Name: ______________________________ Date: __________

AP Calc AB: Derivatives of Inverse Functions Ms. Loughran

Do Now:

1. If $y = 6^{\tan^2 x}$, find $\frac{dy}{dx}$.

$\frac{dy}{dx} = 6^{\tan^2 x} \cdot \ln 6 \cdot 2\tan x \sec^2 x$

2. If $y = \log_4(x^2 + 3x)$, find $\frac{dy}{dx}$.

$\frac{dy}{dx} = \frac{2x+3}{(x^2+3x)\ln 4}$

$x^2+3x>0$

$x(x+3)>0$

$+$ -3 $-$ 0 $+$

$(-\infty,-3)\cup(0,\infty)$

Inverses of functions:

Given $f(x)$ is a function, we call its inverse function $f^{-1}(x)$. If (x,y) falls on $f(x)$, then (y,x) falls on $f^{-1}(x)$.

$f(g(x)) = x$ when f and g are inverses of each other.

Example

If $f(x) = x^2,\ x \ge 0$, find :

(a) $g(x)$ the inverse of $f(x)$

$g(x) = \sqrt{x}$

(b) $f'(x) = 2x$

(c) $g'(x) = \frac{1}{2\sqrt{x}}$

Let's take some points that are on $f(x)$ and investigate.

Points on $f(x)$: $(2,4)$ and $(3,9)$

Find: $\overset{x}{f'(2)} = 4$ $\quad \overset{y}{g'(4)} = \frac{1}{4} = \frac{1}{f'(2)}$

$\overset{x}{f'(3)} = 6$ $\quad \overset{y}{g'(9)} = \frac{1}{6} = \frac{1}{f'(3)}$

$\therefore$ If f and g are inverses, then $g'(y) = \frac{1}{f'(x)}$. [(x,y) is a point on $f(x)$.]

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INVERSE FUNCTION THEOREM

Theorem: Version I: If f and g are inverses, then $g'(y) = \frac{1}{f'(x)}$.

Version II: $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$. What is the "hazard" in writing the theorem this way?

$h'(y) = \frac{1}{f'(x)}$

1. Let $f(x) = x^3 + x$. If h is the inverse of f, then $h'(2) =$

   A) 1/13 B) 1/4 C) 1 D) 4 E) 13

$f(x) = 2$

$x^3 + x = 2$

$x^3 + x - 2 = 0$

$f'(x) = 3x^2 + 1$

$h'(2) = \frac{1}{f'(?)}$ need x

1	1	0	1	-2
		1	1	2
	1	1	2	0

$x^2 + x + 2 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(2)}}{2(1)}$ imaginary

$h'(2) = \frac{1}{f'(1)} = \frac{1}{3(1)^2 + 1} = \frac{1}{4}$

2. Suppose $f(x) = x^2 + 1$ for $x > 0$, and $f[g(x)] = x$. ↓ tells us that f and g are inverses

   a) Find $g'(10)$. $= \frac{1}{f'(?)}$

   b) Find $g'(3)$. $= \frac{1}{f'(?)}$

$x^2 + 1 = 10$

$x^2 = 9$

$x = \cancel{\pm} 3$

$f'(x) = 2x$

$g'(10) = \frac{1}{f'(3)} = \frac{1}{2(3)} = \frac{1}{6}$

$x^2 + 1 = 3$

$x^2 = 2$

$x = \cancel{\pm}\sqrt{2}$

$g'(3) = \frac{1}{2\sqrt{2}}$

$f'(x) = 3x^2 - 2$ and $x \in \mathbb{Z}$ (x has to be an integer)

3. Suppose $f(x) = x^3 - 2x + 1$ and $g(x) = f^{-1}(x)$.

   a) Find $g'(0)$. $= \frac{1}{f'(x)} = \frac{1}{f'(1)} = \frac{1}{3(1)^2 - 2} = 1$

   b) Find $g'(5)$. $= \frac{1}{f'(?)} = \frac{1}{f'(2)} = \frac{1}{3(2)^2 - 2} = \frac{1}{10}$

$x^3 - 2x + 1 = 0$

1	1	0	-2	1
		1	1	-1
	1	1	-1	0

$x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$

$x^3 - 2x + 1 = 5$

$x^3 - 2x - 4 = 0$

2	1	0	-2	-4
		2	4	4
	1	2	2	0

$x^2 + 2x + 2 = 0$

$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$ imaginary

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Version II: $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$. What is the "hazard" in writing the theorem this way?

1. Let $f(x) = x^3 + x$. If h is the inverse of f, then $h'(2) =$

   A) 1/13 B) 1/4 C) 1 D) 4 E) 13

2. Suppose $f(x) = x^2 + 1$ for $x > 0$, and $f[g(x)] = x$.

   a) Find $g'(10)$.

   b) Find $g'(3)$.

