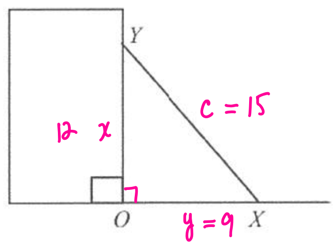
Do Now: From Related Rates packet 3 1982 AB4, 1994 AB4

1982 AB 4



$c = 15 ft$

$\frac{dy}{dt} = \frac{1}{2}$ ft/s

A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

(a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building. $\frac{dx}{dt} = ?$ $y = 9ft$

(b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

$A = \frac{1}{2}bh$

a) $x^2 + y^2 = c^2$

$x\frac{dx}{dt} + y\frac{dy}{dt} = c\frac{dc}{dt}$

$12\frac{dx}{dt} + 9\left(\frac{1}{2}\right) = 15(0)$

$\frac{dx}{dt} = \frac{-9\left(\frac{1}{2}\right)}{12}$ ft/s

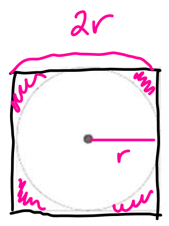
b) $A = \frac{1}{2}yx$

$\frac{dA}{dt} = \frac{1}{2}\left(y\frac{dx}{dt} + \frac{dy}{dt}x\right)$

$\frac{dA}{dt} = \frac{1}{2}\left(9\left(\frac{-9(\frac{1}{2})}{12}\right) + \frac{1}{2}(12)\right)$ ft^2/s

1994 AB 4, BC 2

$\frac{dC}{dt} = 6$ in/s



a) $P = 4(2r) = 8r$

$\frac{dP}{dt} = 8\frac{dr}{dt}$ * need $\frac{dr}{dt}$ *

$\frac{dP}{dt} = 8\left(\frac{3}{\pi}\right) = \frac{24}{\pi}$ in/s

$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$

$6 = 2\pi\frac{dr}{dt}$

$\frac{3}{\pi} = \frac{dr}{dt}$

A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$)

(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

$A = 25\pi$ in^2

$EA = A_{square} - A_{circle}$

$EA = (2r)^2 - \pi r^2$

$EA = 4r^2 - \pi r^2$

$\frac{dEA}{dt} = 8r\frac{dr}{dt} - 2\pi r\frac{dr}{dt}$ * need r

$25\pi = \pi r^2$

$25 = r^2$

$\pm 5 = r$

$\frac{dEA}{dt} = 8(5)\left(\frac{3}{\pi}\right) - 2\pi(5)\left(\frac{3}{\pi}\right)$ in^2/s

