during the last 2 secs. [2, 4]

$\frac{s(4)-s(2)}{4-2}=-32$ ft/s

j) Find the ball's IMPACT VELOCITY (when it hits ground).

$v(4)=64-32(4)=-64$ ft/s

2. **THE FALLING ROCK PROBLEM.** A rock falls from a building, its height y in feet from the ground after t seconds being given by $y = 100 - 16t^2$.

a) Find the INITIAL HEIGHT from which the rock fell.

$y(0) = 100 - 16(0)^2 = 100$ ft

b) Find the IMPACT TIME (the time it takes the rock to strike ground).

$100 - 16t^2 = 0$ $\quad t^2 = \frac{100}{16}$

$100 = 16t^2$ $\quad t = \pm\frac{10}{4}$ or $\pm\frac{5}{2}$

c) Find the IMPACT VELOCITY (at the impact time).

$v(t) = -32t$

$v(\frac{5}{2}) = -32(\frac{5}{2}) = -80$ ft/s

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19. ***Particle Motion*** A particle moves along a line so that its position at any time $t \geq 0$ is given by the function

$$s(t) = t^2 - 3t + 2,$$

$s(t) = (t-2)(t-1)$

where s is measured in meters and t is measured in seconds.

(a) Find the displacement during the first 5 seconds. a) $s(5) - s(0) = 12 - 2 = 10$ m

(b) Find the average velocity during the first 5 seconds. b) $\frac{s(5)-s(0)}{5-0} = 2$ m/s

(c) Find the instantaneous velocity when $t = 4$. c) $v(t) = 2t - 3$, $v(4) = 8 - 3 = 5$ m/s

(d) Find the acceleration of the particle when $t = 4$.

(e) At what values of t does the particle change direction?

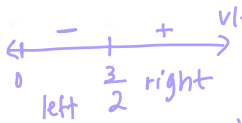
(f) Where is the particle when s is a minimum?

(e) $v(t) = 0$

$2t - 3 = 0$

$t = 3/2$

$t = 3/2$ s



left: $[0, \frac{3}{2})$

right: $(\frac{3}{2}, \infty)$

d) $a(t) = 2$

$a(4) = 2$ m/s^2

f) $s(\frac{3}{2}) = (\frac{3}{2} - 2)(\frac{3}{2} - 1)$

$(-\frac{1}{2})(\frac{1}{2}) = -\frac{1}{4}$ m

(58) (a) $5f'(x) - g'(x)\Big|_{x=1} = 5f'(1) - g'(1) = 5\left(-\frac{1}{3}\right) - \left(-\frac{8}{3}\right) = -\frac{5}{3} + \frac{8}{3} = 1$

(b) $\left(f(x)g^3(x)\right)'\Big|_{x=0}$ $\quad f(x)\cdot 3g^2(x)\cdot g'(x) + g^3(x)f'(x)\Big|_{x=0}$

$f(0)\cdot 3g^2(0)\cdot g'(0) + g^3(0)f'(0)$

$1\cdot 3(1)^2\cdot\frac{1}{3} + (1)^3\cdot 5$

$1 + 5 = 6$

(c) $\left(\frac{f(x)}{g(x)+1}\right)'\Big|_{x=1}$

$\frac{(g(x)+1)f'(x) - f(x)(g'(x))}{(g(x)+1)^2}\Big|_{x=1}$ $\quad \frac{(g(1)+1)f'(1) - f(1)g'(1)}{(g(1)+1)^2}$

$= \frac{(\overset{-3}{-4+1})(-\frac{1}{3}) - (3)(-\frac{8}{3})}{(-4+1)^2}$

$= \frac{1+8}{9} = \frac{9}{9} = 1$

(d) $\left(f(g(x))\right)'\Big|_{x=0}$ $\quad f'(g(x))\cdot g'(x)\Big|_{x=0}$

$f'(g(0))\cdot g'(0) = f'(1)\cdot g'(0) = -\frac{1}{3}\cdot\frac{1}{3} = -\frac{1}{9}$

(e) $\left(g(f(x))\right)'\Big|_{x=0}$ $\quad g'(f(x)) \cdot f'(x)\Big|_{x=0}$ $\quad g'(f(0)) \cdot f'(0) = -\frac{7}{3} \cdot 5$

$= \frac{-40}{3}$

(f) $\left[\left(g(x)+f(x)\right)^{-2}\right]'\Big|_{x=1}$

$-2\left(g(x)+f(x)\right)^{-3} \cdot \left(g'(x)+f'(x)\right)$

$-2\left(g(1)+f(1)\right)^{-3} \cdot \left(g'(1)+f'(1)\right)$

$= -2(-4+3)^{-3} \cdot (-8/3 + -1/3)$

$-2(-1)^{-3} \qquad (-3)$

$-2(-1)(-3) = -6$

(g) $\left(f(x+g(x))\right)' \quad x=0$

$f'(x+g(x)) \cdot (1+g'(x))$

$f'(0+g(0)) \cdot (1+g'(0))$

$f'(0+1) \cdot (1+\frac{1}{3})$

$f'(1) \cdot (\frac{4}{3})$

$-\frac{1}{3} \cdot \frac{4}{3} = -\frac{4}{9}$

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71.

Vertical Motion On Earth, if you shoot a paper clip 64 ft straight up into the air with a rubber band, the paper clip will be $s(t) = 64t - 16t^2$ feet above your hand at t sec after firing.

(a) Find ds/dt and d^2s/dt^2.

(b) How long does it take the paper clip to reach its maximum height?

(c) With what velocity does it leave your hand?

(d) On the moon, the same force will send the paper clip to a height of $s(t) = 64t - 2.6t^2$ ft in t sec. About how long will it take the paper clip to reach its maximum height, and how high will it go?

72.

Free Fall Suppose two balls are falling from rest at a certain height in centimeters above the ground. Use the equation $s = 490t^2$ to answer the following questions.

(a) How long does it take the balls to fall the first 160 cm? What is their average velocity for the period?

(b) How fast are the balls falling when they reach the 160-cm mark? What is their acceleration then?