Name:____ Date: ______________

AP Calculus AB Ms. Loughran

Do Now:

1. The position of a particle moving along the x-axis is given by: $x(t) = e^{2t} - e^{t}$ for all $t \geq 0$. When the particle is at rest, the acceleration of the particle is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\ln\frac{1}{2}$ (D) 2 (E) 4

2. If $\cos x = e^{y}$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

$y = \ln \cos x$

$\frac{dy}{dx} = \frac{1}{\cos x} \cdot -\sin x = \frac{-\sin x}{\cos x}$

① $v(t) = 0$

$2e^{2t} - e^{t} = 0$

$e^{t}(2e^{t} - 1) = 0$

$e^{t} \neq 0$ | $2e^{t} - 1 = 0$

$2e^{t} = 1$

$e^{t} = \frac{1}{2}$

$t = \ln\frac{1}{2}$

$\text{base}^{\text{exponent}} = \text{answer}$

$\text{exponent} = \log_{\text{base}} \text{answer}$

$a(t) = 4e^{2t} - e^{t}$

$a(\ln\frac{1}{2}) = 4e^{2\ln\frac{1}{2}} - e^{\ln\frac{1}{2}}$

$a(\ln\frac{1}{2}) = 4e^{\ln\frac{1}{2}^{2}} - \frac{1}{2}$

$= 4\left(\frac{1}{2}\right)^{2} - \frac{1}{2}$

$= 1 - \frac{1}{2}$

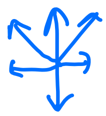
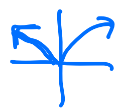
$\frac{1}{2}$

$* \ln e^{x} = x$

$e^{\ln x} = x$

A function will not have a derivative at a point, $P(a, f(a))$ where the slopes of the secant lines, $\frac{f(x)-f(a)}{x-a}$ fail to approach a limit as $x \to a$.

A function whose graph is otherwise smooth will fail to have a derivative at a point where the graph has:

① a corner or a cusp  

② a point of discontinuity

③ a vertical tangent

Differentiability implies continuity.

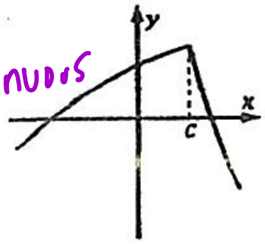
Converse is not true.

Name: ____ Date: __________

AP Calculus AB – Differentiability

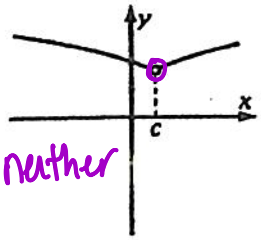
A. For # 1 – 12, state whether the function is continuous, differentiable, both, or neither at $x = c$.

1.



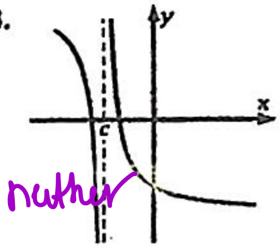
continuous

2.



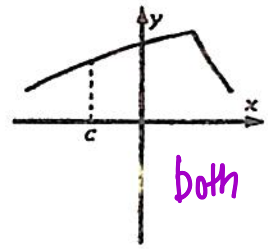
neither

3.



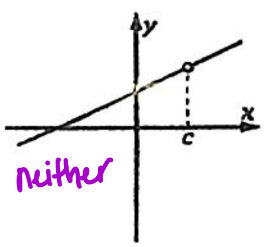
neither

4.



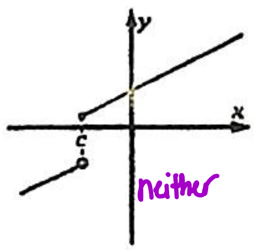
both

5.



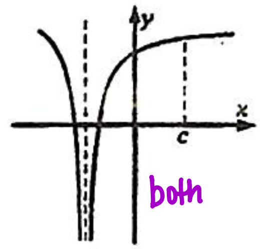
neither

6.