$\frac{120}{\pi} - 30$ in^2/s

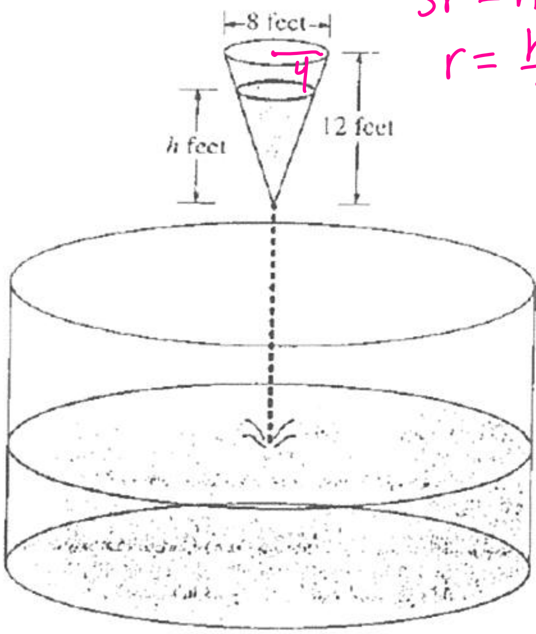
$$\frac{r}{h} = \frac{4}{12}$$

$$\frac{r}{h} = \frac{1}{3}$$

$$3r = h$$

$$r = \frac{h}{3}$$

1995 AB 5, BC 3



a) $V = \frac{1}{3}\pi r^2 h$

$$V(h) = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$$

$$V(h) = \frac{1}{3}\pi\left(\frac{h^2}{9}\right)\cdot h$$

$$V(h) = \frac{\pi h^3}{27} = \frac{\pi}{27}h^3$$

$A = 400\pi\ ft^2$

As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

$\frac{dh}{dt} = (h-12)$ ft/min

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

$\frac{dy}{dt} = ?\quad h = 3$

(b) $\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$

$$\frac{dV}{dt} = \frac{\pi}{\cancel{9}}\cancel{(3)^2}(3-12) = -9\pi\ ft^3/min$$

c) $V = \pi r^2 h$

$V = \pi r^2 y$

$\frac{dV}{dt} = \pi\left(r^2\frac{dy}{dt} + 2r\frac{dr}{dt}y\right)$

* need r
$A = \pi r^2$
$400\pi = \pi r^2$
$400 = r^2$
$* 20 = r$

$9\pi = \pi\left(20^2\frac{dy}{dt} + \cancel{2(20)(0)y}\right)$

the water that is draining out of the cone is filling up the cylinder

$9\pi = 400\pi\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{9\pi}{400\pi}$ or $\frac{9}{400}$ ft/min

Homework 11-17

Homework 11-15

Some Practice

1. Water runs into a conical tank at a rate of $9\,\text{ft}^3/\text{min}$. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?

$$\frac{dh}{dt} = \frac{1}{\pi}\ \text{ft/min}$$

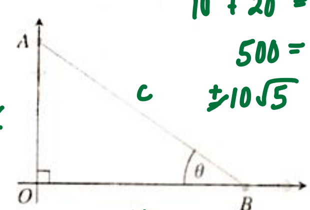
2. A spherical iron ball is coated with a layer of ice of uniform thickness. If the ice melts at the rate of 8 mL/min, how fast is the outer surface area of ice decreasing when the outer diameter (ball plus ice) is 20 cm? The surface area is decreasing at a rate of $\frac{8}{5}\ \text{cm}^2/\text{min}$

$$\frac{dS}{dt} = -\frac{8}{5}\ \text{cm}^2/\text{min}$$

3. A man 6 ft tall walks at the rate of 5 ft/sec toward a streetlight that is 16 ft above the ground. At what rate is the length of his shadow changing when he is 10 ft from the base of the light?

$$\frac{ds}{dt} = -3\ \text{ft/s}$$

4. *A* and *B* are walking on straight streets that meet at right angles. *A* approaches the intersection at 2 m/sec and *B* moves away from the intersection at 1 m/sec, as shown in the figure. At what rate is the angle θ changing when *A* is 10 m from the intersection and *B* is 20 m from the intersection?



10, x, c, y, 20

$10^2 + 20^2 = c^2$

$500 = c^2$

$\pm 10\sqrt{5} = c$

$$\frac{d\theta}{dt} = -\frac{1}{10}\ \text{rad/s}$$

$\frac{dx}{dt} = -2\ \text{m/s}$

$\frac{dy}{dt} = 1\ \text{m/s}$

$\frac{d\theta}{dt} = ?$

$x = 10,\ y = 20$

$\tan\theta = \frac{x}{y}$

$x = y\tan\theta$

$\frac{dx}{dt} = y\sec^2\theta\frac{d\theta}{dt} + \frac{dy}{dt}\tan\theta$

$-2 = 20\left(\frac{10\sqrt{5}}{20}\right)^2\frac{d\theta}{dt} + 1\left(\frac{10}{20}\right)$

$-2 = 20\left(\frac{5}{4}\right)\frac{d\theta}{dt} + \frac{1}{2}$

$-2 - \frac{1}{2} = 20\left(\frac{5}{4}\right)\frac{d\theta}{dt}$

$\frac{-2-\frac{1}{2}}{20\left(\frac{5}{4}\right)}\ \text{rad/s} = \frac{d\theta}{dt}$

