

Name: _____
AP Calculus AB

Date: _____
Ms. Loughran

Do Now;

1. Find $\frac{dy}{dx}$, if $e^y = \sec x$

$$e^y \frac{dy}{dx} = \sec x \tan x$$

$$\frac{dy}{dx} = \frac{\sec x \tan x}{e^y} = \frac{\sec x \tan x}{\sec x} = \tan x$$

Also #3 from Related Rates packet 2

3. If the volume of an expanding cube is increasing at the rate of $4 \text{ cm}^3 / \text{min}$, how fast is its surface area increasing when the surface area is 24 cm^2 ?

$$\frac{dV}{dt} = 4 \text{ cm}^3 / \text{min}$$

$$\frac{dSA}{dt} = ?$$

$$SA = 24 \text{ cm}^2$$

$$SA = 6s^2$$

$$\frac{dSA}{dt} = 12s \frac{ds}{dt}$$

$$\frac{dSA}{dt} = 12(2)\left(\frac{1}{3}\right) = 8 \text{ cm}^2 / \text{min}$$

needs s and $\frac{ds}{dt}$

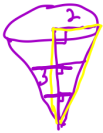
$$24 = 6s^2$$

$$4 = s^2$$

$$\pm 2 = s$$

$$\begin{aligned} V &= s^3 \\ \frac{dV}{dt} &= 3s^2 \frac{ds}{dt} \\ 4 &= 3(2)^2 \frac{ds}{dt} \\ \frac{4}{12} &= \frac{ds}{dt} \\ \frac{1}{3} &= \frac{ds}{dt} \end{aligned}$$

Wrapping up from yesterday...



$$\frac{2}{3} = \frac{r}{h}$$

$$3r = 2h$$

$$r = \frac{2}{3}h$$

6. A conical tank has a height of 3 m and a radius of 2 m at the top. Water flows in at a rate of $3 \text{ m}^3/\text{min}$. How fast is the water level rising when the height is 2m?

$$\frac{dV}{dt} = 3 \text{ m}^3/\text{min}$$

$$h = 2 \text{ m}$$

$$\frac{dh}{dt} = ?$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{2}{3}h\right)^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{4}{9}h^2\right) h$$

$$V = \frac{4}{27} \pi h^3$$

$$\frac{dV}{dt} = 3 \cdot \frac{4}{27} \pi h^2 \frac{dh}{dt}$$

$$3 = \frac{4}{9} \pi (2)^2 \frac{dh}{dt}$$

easier way

$$\frac{3}{\frac{4}{9} \pi (2)^2} \text{ m/min} = \frac{dh}{dt}$$

$$\frac{3}{\frac{16}{9} \pi} \text{ m/min}$$

$$\frac{27}{16\pi} \text{ m/min}$$

$$\frac{dV}{dt} = 3 \text{ m}^3/\text{min}$$

$$h = 2 \text{ m}$$

$$\frac{dh}{dt} = ?$$

$$V = \frac{1}{3} \pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3} \pi \left(r^2 \frac{dh}{dt} + 2r \frac{dr}{dt} h \right)$$

$$3 = \frac{1}{3} \pi \left(\left(\frac{4}{3}\right)^2 \frac{dh}{dt} + 2 \left(\frac{4}{3}\right) \left(\frac{2}{3} \frac{dr}{dt}\right) 2 \right)$$

need r
and
 $\frac{dr}{dt}$

$$\frac{2}{3} = \frac{r}{h}$$

$$3r = 2h$$

$$r = \frac{2}{3}h$$

$$\text{when } h = 2$$

$$r = \frac{2}{3}(2) = \frac{4}{3}$$

and if

$$r = \frac{2}{3}h$$

$$\frac{dr}{dt} = \frac{2}{3} \frac{dh}{dt}$$

$$\frac{9}{\pi} = \frac{16}{9} \frac{dh}{dt} + \frac{32}{9} \frac{dh}{dt}$$

$$\frac{9}{48} \cdot \frac{9}{\pi} = \frac{48}{9} \frac{dh}{dt} \cdot \frac{9}{48}$$

$$\frac{81}{48\pi} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{81}{48\pi} \text{ or } \frac{27}{16\pi} \text{ m/min}$$

longer way

Back to packet 2

2. The radius of a right circular cone increases at 3 m/sec while the height decreases so that the volume is always $12\pi \text{ m}^3$. How fast is the height changing when $r = 3\text{m}$?