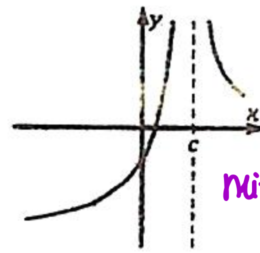
neither

7.



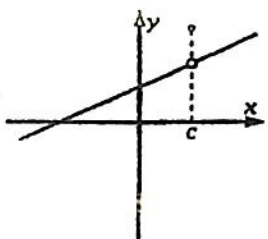
both

8.



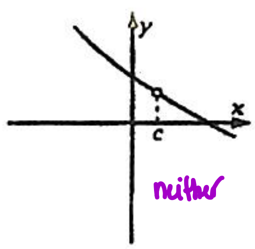
neither

9.



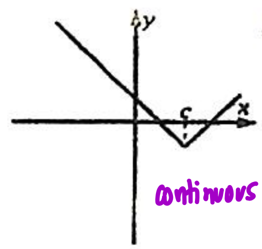
neither

10.



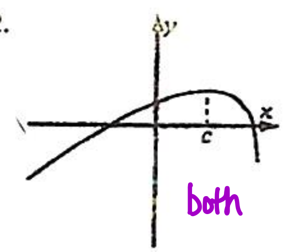
neither

11.



continuous

12.

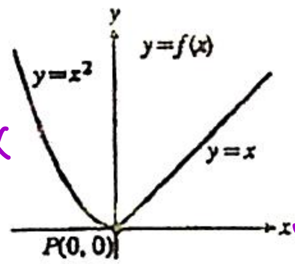


both

B. For # 1 – 4, compare the left-hand and right-hand derivatives to show that the function is not differentiable at P.

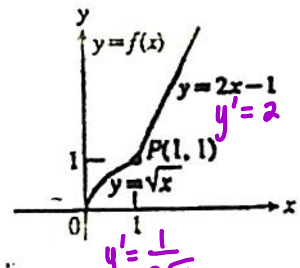
1.

$y' = 2x$



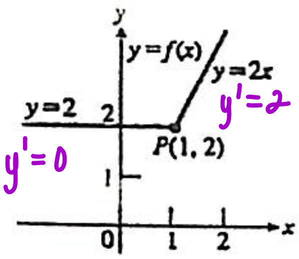
$y' = 1$

3.

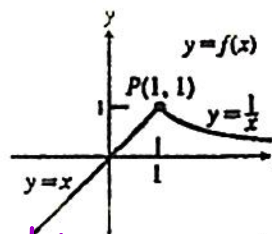


$y' = \frac{1}{2\sqrt{x}}$

2.



4.



$y' = -\frac{1}{x^2}$

$y' = 1$

For # 5 – 8, determine if $f(x)$ is differentiable at the value mentioned.

5. $f(x) = \begin{cases} x^2 - 4x + 8, & x \le 3 \\ 11 - x, & x > 3 \end{cases}$ Is $f(x)$ differentiable at $x = 3$?

cont $x^2 - 4x + 8 \overset{?}{=} 11 - x$ @$x=3$

$9 - 12 + 8 \overset{?}{=} 8$

$5 = 8$ ✗

Not cont. @$x=3$, so it can not be differentiable at $x=3$.

6. $f(x) = \begin{cases} x^2 - 6x + 8, & x \ge 1 \\ 7 - 4x, & x < 1 \end{cases}$ Is $f(x)$ differentiable at $x = 1$?

cont: $x^2 - 6x + 8 \overset{?}{=} 7 - 4x$ @$x=1$

$1 - 6 + 8 \overset{?}{=} 7 - 4$ ✓

$f'(x) = \begin{cases} 2x - 6 & x \geq 1 \\ -4 & x < 1 \end{cases}$

diff $2x - 6 \overset{?}{=} -4$ @$x=1$

$2 - 6 \overset{?}{=} -4$ ✓

yes

7. $f(x) = \begin{cases} x^2 - x + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$ Is $f(x)$ differentiable at $x = 1$?

not differentiable b/c it's not continuous at $x=1$

cont:

$x^2 - x + 1 \overset{?}{=} x + 1$ @$x=1$

$1 - 1 + 1 \overset{?}{\neq} 1 + 1$

$$f'(x) = \begin{cases} 2x-1 & x \le 1 \\ -1 & x > 1 \end{cases}$$

8. $f(x) = \begin{cases} x^2 - x + 1, & x \le 1 \\ 2 - x, & x > 1 \end{cases}$ Is $f(x)$ differentiable at $x = 1$?

no

cont ?