3. Suppose $f(x) = x^3 - 2x + 1$ and $g(x) = f^{-1}(x)$.

   a) Find $g'(0)$.

   b) Find $g'(5)$.

4. If $f(x) = \sin x + \cos x$, $0 \le x \le \pi$, and $g = f^{-1}$, find $g'(-1)$.

5. If $f(x) = (x-1)(x-2)(x-3) = x^3-6x^2+11x+6$ and $g = f^{-1}$, find $g'(6)$

6. If $f(x) = 5x^2 + 1$ for $x \ge 0$, and $g = f^{-1}$, find $g'(11)$

7. If $F(x) = 3x^2 - x$ for $x > 1$, and $F[t(x)] = x$, find $t'(10)$.

8. If $f(x) = (x-2)\sqrt{x+1}$. Let g be the inverse of f. Find $g'(18)$.

9. Find an equation of the line tangent to the inverse of $f(x) = \frac{x}{x-2}$ at the point (2,4).

USE OF CHARTS

x	f(x)	g(x)	f'(x)	g'(x)
2	2	-1	5	-4
3	4	2	1	0
4	-2	6	-3	2

10. Find the derivative of f^{-1} at $x = 4$.

11. Find the derivative of g^{-1} at $x = 6$.

12. Find the derivative of g^{-1} at $x = 2$.

Homework 11-09

Name: Key

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AP Calc AB: Derivatives of Inverse Trig Functions Homework

Ms. Loughran

For 1–7, find $\frac{dy}{dx}$.

1. $y=\sin^{-1}\left(\frac{1}{3}x\right)$

$y' = \frac{1}{\sqrt{1-(\frac{1}{3}x)^2}} \cdot \frac{1}{3} = \frac{1}{3\sqrt{1-\frac{x^2}{9}}} = \frac{1}{\sqrt{9-x^2}}$

2. $y=\cos^{-1}(2x+1)$

$y' = -\frac{1}{\sqrt{1-(2x+1)^2}} \cdot 2 = \frac{-\cancel{2}}{\sqrt{-4x^2-4x}} = \frac{-1}{\sqrt{-x^2-x}}$

$4x^2+4x+1$

$\sqrt{4}\sqrt{-x^2-x}$

3. $y=\tan^{-1}(x^2)$

$y' = \frac{1}{1+(x^2)^2} \cdot 2x = \frac{2x}{1+x^4}$

4. $y=\sin^{-1}\left(\frac{1}{x}\right)$

$y' = \frac{1}{\sqrt{1-(\frac{1}{x})^2}} \cdot \frac{-1}{x^2} = \frac{-1}{x^2\sqrt{1-\frac{1}{x^2}}} = \frac{-1}{\sqrt{x^4-x^2}}$

$1+\frac{1}{x^2}$ $\quad \sqrt{x^4}$

5. $y=\ln(\cos^{-1}x)$

$y' = \frac{1}{\cos^{-1}(x)} \cdot \frac{-1}{\sqrt{1-x^2}} = \frac{-1}{\cos^{-1}(x)\sqrt{1-x^2}}$

6. $y=x^2(\sin^{-1}x)^3$

7. $y=\sin^{-1}x+\cos^{-1}x$

$y' = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}} = 0$

8. Find $\frac{dy}{dx}$, if $x^3+x\tan^{-1}y=e^y$.

⑥ $y' = x^2 \cdot 3(\sin^{-1}x)^2 \cdot \frac{1}{\sqrt{1-x^2}} + 2x(\sin^{-1}(x))^3$

$\frac{3x^2(\sin^{-1}x)^2}{\sqrt{1-x^2}} + 2x(\sin^{-1}(x))^3$

⑧ $3x^2 + x \cdot \frac{1}{y^2+1} \cdot \frac{dy}{dx} + \tan^{-1}y = e^y\frac{dy}{dx}$

$3x^2 + \frac{x}{y^2+1}\frac{dy}{dx} + \tan^{-1}y = e^y\frac{dy}{dx}$

$3x^2+\tan^{-1}y = e^y\frac{dy}{dx} - \frac{x}{y^2+1}\frac{dy}{dx}$

$\frac{3x^2+\tan^{-1}y}{e^y-\frac{x}{y^2+1}} = \frac{\frac{dy}{dx}\left(e^y-\frac{x}{y^2+1}\right)}{e^y-\frac{x}{y^2+1}}$

or

$\frac{(3x^2+\tan^{-1}y)(y^2+1)}{e^y(y^2+1)-x}$

or

$\frac{(3x^2+\tan^{-1}y)(y^2+1)}{(x^3+x\tan^{-1}y)(y^2+1)-x}$

subbing in what e^y = from beginning