$g'(y) = \frac{1}{f'(x)}$

5. If $f(x) = \frac{8}{x^3}$ and $f(g(x)) = x$, what is the value of $g'(1)$?

y

$$g'(1) = -\frac{2}{3}$$

$g'(1) = \frac{1}{f'(2)} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$

$f(x) = 8x^{-3}$

$f'(x) = -24x^{-4}$

$f'(2) = -24(2)^{-4} = \frac{-24}{16} = \frac{-3}{2}$

$\frac{8}{x^3} = 1$

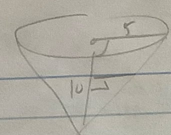
$x^3 = 8$

$x^3 - 8 = 0$

$(x-2)(x^2+2x+4) = 0$

$x = 2$ | $\cancel{x = \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2}}$ imaginary

Key



$\frac{r}{h} = \frac{5}{10} = \frac{1}{2}$

$2r = h$

$r = h/2$

1. $\frac{dV}{dt} = 9\ ft^3/min$

$h = 6ft$

dh/dt

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\pi \frac{h^3}{4} = \frac{\pi h^3}{12}$

$\frac{dV}{dt} = \frac{3}{12}\pi h^2\ dh/dt$

$9 = 1/4\ \pi\ (6)^2\ dh/dt$

$9 = 1/4\ \pi\ (\frac{9}{36})\ dh/dt$

$9 = 9\pi\ dh/dt$

$\frac{9}{9\pi} = dh/dt$

$dh/dt = \frac{1}{\pi}\ ft/min$

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* 1ml = 1cm³

2. $SA = 4\pi r^2$

$\frac{dSA}{dt} = ?$

$\frac{dV}{dt} = -8\ ml/min$

$d = 20cm$

$\therefore r = 10cm$

$dSA/dt = 8\pi r\ dr/dt$

$dSA/dt = 8\pi (10)(dV/dt)$

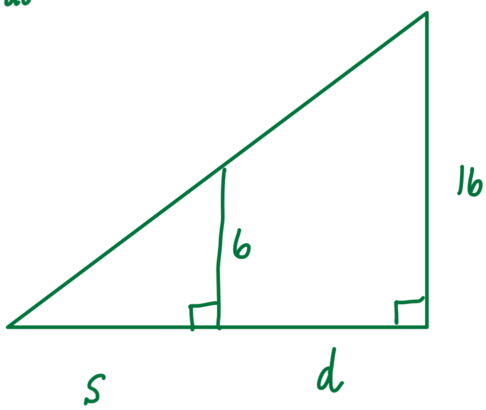
dr/dt

$\longrightarrow V = \frac{4}{3}\pi r^3$

$\frac{ds}{dt} = -\frac{8}{5}\ cm^2/min$

3. A man 6 ft tall walks at the rate of 5 ft/sec toward a streetlight that is 16 ft above the ground. At what rate is the length of his shadow changing when he is 10 ft from the base of the light?

$\frac{ds}{dt} = -3\ ft/s$



$\frac{dd}{dt} = -5\ ft/s$

$\frac{ds}{dt} = ?$

$d = 10$

$\frac{6}{16} = \frac{s}{s+d}$

$16s = 6s + 6d$

$10s = 6d$

$s = \frac{3}{5}d$

$s = \frac{3}{5}d$

$\frac{ds}{dt} = \frac{3}{5}\frac{dd}{dt}$

$\frac{ds}{dt} = \frac{3}{\cancel{5}}(-\cancel{5}) = -3\ ft/s$

2. A spherical iron ball is coated with a layer of ice of uniform thickness. If the ice melts at the rate of 8 mL/min, how fast is the outer surface area of ice decreasing when the outer diameter (ball plus ice) is 20 cm?

$\frac{ds}{dt} = -\frac{8}{5}\ cm^2/min$

$\frac{dV}{dt} = -8\ cm^3/min$

$\frac{ds}{dt} = ?$

$d = 20cm$

$r = 10$

$S = 4\pi r^2$

$\frac{ds}{dt} = 8\pi r \frac{dr}{dt}$ * need $\frac{dr}{dt}$

$\frac{ds}{dt} = 8\pi(10)\left(\frac{-8}{400\pi}\right)\ cm^2/min$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-8 = 4\pi(10)^2 \frac{dr}{dt}$

$\frac{-8}{400\pi} = \frac{dr}{dt}$