Name: ______________________________________ Date: ___________

AP Calculus AB: Related Rates Packet 1 Ms. Loughran

1. Consider a rectangular prism bathtub that has a base whose area is 18ft^2. How fast is the water level rising if water is filling the tub at a rate of $0.7\text{ft}^3/\text{min}$?

$V = lwh$

$V = Bh$

$\frac{dV}{dt} = .7 \text{ ft}^3/\text{min}$

$\frac{dh}{dt} = ?$

$B = 18 \text{ft}^2$

$\frac{dV}{dt} = h\frac{dB}{dt} + B\frac{dh}{dt}$

$.7 = h(0) + 18\frac{dh}{dt}$

$.7 = 18\frac{dh}{dt}$

$\frac{.7}{18}$ or $\frac{7}{180}$ ft/min $= \frac{dh}{dt}$

2. Assume that the radius of a sphere is expanding at a rate of 14 in / min. Determine the rate at which the surface area is changing when the radius is 8 in.

$\frac{dr}{dt} = 14 \text{ in/min}$

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

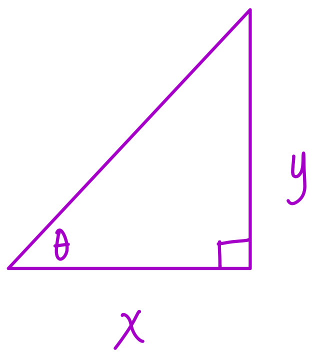
$\frac{dS}{dt} = ?$

$\frac{dS}{dt} = 8\pi(8)(14) \text{ in}^2/\text{min}$

$r = 8 \text{ in}$

$= 896\pi \text{ in}^2/\text{min}$

3. A hot air balloon rising vertically is tracked by an observer who is located 2 miles from the lift-off point. At a certain moment, the angle between the observer's line of sight and the horizontal is $\frac{\pi}{6}$, and it is changing at a rate of 0.2 radians/min. How fast is the balloon rising at this moment?



$x = 2$ miles

$\theta = \frac{\pi}{6}$

$\frac{d\theta}{dt} = .2$ radians/min

$\frac{dy}{dt} = ?$

$$\tan\theta = \frac{y}{x}$$

$$y = x\tan\theta$$

$$\frac{dy}{dt} = x\sec^2\theta\frac{d\theta}{dt} + \frac{dx}{dt}\tan\theta$$

$$\frac{dy}{dt} = 2\sec^2\left(\frac{\pi}{6}\right)(.2) + 0\cancel{\left(\tan\frac{\pi}{6}\right)}$$

$$\frac{dy}{dt} = 2\left(\frac{2}{\sqrt{3}}\right)^2(.2) \text{ miles/min}$$

4. Assume that the radius of the sphere is expanding at a rate of 14 in/min. Determine the rate at which the volume is changing with respect to time when the radius is 8 in.

$\frac{dr}{dt} = 14$ in/min

$\frac{dV}{dt} = ?$

$r = 8$ in

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi(8)^2(14) \text{ in}^3\text{/min}$$

5. A jogger runs around a circular track of radius 60ft. Let (x, y) be her coordinates where the origin is the center of the track. When the jogger's coordinates are $(36, 48)$, her x-coordinate is changing at a rate of 14 ft/s. Find $\frac{dy}{dt}$.

$\frac{dy}{dt} = ?$

$x = 36$

$y = 48$

$\frac{dx}{dt} = 14 \text{ft/s}$

$r = 60 \text{ft}$

$$x^2 + y^2 = r^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}r\frac{dr}{dt}$$

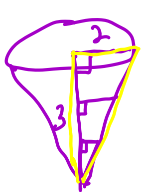
$$36(14) + 48\frac{dy}{dt} = 60(0)$$

$$36(14) + 48\frac{dy}{dt} = 0$$

$$48\frac{dy}{dt} = -36(14)$$

$$\frac{dy}{dt} = \frac{-36(14)}{48} \text{ ft/s}$$

6. A conical tank has a height of 3 m and a radius of 2 m at the top. Water flows in at a rate of 3 m^3/min. How fast is the water level rising when the height is 2m?



