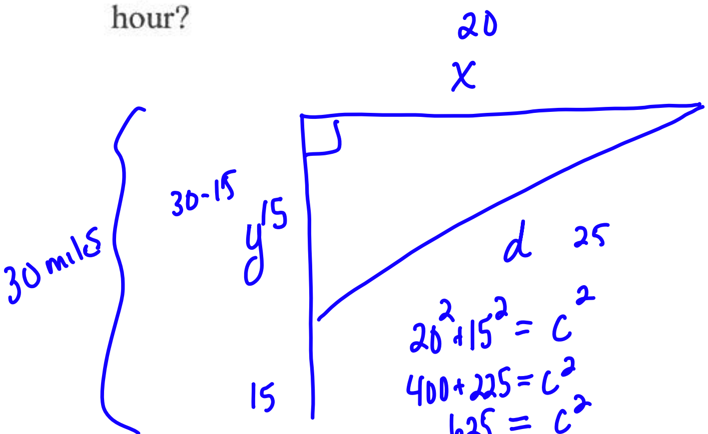
Do Now: #8 from Related Rates packet 2

8. At noon a ship is sailing due north at the uniform rate of 15 mph. Another ship, 30 miles due north of the first ship, is sailing due east at the constant rate of 20 mph. At what rate is the distance between the ships changing at the end of 1 hour?



$20^2+15^2=c^2$

$400+225=c^2$

$625=c^2$

$c=\pm 25$

$\frac{dy}{dt} = -15\text{ mph}$

$\frac{dx}{dt} = 20\text{ mph}$

$x^2+y^2=d^2$

$\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt}=\cancel{2}d\frac{dd}{dt}$

$20(20)+15(-15)=25\frac{dd}{dt}$

$\frac{dd}{dt}=\frac{20(20)+15(-15)}{25}\text{ mph}$

$\frac{dd}{dt}=7\text{ mph}$

Wrapping up Related Rates packet 2…

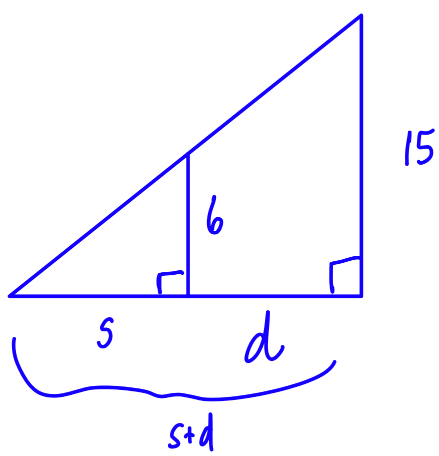
7. A 6 ft man walks away from a 15ft lamppost at 4 ft/sec..

(a) How fast is the far end of the man's shadow moving away from the lamppost?

(b) How fast is the length of his shadow increasing?

$d(d+s)/dt = ?$ $\frac{dd}{dt}+\frac{ds}{dt}=4+\frac{8}{3}=\frac{20}{3}\text{ ft/sec}$

$\frac{ds}{dt}=8/3\text{ ft/sec}$



$\frac{dd}{dt}=4\text{ft/s}$

$\frac{6}{s}=\frac{15}{s+d}$

$6s+6d=15s$

$6d=9s$

$\frac{6d}{9}=s$

$s=\frac{2}{3}d$

$\frac{ds}{dt}=\frac{2}{3}\frac{dd}{dt}$

b) $\frac{ds}{dt}=\frac{2}{3}(4)=\frac{8}{3}$

Homework 11-16

9) $\frac{r}{h} = \frac{1}{4}$

$4r = h$

$r = \frac{h}{4}$

$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$

$V = \frac{\pi}{48} h^3$

$\frac{dV}{dt} = \frac{\pi}{16} h^2 \frac{dh}{dt}$

$2 = \frac{\pi}{\cancel{16}} (\cancel{4})^2 \frac{dh}{dt}$

$2 = \pi \frac{dh}{dt}$

$\frac{2}{\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{2}{\pi}$ m/min

b) $\frac{dr}{dt} = ?$

$V = \frac{1}{3}\pi r^2 (4r)$

$V = \frac{4}{3}\pi r^3$

$4.5\cancel{\pi} = \frac{4}{3}\cancel{\pi} r^3$

$\frac{3}{4} \cdot \frac{9}{2} = \frac{4}{3} r^3 \cdot \frac{3}{4}$

$\frac{27}{8} = r^3$

$r = 3/2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$2 = 4\pi (3/2)^2 \frac{dr}{dt}$