$1^2 - 1 + 1 = 2 - 1$ ✓ @ $x=1$

diff

$2x - 1 \ne -1$ @ $x = 1$

For # 9 – 12, find the values of a and b that make $f(x)$ differentiable at the value stated.

9. $f(x) = \begin{cases} ax^2 + 10, & x < 2 \\ x^2 - 6x + b, & x \ge 2 \end{cases}$ at $x = 2$

$$f'(x) = \begin{cases} 2ax & x < 2 \\ 2x - 6 & x \ge 2 \end{cases}$$

cont

$ax^2 + 10 = x^2 - 6x + b$ @ $x = 2$

$a(2)^2 + 10 = 2^2 - 6(2) + b$

$4a + 10 = -8 + b$

$4a + 18 = b$

$4(-\frac{1}{2}) + 18 = b$

$16 = b$

diff

$2ax = 2x - 6$ @ $x = 2$

$2a(2) = 2(2) - 6$

$4a = -2$

$a = -\frac{1}{2}$

10. $f(x)=\begin{cases} x^3, & x<1 \\ a(x-2)^2+b, & x\geq 1 \end{cases}$ at $x=1$

$f'(x)=\begin{cases} 3x^2 & x<1 \\ 2a(x-2) & x\geq 1 \end{cases}$

cont

$x^3 = a(x-2)^2+b$ @ $x=1$

$1 = a(1-2)^2+b$

$1 = a+b$

diff

$3x^2 = 2a(x-2)$ @ $x=1$

$3 = 2(a)(-1)$

$3 = -2a$

$\frac{-3}{2} = a$

$1 = -\frac{3}{2}+b$

$\frac{5}{2} = b$

11. $f(x)=\begin{cases} -(x-3)^2+7, & x\geq 2 \\ ax^3+b, & x<2 \end{cases}$ at $x=2$

$f'(x)=\begin{cases} -2(x-3) & x\geq 2 \\ 3ax^2 & x<2 \end{cases}$

cont:

$-(x-3)^2+7 = ax^3+b$ @ $x=2$

$-(2-3)^2+7 = a(2)^3+b$

$-1+7 = 8a+b$

$6 = 8a+b$

diff

$-2(x-3) = 3ax^2$ @ $x=2$

$-2(2-3) = 3a(2)^2$

$2 = 12a$

$\frac{1}{6} = a$

$6 = 8\left(\frac{1}{6}\right)+b$

$6 = \frac{4}{3}+b$

$b = \frac{14}{3}$

Homework 10-24

Name: ________________________________ Date: __________

AP Calculus AB: More Derivative Multiple Choice Practice — Ms. Loughran

$g'(x) = 3 \cdot 2\tan(2x) \cdot \sec^2(2x) \cdot 2$

$g'(x) = 12\tan(2x)\sec^2(2x)$

$g'(\frac{\pi}{8}) = 12\tan(\frac{\pi}{4})\sec^2(\pi/4) = 12(1)(\sqrt{2})^2 = 24$

1. If $g(x) = 3\tan^2(2x)$, then $g'\left(\frac{\pi}{8}\right)$ is

   (A) 6 (B) $6\sqrt{2}$ (C) 12 (D) $12\sqrt{2}$ (E) 24

$m_\perp = \frac{2}{3}$ $y - \frac{3}{4} = \frac{2}{3}(x - \ln 2)$ $(\ln 2, \frac{3}{4})$

