

Homework 11-21

1984 AB5

$$G: \frac{dv}{dt} = 28\pi \text{ units}^3/\text{sec}$$

$$\text{When } r = 3, V = 12\pi$$

$$\frac{dv}{dt} = \frac{1}{2} \text{ unit}/\text{sec}$$

(a) $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(3)\left(\frac{1}{2}\right)$$

$$\frac{dA}{dt} = 3\pi \text{ units}^2/\text{sec}$$

(b) $\frac{dh}{dt}?$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dv}{dt} = \frac{1}{3}\pi(2r \frac{dr}{dt} \cdot h + r^2 \frac{dh}{dt})$$

* need h

So... when $r = 3, V = 12\pi$ so

$$12\pi = \frac{1}{3}\pi(3)^2 h$$

$$12 = 3h$$

$$4 = h$$

$$28\pi = \frac{1}{3}\pi(2(3)\left(\frac{1}{2}\right)(4) + (3)^2 \frac{dh}{dt})$$

$$28 = \frac{1}{3}(12 + 9 \frac{dh}{dt})$$

$$84 = 12 + 9 \frac{dh}{dt}$$

$$72 = 9 \frac{dh}{dt}$$

$$8 \text{ units}/\text{sec} = \frac{dh}{dt}$$

(c) $\frac{dA}{dh} = \frac{dA}{dt} \cdot \frac{dt}{dh}$
 $\frac{dA}{dh} = \frac{3\pi}{8} \text{ units}^2/\text{unit}$
 $\frac{dh}{dt} = 8$
so...

$$\frac{dA}{dh} = \frac{3\pi}{8} \text{ units}^2/\text{unit}$$

$$\frac{\text{units}^2}{8} \cdot \frac{8}{\text{units}}$$

1985 AB 5, BC 2

$$dV/dt = 261\pi \text{ cm}^3/\text{min}$$

When $r=3$, $V=144\pi \text{ cm}^3$, $dr/dt = 2 \text{ cm}/\text{min}$

$$(a) V = \pi r^2 h + \frac{4}{3}\pi r^3$$

When $r=3$, $V=144\pi \text{ cm}^3$ so

$$144\pi = \pi(3)^2 h + \frac{4}{3}\pi(3)^3$$

$$144 = 9h + 36$$

$$108 = 9h$$

$$h = 12 \text{ cm}$$

$$(b) dh/dt = ?$$

$$\frac{dV}{dt} = \pi \left(r^2 \frac{dh}{dt} + 2r \frac{dr}{dt} h \right) + 4\pi r^2 \frac{dr}{dt}$$

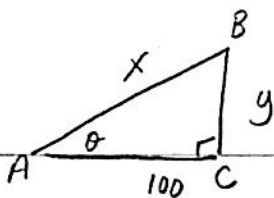
$$261\pi = \pi \left((3)^2 \frac{dh}{dt} + 2(3)(2)(12) \right) + 4\pi(3)^2(2)$$

$$261 = 9 \frac{dh}{dt} + 144 + 72$$

$$261 = 9 \frac{dh}{dt} + 216$$

$$5 \text{ cm}/\text{min} = \frac{dh}{dt}$$

1988 BC 3



$$dy/dt = 3 \text{ m/sec}$$

$$AC = 100$$

(a) $dx/dt = ?$

$$2x \frac{dx}{dt} = 2b \frac{db}{dt} + 2y \frac{dy}{dt}$$

$$x^2 = b^2 + y^2 \Rightarrow$$

$$2x \frac{dx}{dt} = 0 + 2y \frac{dy}{dt}$$

$$2(50\sqrt{5}) \frac{dx}{dt} = 2(50)(3)$$

$$100\sqrt{5} \frac{dx}{dt} = 300$$

$$\frac{dx}{dt} = \frac{3}{\sqrt{5}}$$

$$\frac{dx}{dt} = \frac{3\sqrt{5}}{5} \text{ m/sec}$$

if $y = 50$, $x^2 = 100^2 + y^2$

$$x^2 = 100^2 + 50^2$$

$$x^2 = 12500$$

$$x = 50\sqrt{5}$$

(b) $dA/dt = ?$

$$y = 50$$

$$A = \frac{1}{2} y b$$

$$\frac{dA}{dt} = \frac{1}{2} (y \frac{db}{dt} + b \frac{dy}{dt})$$

$$\frac{dA}{dt} = \frac{1}{2} b \frac{dy}{dt}$$

$$\frac{dA}{dt} = \frac{1}{2} (100) \frac{dy}{dt}$$

$$\frac{dA}{dt} = 50(3)$$

$$\frac{dA}{dt} = 150 \text{ m}^2/\text{sec}$$

(c) $\tan \theta = \frac{y}{b}$, $d\theta/dt = ?$ $y = 50$

$$y = b \tan \theta$$

$$\frac{dy}{dt} = \tan \theta \frac{db}{dt} + b \sec^2 \theta \frac{d\theta}{dt}$$

$$\frac{dy}{dt} = 100 \cdot \sec^2 \theta \frac{d\theta}{dt}$$

$$3 = 100 \cdot \sec^2 \theta \frac{d\theta}{dt}$$

$$3 = 100 \cdot \left(\frac{50\sqrt{5}}{100}\right)^2 \frac{d\theta}{dt}$$

$$3 = 100 \cdot \frac{12500}{100^2} \frac{d\theta}{dt}$$

$$3 = 125 \frac{d\theta}{dt}$$

$$\frac{3}{125} \text{ rad/sec} = \frac{d\theta}{dt}$$

1990 AB 4

$$dr/dt = 0.04 \text{ cm/sec}$$

(a) $r=10$, $dv/dt = ?$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi(10)^2(0.04)$$

$$\frac{dv}{dt} = 16\pi \text{ cm}^3/\text{sec}$$

(b) $V = 36\pi \text{ cm}^3$

$$36\pi = \frac{4}{3}\pi r^3$$

$$36 = \frac{4}{3}r^3$$

$$27 = r^3$$

$$3 = r$$

A cross section \rightarrow circle

$$A = \pi r^2$$

$$dA/dt = 2\pi r \frac{dr}{dt}$$

$$dA/dt = 2\pi(3)(0.04)$$

$$dA/dt = 0.24\pi \text{ cm}^2/\text{sec}$$

(c) $dv/dt = dr/dt$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

if $\frac{dv}{dt} = \frac{dr}{dt}$

$$1 = 4\pi r^2$$

$$\frac{1}{4\pi} = r^2$$

$$r = \frac{1}{2\sqrt{\pi}} \text{ cm}$$