$\frac{2}{3} = \frac{r}{h}$

$3r = 2h$

$r = \frac{2}{3}h$

easier way

$\frac{dV}{dt} = 3 \text{ m}^3/\text{min}$

$h = 2\text{m}$

$\frac{dh}{dt} = ?$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{4}{9}h^2\right)h$$

$$V = \frac{4}{27}\pi h^3$$

$$\frac{dV}{dt} = \cancel{3}\cdot\frac{4}{\cancel{27}_{9}}\pi h^2\frac{dh}{dt}$$

$$3 = \frac{4}{9}\pi(2)^2\frac{dh}{dt}$$

$$\frac{3}{\frac{4}{9}\pi(2)^2} \text{ m/min} = \frac{dh}{dt}$$

$$\frac{3}{\frac{16}{9}\pi} \text{ m/min}$$

$$\frac{27}{16\pi} \text{ m/min}$$

$\frac{dV}{dt} = 3\ m^3/min$

$h = 2m$

$\frac{dh}{dt} = ?$

longer way

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2r\frac{dr}{dt}h\right)$$

need r and $\frac{dr}{dt}$

$$3 = \frac{1}{3}\pi\left(\left(\frac{4}{3}\right)^2\frac{dh}{dt} + 2\left(\frac{4}{3}\right)\left(\frac{2}{3}\frac{dh}{dt}\right)2\right)$$

$\frac{2}{3} = \frac{r}{h}$

$3r = 2h$

$r = \frac{2}{3}h$

when $h = 2$

$r = \frac{2}{3}(2) = \frac{4}{3}$

and if

$r = \frac{2}{3}h$

$\frac{dr}{dt} = \frac{2}{3}\frac{dh}{dt}$

$$\frac{9}{\pi} = \frac{16}{9}\frac{dh}{dt} + \frac{32}{9}\frac{dh}{dt}$$

$$\frac{9}{48}\cdot\frac{9}{\pi} = \frac{48}{9}\frac{dh}{dt}\cdot\frac{9}{48}$$

$$\frac{81}{48\pi} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{81}{48\pi} \text{ or } \frac{27}{16\pi}\ m/min$$

Homework 11-15

AP Calculus AB: Related Rates Intro Homework

1. A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/s. How rapidly is the area enclosed by the ripple increasing at the end of 10s?

$\frac{dr}{dt} = 3 \text{ ft/s}$

at the end of 10 s

$r = 10(3) = 30 \text{ ft}$

$\frac{dA}{dt} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(30)(3)$

$\frac{dA}{dt} = 180\pi \text{ ft}^2/\text{s}$

2. A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm?

$V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = -15 \text{ cm/min}$

$r = 9 \text{ cm}$

$\frac{dV}{dt} = ?$

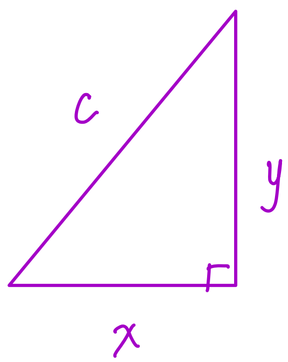
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi (9)^2(-15)$

$\frac{dV}{dt} = -4,860\pi \text{ cm}^3/\text{min}$

Air must be removed at a rate of $4,860\pi \text{ cm}^3/\text{min}$

3. A 13-ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the foot be moving away from the wall when the top is 5 ft above the ground?



$\frac{dy}{dt} = -2 ft/s$

$\frac{dx}{dt} = ?$

$c = 13$ ft

$y = 5$ ft

if $c = 13, y = 5$

$13^2 = x^2 + 5^2$

$144 = x^2$

$\pm 12 = x$

$c\frac{dc}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$

$13(0) = 12\frac{dx}{dt} + 5(-2)$

$0 = 12\frac{dx}{dt} - 10$

$10 = 12\frac{dx}{dt}$

$\frac{dx}{dt} = \frac{5}{6}$ ft/s