Do Now: Question 3 from Related Rates packet 2

3. If the volume of an expanding cube is increasing at the rate of $4\text{ cm}^3/\text{min}$, how fast is its surface area increasing when the surface area is 24 cm^2?

$\frac{dV}{dt} = 4\text{cm}^3/\text{min}$

$\frac{dSA}{dt} = ?$

$SA = 24\text{ cm}^2$

$SA = 6e^2$

$\frac{dSA}{dt} = 12e\frac{de}{dt}$

$= 12(2)\left(\frac{1}{3}\right)$

$\boxed{\frac{dSA}{dt} = 8\text{ cm}^2/\text{min}}$

* neede ond $\frac{de}{dt}$

$SA = 24$

$6e^2 = 24$

$e^2 = 4$

$e = \pm 2$

$V = e^3$

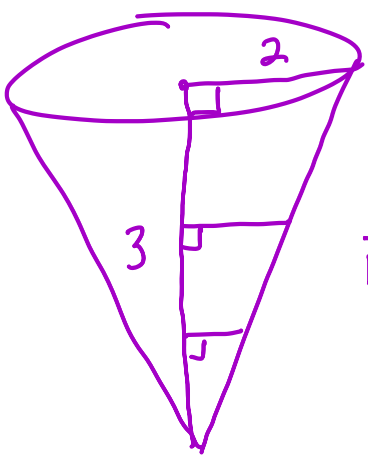
$\frac{dV}{dt} = 3e^2\frac{de}{dt}$

$4 = 3(2)^2\frac{de}{dt}$

$\frac{1}{3} = \frac{de}{dt}$

Continuing from Thursday…

6. A conical tank has a height of 3 m and a radius of 2 m at the top. Water flows in at a rate of $3\text{ m}^3/\text{min}$. How fast is the water level rising when the height is 2m?



$\frac{r}{h} = \frac{2}{3}$

$3r = 2h$

$r = \frac{2h}{3}$

$\frac{dV}{dt} = 3\text{m}^3/\text{min}$

$\frac{dh}{dt} = ?$ $\quad h = 2\text{m}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{2h}{3}\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{4h^2}{9}\right)h$

$V = \frac{4}{27}\pi h^3$

$\frac{dV}{dt} = \frac{4}{9}\pi h^2\frac{dh}{dt}$

$3 = \frac{4}{9}\pi(2)^2\frac{dh}{dt}$

$3 = \frac{16\pi}{9}\frac{dh}{dt}$

$\frac{9}{16\pi}\cdot 3 = \frac{27}{16\pi}\text{m/min} = \frac{dh}{dt}$

can write V formula with respect to one variable only if given the original dimensions of cone

If you chose not to write in terms of one variable...

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2r\frac{dr}{dt}h\right)$$

*need r

$r = \frac{2}{3}h$

$r = \frac{2}{3}(2)$

$$3 = \frac{1}{3}\pi\left(\left(\frac{4}{3}\right)^2\frac{dh}{dt} + 2\left(\frac{4}{3}\right)\left(\frac{2}{3}\frac{dh}{dt}\right)(2)\right)$$

need to solve for $\frac{dh}{dt}$

Back to packet 2 …

2. The radius of a right circular cone increases at 3 m/sec while the height decreases so that the volume is always 12π m^3. How fast is the height changing when r = 3m?

$\frac{dr}{dt} = 3$ m/s

$V = 12\pi m^3$

$\frac{dh}{dt} = ?$

$r = 3m$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$

$0 = \frac{1}{3}\pi\left(2(3)(3)(4) + 3^2\frac{dh}{dt}\right)$

$0 = 72 + 9\frac{dh}{dt}$

-8 m/s $= \frac{dh}{dt}$

*need h

$12\pi = \frac{1}{3}\pi(3)^2 h$

$12\pi = 3\pi h$

$4 = h$

4. Point P moves from the origin along a curve.
   (a) If the equation of the curve is $y = x^3 - 3x^2$, and P's x-coordinate increases at the rate of 3 units/sec., find the rate at which the distance from P to the origin is increasing when P is at (1, -2).

$\frac{dx}{dt} = 3$ units/sec

$\frac{dd}{dt} = ?$

$x = 1$

$y = -2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2}$ * from the origin

$d = \sqrt{x^2 + y^2}$

$d^2 = x^2 + y^2$

$d\frac{dd}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$

$\sqrt{5}\frac{dd}{dt} = 1(3) + (-2)(-9)$

$\frac{dd}{dt} = \frac{21}{\sqrt{5}}$ units/sec

* need d

$d^2 = 1^2 + (-2)^2$

$d^2 = 5$

$d = \pm\sqrt{5}$

need $\frac{dy}{dt}$

$y = x^3 - 3x^2$

$\frac{dy}{dt} = 3x^2\frac{dx}{dt} - 6x\frac{dx}{dt}$

$= 3(1)^2(3) - 6(1)(3)$

$= 9 - 18 = -9$

(b) If the equation of the curve is $y^2 = x^3$, and P's distance from the origin increases at the rate of 2 units/sec., find dx/dt at $(2, 2\sqrt{2})$.