$\frac{dr}{dt} = 3 \text{ m/sec}$
 $V = 12\pi \text{ m}^3$
 $\frac{dh}{dt} = ?$
 $r = 3 \text{ m}$

Volume is always $\pi r^2 h$ so it is not changing
 $V = \frac{1}{3}\pi r^2 h$
 $\frac{dV}{dt} = \frac{1}{3}\pi \left(r^2 \frac{dh}{dt} + 2r \frac{dr}{dt} h \right)$
 $0 = \frac{1}{3}\pi \left(3^2 \frac{dh}{dt} + 2(3)(3)(4) \right)$
 $0 = 9 \frac{dh}{dt} + 72$
 $-72 = 9 \frac{dh}{dt}$
 $-8 \text{ m/sec} = \frac{dh}{dt}$

need h
 $V = 12\pi$
 $12\pi = \frac{1}{3}\pi(3)^2 h$
 $12 = 3h$
 $4 = h$

4. Point P moves from the origin along a curve.

- (a) If the equation of the curve is $y = x^3 - 3x^2$, and P's x-coordinate increases at the rate of 3 units/sec., find the rate at which the distance from P to the origin is increasing when P is at (1, -2).

$\frac{dx}{dt} = 3 \text{ units/sec}$
 $\frac{dd}{dt} = ?$
 $x = 1$
 $y = -2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
 $d^2 = (x_2 - 0)^2 + (y_2 - 0)^2$
 $d^2 = x^2 + y^2$
 $2d \frac{dd}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$
 $\sqrt{5} \frac{dd}{dt} = (1)(3) + (-2)(-9)$
 $\frac{dd}{dt} = \frac{21}{\sqrt{5}} \text{ units/sec}$

$d^2 = 1^2 + (-2)^2$
 $d^2 = 5$
 $d = \pm\sqrt{5}$

need $\frac{dy}{dt} + d$
 $y = x^3 - 3x^2$
 $\frac{dy}{dt} = 3x^2 \frac{dx}{dt} - 6x \frac{dx}{dt}$
 when $x = 1, \frac{dx}{dt} = 3$
 $\frac{dy}{dt} = 3(1)^2(3) - 6(1)(3)$
 $= -9$

(b) If the equation of the curve is $y^2 = x^3$, and P's distance from the origin increases at the rate of 2 units/sec., find dx/dt at $(2, 2\sqrt{2})$.

$$\frac{dd}{dt} = 2 \text{ units/sec}$$

$$x = 2$$

$$y = 2\sqrt{2}$$

$$d^2 = x^2 + y^2$$

$$2d \frac{dd}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\sqrt{12} (2) = 2 \frac{dx}{dt} + 2\sqrt{2} \left(\frac{12}{4\sqrt{2}} \frac{dx}{dt} \right)$$

$$2\sqrt{12} = 2 \frac{dx}{dt} + 6 \frac{dx}{dt}$$

$$2\sqrt{12} = 8 \frac{dx}{dt}$$

$$\frac{2\sqrt{12}}{8} = \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{\sqrt{12}}{4} \text{ or } \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \text{ units/sec}$$

need $d + \frac{dy}{dt}$

$$d^2 = 2^2 + (2\sqrt{2})^2$$

$$d^2 = 12$$

$$d = \pm\sqrt{12}$$

$$y^2 = x^3$$

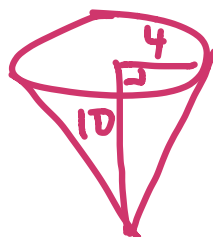
$$2y \frac{dy}{dt} = 3x^2 \frac{dx}{dt}$$

$$2(2\sqrt{2}) \frac{dy}{dt} = 3(2)^2 \frac{dx}{dt}$$

$$4\sqrt{2} \frac{dy}{dt} = 12 \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{12}{4\sqrt{2}} \frac{dx}{dt}$$

