2012

Do Now: From 2012 mc #s 1,4,5,7 and 9

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1. If $y = x\sin x$, then $\frac{dy}{dx} =$ $x\cos x + \sin x$

(A) $\sin x + \cos x$

[✓ (B) $\sin x + x\cos x$]

(C) $\sin x - x\cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

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2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

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4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

(A) 4 (B) 5 (C) 6 (D) 7 [✓ (E) 8]

$$f'(x) = 7x + \frac{1}{x}$$

$$f'(1) = 7(1) + \frac{1}{1}$$

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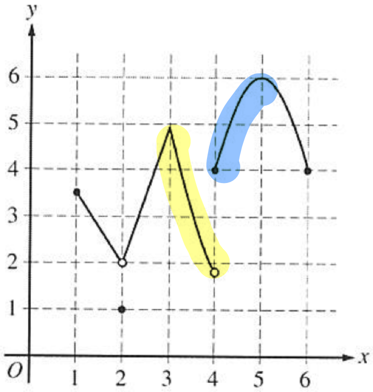


Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim_{x \to 2} f(x)$ exists.

(B) $\lim_{x \to 3} f(x)$ exists.

[✓ (C)] $\lim_{x \to 4} f(x)$ exists.

$\lim_{x \to 4} f(x)$ dne

(D) $\lim_{x \to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

7. If $y = \left(x^3 - \cos x\right)^5$, then $y' =$ $5(x^3 - \cos x)^4\,(3x^2 + \sin x)$

(A) $5\left(x^3 - \cos x\right)^4$

(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

[✓ (E)] $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

$$f(x) = \begin{cases} \dfrac{(2x+1)\cancel{(x-2)}}{\cancel{x-2}} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

(A) 0 (B) 1 (C) 2 (D) 3 [✓ (E)] 5

$2x + 1 = k$ @ $x = 2$

$2(2) + 1 = k$

$5 = k$

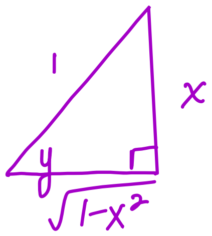
$\sin^{-1} x = \arcsin x$

Classwork:

1. $y = \sin^{-1}(x)$

y is the angle whose sine is x

$$[\sin y = x]'$$



$$b^2 + x^2 = 1^2$$
$$b^2 + x^2 = 1$$
$$b^2 = 1 - x^2$$
$$b = \cancel{\pm}\sqrt{1-x^2}$$

$$\cos y \frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{\cos y}$$
$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

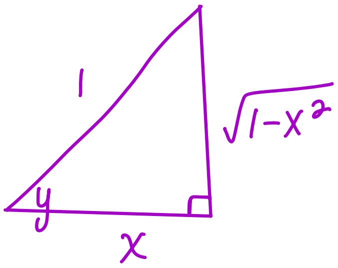
$$\boxed{\therefore \frac{dy}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}}$$

More generally $\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \cdot u'$

2. $y = \cos^{-1}(x)$

y is the angle whose cosine is x

$$[\cos y = x]'$$



$$-\sin y \frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = -\frac{1}{\sin y}$$
$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

More generally $\frac{d}{dx}(\cos^{-1} u) = -\frac{1}{\sqrt{1-u^2}} \cdot u'$

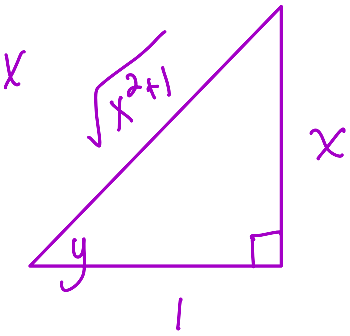
3. $y = \tan^{-1}(x)$

y is the angle whose tangent is x

$$[\tan y = x]'$$

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y = \left(\frac{1}{\sqrt{x^2+1}}\right)^2 = \frac{1}{x^2+1}$$



$$x^2 + 1^2 = c^2$$

$$x^2 + 1 = c^2$$

$$\cancel{\pm}\sqrt{x^2+1} = c$$

$\therefore \frac{dy}{dx}(\tan^{-1} x) = \frac{1}{x^2+1}$

More generally $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{u^2+1} \cdot u'$

These are not covered on the AP exam but for your information…

$$\frac{d}{dx}[\csc^{-1}u] = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}[\sec^{-1}u] = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}[\cot^{-1}u] = -\frac{1}{1+u^2}\frac{du}{dx}$$

Examples. For each of the following, find $\frac{dy}{dx}$.

1. $y = \sin^{-1}(\overset{u}{3x})$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3 = \frac{3}{\sqrt{1-9x^2}}$$

2. $y = \cos^{-1}\overset{u}{\left(\frac{x}{2}\right)}$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-\frac{x^2}{4}}} = \frac{-1}{\sqrt{4}\sqrt{1-\frac{x^2}{4}}} = \frac{-1}{\sqrt{4\left(1-\frac{x^2}{4}\right)}} = \frac{-1}{\sqrt{4-x^2}}$$

3. $y = \tan^{-1}\overset{u}{\left(\frac{3}{x}\right)}$

$$\frac{dy}{dx} = \frac{1}{\left(\frac{3}{x}\right)^2+1} \cdot -3x^{-2}$$

$$\frac{1}{\left(\frac{9}{x^2}+1\right)} \cdot \frac{-3}{x^2} = \frac{-3}{9+x^2}$$

4. $y = \sin^{-1}(e^x)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x = \frac{e^x}{\sqrt{1-e^{2x}}}$$

5. $y = \cos^{-1}(3x+2)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(3x+2)^2}} \cdot 3$$

$$= \frac{-1}{\sqrt{1-(9x^2+12x+4)}} \cdot 3$$

$$\frac{-3}{\sqrt{-9x^2-12x-3}}$$