Do Now: #s 1 and 2

IMPLICIT DIFFERENTIATION

1. If $x^2 + y^2 = 5$, find $\frac{dy}{dx}$.

$$2x + 2y\frac{dy}{dx} = 0$$
$$2y\frac{dy}{dx} = -2x$$
$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

2. If $x^2 - xy + y^3 = 1$,

a) Find $\frac{dy}{dx}$

b) Find y' when $x = 1$. Explain.

a) $$2x - x\frac{dy}{dx} - 1y + 3y^2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}\left(3y^2 - x\right) = y - 2x$$
$$\frac{dy}{dx} = \frac{y-2x}{3y^2-x}$$

b) $x=1$ $\quad 1^2 - 1y + y^3 = 1$
$$y^3 - y = 0$$
$$y(y^2-1) = 0$$
$$y = 0, \pm 1$$

$$\left.\frac{dy}{dx}\right|_{(1,0)} = \frac{0-2(1)}{3(0)^2-1} = \frac{-2}{-1} = 2$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{1-2(1)}{3(1)^2-1} = \frac{-1}{2}$$

$$\left.\frac{dy}{dx}\right|_{(+1,-1)} = \frac{-1-2(1)}{3(-1)^2-1} = \frac{-3}{2}$$

$y(4) = \left(3 + 6(4) + \sqrt{4^2+9}\right)^{\frac{1}{5}} = (\cancel{32})^{\frac{1}{5}}$ $(4,2)$

5. If $y = (3 + 6x + \sqrt{x^2 + 9})^{1/5}$, find $y'(4)$. (Hint: It's EASIEST to raise both sides to the 5th power, then differentiate implicitly.)

$$y^5 = 3 + 6x + \sqrt{x^2+9}$$

$$5y^4 \frac{dy}{dx} = 6 + \frac{1}{2}(x^2+9)^{-\frac{1}{2}} \cdot 2x$$

$$5(2)^4 \frac{dy}{dx} = 6 + \frac{1}{\cancel{2}}(4^2+9)^{-\frac{1}{2}} \cdot \cancel{2}(4)$$

$$5(2)^4 \frac{dy}{dx} = 6 + (4^2+9)^{-\frac{1}{2}}(4)$$

$$\frac{dy}{dx} = \frac{6 + (4^2+9)^{-\frac{1}{2}}(4)}{5(2)^4} \quad \text{or} \quad \frac{6 + \frac{4}{5}}{80}$$

$$\frac{30+4}{400} \text{ or } \frac{34}{400}$$

$$\text{or } \frac{17}{200}$$

6. <u>2ND AND 3RD DERIVATIVES</u> If $x^2 - y^2 = 1$, find a) $\frac{dy}{dx}$, b) $\frac{d^2y}{dx^2}$, c) $\frac{d^3y}{dx^3}$.

$$2x - 2y\frac{dy}{dx} = 0$$

$$-2y\frac{dy}{dx} = -2x$$

a) $$\frac{dy}{dx} = \frac{x}{y}$$

b) $$\left[\frac{x}{y}\right]' = \frac{y(1) - x\frac{dy}{dx}}{y^2} = \frac{y - x\left(\frac{x}{y}\right)}{y^2} \frac{y}{y} = \frac{y - \frac{x^2}{\cancel{y}}\cancel{y}}{y^2 \cdot y}$$

b) $$= \frac{y^2 - x^2}{y^3}$$

$$\frac{y^2-x^2}{y^3} = \frac{-1}{y^3} = -y^{-3}$$

$$\left[-y^{-3}\right]' = 3y^{-4} \cdot \frac{dy}{dx}$$

$$\frac{3}{y^4} \cdot \frac{x}{y} = \frac{3x}{y^5}$$

1987 BC 2 a b

9 Given the curve $y^3 + 3x^2y + 13 = 0$.

product rule

a) Find $\frac{dy}{dx}$.

$$3y^2\frac{dy}{dx} + 3\left(x^2\frac{dy}{dx} + 2xy\right) = 0$$

$$3y^2\frac{dy}{dx} + 3x^2\frac{dy}{dx} + 6xy = 0$$

$$\frac{dy}{dx}\left(3y^2 + 3x^2\right) = -6xy$$

$$\frac{dy}{dx} = \frac{-6xy}{3y^2+3x^2} \quad \text{or} \quad \frac{-2xy}{y^2+x^2}$$

b) Write an equation for the line tangent to the curve at the point (2,-1).

$$\left.\frac{dy}{dx}\right|_{(2,-1)} = \frac{-2(2)(-1)}{(-1)^2 + 2^2} = \frac{4}{5}$$

$$y+1 = \frac{4}{5}(x-2)$$

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CALCULUS AB

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$2x-\left(y+x\frac{dy}{dx}\right)+2y\frac{dy}{dx}=0$

1978 BC 1

(10) Given the curve $x^2 - \underbrace{xy}_{\text{product}} + y^2 = 9$.

a) Write a general expression for the slope of the curve. (find $\frac{dy}{dx}$)

$$2x-y-x\frac{dy}{dx}+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(2y-x)=y-2x$$

$$\frac{dy}{dx}=\frac{y-2x}{2y-x}$$

b) Find the coordinates of the points on the curve where the tangents are vertical. (slope is undefined, the denominator of $\frac{dy}{dx}=0$)

$2y-x=0$

$2y=x$

substitute back into the original

$x^2-xy+y^2=9$

$(2y)^2-2y\cdot y+y^2=9$

$4y^2-2y^2+y^2=9$

$3y^2=9$

$y^2=3$

$y=\pm\sqrt{3}$

Remember $x=2y$

$(2\sqrt{3},\sqrt{3})$

$(-2\sqrt{3},-\sqrt{3})$

c) At the point (0,3), $\underbrace{\text{find the rate of change}}_{\text{derivative}}$ in the slope of the curve with respect to x.

$\left(\frac{d^2y}{dx^2}\right)$

$\left.\frac{dy}{dx}\right|_{(0,3)}=\frac{3-2(0)}{2(3)-0}=\frac{1}{2}$

$$\left[\frac{y-2x}{2y-x}\right]'=\frac{(2y-x)\left(\frac{dy}{dx}-2\right)-(y-2x)\left(2\frac{dy}{dx}-1\right)}{(2y-x)^2}$$

$$=\frac{(2(3)-0)\left(\frac{1}{2}-2\right)-(3-2(0))\left(2\left(\frac{1}{2}\right)-1\right)}{(2(3)-0)^2}$$

Homework

Homework 10-31

For 1-2, find $\frac{dy}{dx}$.

1. $x^2 + y^2 = 100$

2. $x^2 y + 3xy^3 - x = 3$

3. Find the slope of the tangent line to the curve $x^2 + y^2 = 1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and at $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

① $2x + 2y\frac{dy}{dx} = 0$

$2y\, dy/dx = -2x$

$dy/dx = \frac{-2x}{2y} = \frac{-x}{y}$

② $2xy + x^2\frac{dy}{dx} + 3x \cdot 3y^2\frac{dy}{dx} + 3y^3 - 1 = 0$

$2xy + x^2\frac{dy}{dx} + 9xy^2\frac{dy}{dx} + 3y^3 - 1 = 0$

$\frac{dy}{dx}(x^2 + 9xy^2) = 1 - 2xy - 3y^3$

$\frac{dy}{dx} = \frac{1 - 2xy - 3y^3}{x^2 + 9xy^2}$

③ $x^2 + y^2 = 1$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

$\left.\frac{dy}{dx}\right|_{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$ $\quad\left.\frac{dy}{dx}\right|_{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)} = \frac{-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = 1$