Homework 11-15



$$\frac{4}{10} = \frac{r}{h}$$

$$4h = 10r$$

$$\frac{2}{5}h = r$$

7. A water tank in the shape of a right circular cone has a height of 10 feet. The top rim of the tank is a circle with a radius of 4 feet. If water is being pumped into the tank at the rate of 2 cubic feet per minute, what is the rate of change of the water depth, in feet per minute, when the depth is 5 feet?

$$\frac{dh}{dt} = \frac{1}{2\pi} \text{ ft/min}$$

$$\frac{dV}{dt} = 2 \text{ ft}^3/\text{min} \quad \frac{dh}{dt}?$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{2}{5}h\right)^2 h$$

$$V = \frac{1}{3} \pi \frac{4}{25} h^2 \cdot h$$

$$V = \frac{4\pi}{75} h^3$$

$$\frac{dV}{dt} = \frac{4\pi}{25} h^2 \frac{dh}{dt}$$

$$2 = \frac{4\pi}{25} (5)^2 \frac{dh}{dt}$$

$$2 = 4\pi \frac{dh}{dt}$$

$$\frac{1}{2\pi} \text{ ft/min} = \frac{dh}{dt}$$

1. A stone thrown into a pond produces a circular ripple which expands from the point of impact. If the radius of the ripple increases at the rate of 1.5m/sec, how fast is the disturbed area growing when $r = 8\text{m}$?

$$\frac{dA}{dt} = 24\pi \text{ m}^2/\text{s}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi (8) \left(\frac{3}{2}\right) = 24\pi \text{ m}^2/\text{s}$$

$$\frac{dr}{dt} = 1.5 \text{ m/s} \quad \frac{dA}{dt} = ?$$

$$r = 8\text{m}$$

5. A large spherical ball is inflated by a pump which injects helium into the balloon at the rate of $10 \text{ m}^3/\text{sec}$. At the instant when the balloon contains $972\pi \text{ m}^3$ of gas, how fast is its radius increasing?

$$\frac{dr}{dt} = \frac{10}{324\pi} \text{ m/s}$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$10 = 4\pi 9^2 \frac{dr}{dt}$$

$$10 = 81\pi (4) \frac{dr}{dt}$$

* need r

$$972\pi = \frac{4}{3} \pi r^3$$

$$729 = r^3$$

$$r = 9$$

$$\frac{dV}{dt} = 10 \text{ m}^3/\text{s}$$

$$V = 972\pi \text{ m}^3$$

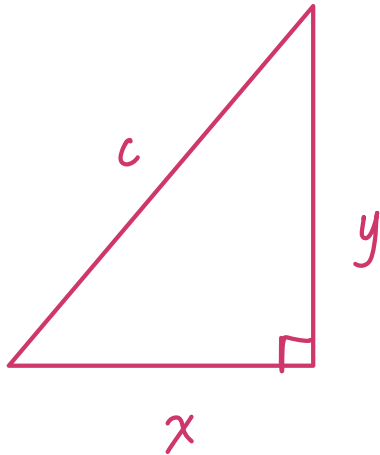
$$\frac{dr}{dt} = ?$$

$$\frac{dr}{dt} = \frac{10}{324\pi} \text{ m/s}$$

6. A 10ft ladder is leaning against a wall. The foot of the ladder is moving away from the wall at 2 ft/sec.

(a) How fast is the ladders top falling when the ladder is 6 feet from the wall?

a) falling at $\frac{3}{2}$ ft/s



$$\begin{aligned}6^2 + y^2 &= 10^2 \\y^2 &= 64 \\y &= \pm 8\end{aligned}$$

$$\begin{aligned}c &= 10 \text{ ft} \\ \frac{dx}{dt} &= 2 \text{ ft/sec} \quad \frac{dy}{dt} = ?\end{aligned}$$

$$x = 6 \text{ ft}$$

$$c \frac{dc}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$$

$$10(0) = 6(2) + 8 \frac{dy}{dt}$$

$$-12 = 8 \frac{dy}{dt}$$

$$\frac{-12}{8} = \frac{-3}{2} \text{ ft/s} = \frac{dy}{dt}$$