$2 = 9\pi \frac{dr}{dt}$

$\frac{2}{9\pi}$ m/min $= \frac{dr}{dt}$

(10) $SA = 4\pi r^2$

$$\frac{dSA}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

Need r and dr/dt

To find r:

$$V = \frac{4}{3}\pi r^3$$

$$36\pi = 4/3\, \pi r^3$$

$$27 = r^3$$

$$3_m = r$$

To find dr/dt

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2\, dr/dt$$

$$10 = 4\pi (3)^2\, dr/dt$$

$$\frac{5}{18\pi}\ m/sec = \frac{dr}{dt}$$

So...

$$\frac{dSA}{dt} = 8\pi(3)\left(\frac{5}{18\pi}\right)$$

$$\frac{dSA}{dt} = \frac{120}{18} = \frac{20}{3}\ m^2/sec$$

(11) (a) $y^2 = x^3 - x^2$

$(2,2)$
$dx/dt = 3 \text{ units/sec}$
$dy/dt = ?$

$2y\, dy/dt = 3x^2\, dx/dt - 2x\, dx/dt$

$2(2)\, dy/dt = 3(2)^2(3) - 2(2)(3)$

$4\, dy/dt = 24$

$\boxed{dy/dt = 6 \text{ units/sec}}$

(b) How $\underbrace{\text{fast is the slope}}_{\text{rate of change of slope}}$ of the graph at the point changing at that instant?

$$\left. \frac{d}{dt}\left(\frac{dy}{dx}\right) \right|_{(2,2)}$$

$$\frac{dy}{dx} \Rightarrow 2y\, dy/dx = 3x^2 - 2x$$

$$\frac{dy}{dx} = \frac{3x^2 - 2x}{2y}$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{2y\left(6x\, dx/dt - 2\, dx/dt\right) - (3x^2 - 2x)(2\, dy/dt)}{4y^2}$$

$$= \frac{2(2)\big(6(2)(3) - 2(3)\big) - \big(3(2)^2 - 2(2)\big)\big(2(6)\big)}{4(2)^2}$$

$$= \frac{120 - 96}{16} = \frac{24}{16} = \frac{3}{2}$$

12 $LSA = 2\pi r h$

$$\frac{dLSA}{dt} = 2\pi\left(r\frac{dh}{dt} + h\frac{dr}{dt}\right)$$

* need $h\,dr/dt$

$$\frac{dLSA}{dt} = 2\pi\left((6)(2) + \underbrace{\quad}_{?}\right)$$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi\left(r^2\, dh/dt + h \cdot 2r\, dr/dt\right)$$

$$96\pi = \pi\left((6)^2(2) + h \cdot 2(6)\, dr/dt\right)$$

$$96 = 72 + 12h\, dr/dt$$

$$24 = 12h\, dr/dt$$

$$2 = h\, dr/dt$$

So...,

$$\frac{dLSA}{dt} = 2\pi(6(2) + 2)$$

$$\boxed{\frac{dLSA}{dt} = 2\pi(14) = 28\pi\ cm^2/sec}$$

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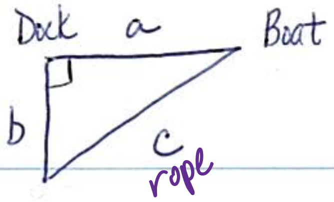
* The radius and the height are both changing. Not just filling a fixed cylinder with water.

Examples:

- Soda can in freezer (expands)
- tubbie guy outside of grand openings flying around (my Halloween costume)

(13) 

$\frac{dc}{dt} = -6 \text{ m/sec}$, $b = 3$

(a) $a^2 + b^2 = c^2$

$2a\, da/dt + 2b\, db/dt = 2c\, dc/dt$

$2(4)\, da/dt + 2(3)(0) = 2(5)(-6)$

if $b=3$ and $a=4$ then $c=5$.

$8\, da/dt + 0 = -60$

$$\boxed{\frac{da}{dt} = \frac{-60}{8} = -\frac{15}{2} \text{ m/sec}}$$

Approaching the dock at $\frac{15}{2}$ m/sec

(b) Now $da/dt = 2$, b is still 3 and $a = 12$, so $c = 3\sqrt{17}$

$2(12)(2) + 2(3)(0) = 2(3\sqrt{17})\, dc/dt$

$48 = 6\sqrt{17}\, dc/dt$

$\frac{48}{6\sqrt{17}} = \frac{dc}{dt}$

$$\frac{8}{\sqrt{17}} = \boxed{\frac{8\sqrt{17}}{17} \text{ m/sec} = \frac{dc}{dt}}$$

Why is $c = 3\sqrt{17}$? Are you serious?!!!

$a^2 + b^2 = c^2$

$12^2 + 3^2 = c^2$

$153 = c^2 \Rightarrow c = \sqrt{153}$

$c = \sqrt{9}\sqrt{17} = 3\sqrt{1}$