$\frac{dd}{dt} = 2$ units/sec

$\frac{dx}{dt} = ?$

$x = 2$

$y = 2\sqrt{2}$

* need d

$d^2 = 2^2 + (2\sqrt{2})^2$

$d^2 = 4 + 8$

$d^2 = 12$

$d = \pm\sqrt{12} = 2\sqrt{3}$

$d^2 = x^2 + y^2$

$d \frac{dd}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$

$2\sqrt{3}(2) = 2 \frac{dx}{dt} + 2\cancel{\sqrt{2}}\left(\frac{3}{\cancel{\sqrt{2}}} \frac{dx}{dt}\right)$

$4\sqrt{3} = 8 \frac{dx}{dt}$

$\frac{4\sqrt{3}}{8} = \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{\sqrt{3}}{2}$ units/sec

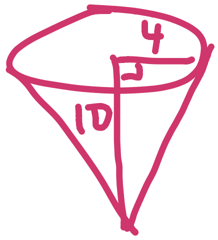
* need $\frac{dy}{dt}$

$y^2 = x^3$

$2y \frac{dy}{dt} = 3x^2 \frac{dx}{dt}$

$2(2\sqrt{2}) \frac{dy}{dt} = 3(2)^2 \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{\cancel{12}^{3}}{\cancel{4}\sqrt{2}} \frac{dx}{dt}$



$\frac{4}{10} = \frac{r}{h}$

$4h = 10r$

$\frac{2}{5}h = r$

Homework 11-10

7. A water tank in the shape of a right circular cone has a height of 10 feet. The top rim of the tank is a circle with a radius of 4 feet. If water is being pumped into the tank at the rate of 2 cubic feet per minute, what is the rate of change of the water depth, in feet per minute, when the depth is 5 feet?

$\frac{dh}{dt} = \frac{1}{2\pi}$ ft/min

$\frac{dV}{dt} = 2 \text{ ft}^3/\text{min}$ $\quad \frac{dh}{dt}?$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h$

$V = \frac{1}{3}\pi \frac{4}{25} h^2 \cdot h$

$V = \frac{4\pi}{75} h^3$

$\frac{dV}{dt} = \frac{4\pi}{25} h^2 \frac{dh}{dt}$

$2 = \frac{4\pi}{25}(5)^2 \frac{dh}{dt}$

$2 = 4\pi \frac{dh}{dt}$

$\frac{1}{2\pi}$ ft/min $= \frac{dh}{dt}$

1. A stone thrown into a pond produces a circular ripple which expands from the point of impact. If the radius of the ripple increases at the rate of 1.5m/sec, how fast is the disturbed area growing when r = 8m?

$\frac{dr}{dt} = 1.5 \text{ m/s}$ $\quad \frac{dA}{dt} = ?$

$r = 8\text{m}$

$\frac{dA}{dt} = 24\pi \text{ m}^2/\text{s}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = \cancel{2}\pi(8)\left(\frac{3}{\cancel{2}}\right) = 24\pi \text{ m}^2/\text{s}$

5. A large spherical ball is inflated by a pump which injects helium into the balloon at the rate of 10 m^3/sec. At the instant when the balloon contains $972\pi \text{ m}^3$ of gas, how fast is its radius increasing?

$\frac{dr}{dt} = \frac{10}{324\pi}$ m/s

$\frac{dV}{dt} = 10 \text{ m}^3/\text{s}$

$V = 972\pi \text{ m}^3$

$\frac{dr}{dt} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$10 = 4\pi 9^2 \frac{dr}{dt}$

$10 = 81\pi(4) \frac{dr}{dt}$

* need r

$972\pi = \frac{4}{3}\pi r^3$

$729 = r^3$

$r = 9$

$\frac{dr}{dt} = \frac{10}{324\pi}$ m/s

6. A 10ft ladder is leaning against a wall. The foot of the ladder is moving away from the wall at 2 ft/sec.

(a) How fast is the ladders top falling when the ladder is 6 feet from the wall?

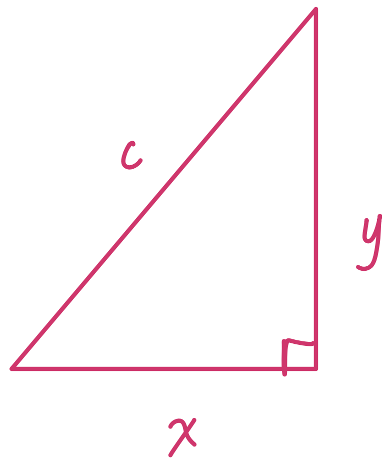
a) falling at $\frac{3}{2}$ ft/s

$c = 10\text{ ft}$

$\frac{dx}{dt} = 2\text{ ft/sec}$

$x = 6\text{ ft}$

$\frac{dy}{dt} = ?$



$c\frac{dc}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$

$10(0) = 6(2) + 8\frac{dy}{dt}$

$-12 = 8\frac{dy}{dt}$

$-\frac{12}{8} = -\frac{3}{2}\text{ ft/s} = \frac{dy}{dt}$

$6^2 + y^2 = 10^2$

$y^2 = 64$

$y = \cancel{\pm}8$