2. Given $y = 3e^{-2x}$, what is an equation of the normal line to the graph at $x = \ln 2$?

   (A) $y = \frac{2}{3}(x - \ln 2) + \frac{3}{4}$

   (B) $y = \frac{2}{3}(x + \ln 2) - \frac{3}{4}$

   (C) $y = -\frac{3}{2}(x - \ln 2) + \frac{3}{4}$

   (D) $y = -\frac{3}{2}(x - \ln 2) - \frac{3}{4}$

   (E) $y = 24(x - \ln 2) + 12$

$y' = 3 \cdot e^{-2x} \cdot -2$

$y' = -6e^{-2x}$

$y'(\ln 2) = -6e^{-2\ln 2}$

$= -6(2^{-2})$

$= -\frac{3}{2}$

$y(\ln 2) = 3e^{-2\ln 2}$

$= 3e^{\ln 2^{-2}} = 3(2)^{-2} = \frac{3}{4}$

3. If $f(x) = \ln(\underbrace{\ln(1-x)}_{u})$, $f'(x)$ is

   (A) $-\frac{1}{\ln(1-x)}$ (B) $\frac{1}{(1-x)\ln(1-x)}$ (C) $\frac{1}{(1-x)^2}$

   (D) $-\frac{1}{(1-x)\ln(1-x)}$ (E) $-\frac{1}{\ln(1-x)^2}$

$f'(x) = \frac{1}{\ln(1-x)} \cdot \frac{1}{1-x} \cdot -1 = -\frac{1}{(1-x)\ln(1-x)}$

$f(x+h) - f(x)$

4. $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{6} + h\right) - \tan\left(\frac{\pi}{6}\right)}{h} =$

   (A) $\frac{\sqrt{3}}{3}$ (B) $\frac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\frac{3}{4}$

Formal Defn of Der.

$f(x) = \tan x$

$x = \frac{\pi}{6}$

$f'(x) = \sec^2 x$

$f'(\frac{\pi}{6}) = \sec^2(\frac{\pi}{6}) = \left(\frac{2}{\sqrt{3}}\right)^2$

$\frac{4}{3}$

$\sin\left(\frac{5\pi}{6}\right)$

Formal Def. of Der

$f(x+h) \quad - f(x)$

$f(x) = \sin x \qquad f'(x) = \cos x$

$x = \frac{5\pi}{6} \qquad f'\left(\frac{5\pi}{6}\right) = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

5. $\lim_{h\to 0}\dfrac{\sin\left(\frac{5\pi}{6}+h\right)-\frac{1}{2}}{h} =$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) $-\frac{\sqrt{3}}{2}$

6. Find the derivative of $g(x) = 5\sin^2(6x) + 5\cos^2(6x)$ with respect to x.

(A) $30\cos^2(6x) - 30\sin^2(6x)$
(B) $5\cos^2(6x) - 5\sin^2(6x)$
(C) $120\sin(6x)\cos(6x)$
(D) 30
(E) 0

$g(x) = 5(\sin^2(6x) + \cos^2(6x))$

$g(x) = 5(1) = 5$

$g'(x) = 0$

7. Let $f(x) = 5x\sec x + x^3\cos x + 17\pi$, find $\frac{d}{dx}f(x)$.

(A) $5\sec x\tan x + 3x^2\cos x + 17\pi$ ← constant
(B) $5\sec^2 x - x^3\sin x$
(C) $5\sec x\tan x - 3x^2\sin x$
(D) $5\sec x + 5x\sec x\tan x + 3x^2\cos x - x^3\sin x$
(E) $5\sec x + 5x\sec x\tan x - 3x^2\cos x + x^3\sin x + 17\pi$

$f'(x) = 5x\sec x\tan x + 5\sec x - x^3\sin x + 3x^2\cos x + 0$

8. $\frac{d}{dx}(\ln e^{2x}) = \frac{d}{dx}(2x) = 2$

(A) $2x$ (B) 2 (C) $\frac{1}{e^{2x}}$ (D) $\frac{2}{e^{2x}}$ (E) $\frac{2x}{e^{2x}}$

9. If $f(x) = e^{4\ln(x^3)}$, then $f'(x) =$

$f(x) = e^{\ln(x^3)^4}$

$f(x) = (x^3)^4 = x^{12}$

$f'(x) = 12x^{11}$

Remember

$e^{\ln x} = \ln e^x$

$x = x$