

Do Now:

1. Find the equation of the tangent line to  $x^2 + y^2 = 25$  at  $(3, -4)$ .

Method 1

circle: center  $(0,0)$   $r=5$

m radius:  $\frac{-4-0}{3-0} = -\frac{4}{3}$

m tan =  $\frac{3}{4}$

$y+4 = \frac{3}{4}(x-3)$

Implicit Differentiation:

\*  $[f^2(x)]' = 2f(x)f'(x)$   
 $[y^2]' = 2y \cdot y'$

Method 2

$x^2 + y^2 = 25$

$y^2 = 25 - x^2$

$y = \pm \sqrt{25 - x^2}$

$(3, -4)$  is on  $y = -\sqrt{25 - x^2}$

$\frac{dy}{dx} = -\frac{1}{2}(25 - x^2)^{-\frac{1}{2}} \cdot -2x$

$\frac{dy}{dx} = \frac{x}{\sqrt{25 - x^2}}$

$\frac{dy}{dx} \Big|_{x=3} = \frac{3}{\sqrt{25 - 3^2}} = \frac{3}{4}$

$y+4 = \frac{3}{4}(x-3)$

$x^2 + y^2 = 25$

$2x + 2y \frac{dy}{dx} = 0$

$2y \frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

$\frac{dy}{dx} \Big|_{(3,-4)} = \frac{-3}{-4} = \frac{3}{4}$

$y+4 = \frac{3}{4}(x-3)$

Implicit differentiation allows you to find the slope of a tangent line when the equation cannot be solved for  $y$ .

↓ product rule

1. Find the slope of the tangent line to the graph of  $x^2 + 3xy - 2y^2 = -4$  at the point  $(1, -1)$ .

$2x + 3x \frac{dy}{dx} + 3y - 4y \frac{dy}{dx} = 0$

$3x \frac{dy}{dx} - 4y \frac{dy}{dx} = -2x - 3y$

$\frac{dy}{dx} (3x - 4y) = -2x - 3y$

$\frac{dy}{dx} = \frac{-2x - 3y}{3x - 4y}$

$\frac{dy}{dx} \Big|_{(1,-1)} = \frac{-2(1) - 3(-1)}{3(1) - 4(-1)} = \frac{-5}{7}$

2. Find the slope(s) of the tangent line(s) to the graph of  $4x + xy - 3y^2 = 6$  at  $x = 3$ .

$$\begin{aligned}
 x=3 \quad 4(3) + 3y - 3y^2 &= 6 \\
 12 + 3y - 3y^2 &= 6 \\
 0 &= 3y^2 - 3y - 6 \\
 0 &= 3(y^2 - y - 2) \\
 0 &= 3(y-2)(y+1) \\
 y &= 2, y = -1
 \end{aligned}$$

$$(3, 2), (3, -1)$$

$$\begin{aligned}
 4x + xy - 3y^2 &= 6 \\
 4 + x \frac{dy}{dx} + y - 6y \frac{dy}{dx} &= 0 \\
 x \frac{dy}{dx} - 6y \frac{dy}{dx} &= -4 - y \\
 \frac{dy}{dx} (x - 6y) &= -4 - y \\
 \frac{dy}{dx} &= \frac{-4 - y}{x - 6y}
 \end{aligned}$$

$$\left. \frac{dy}{dx} \right|_{(3,2)} = \frac{-4-2}{3-6(2)} = \frac{-6}{-9} = \frac{2}{3}$$

$$\left. \frac{dy}{dx} \right|_{(3,-1)} = \frac{-4+1}{3-6(-1)} = \frac{-3}{9} = -\frac{1}{3}$$

### Homework

For 1-2, find  $\frac{dy}{dx}$ .

1.  $x^2 + y^2 = 100$

2.  $x^2y + 3xy^3 - x = 3$

3. Find the slope of the tangent line to the curve  $x^2 + y^2 = 1$  at  $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$  and at  $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ .