Name: ______________________________

Calculus AB: Graphs of Derivatives

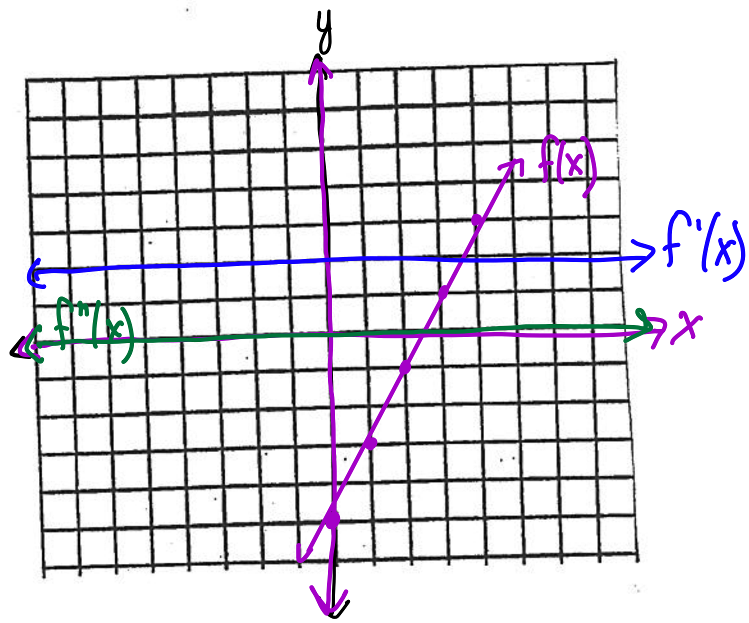
Make a sketch of $f(x)$ and $f'(x)$ on the axes provided.

1. $f(x) = 2x - 5$

$f'(x) = 2$

$f''(x) = 0$

If $f(x)$ is a linear functions ($f(x) = mx + b$), $f'(x)$ is always a constant, namely m. and $f''(x) = 0$

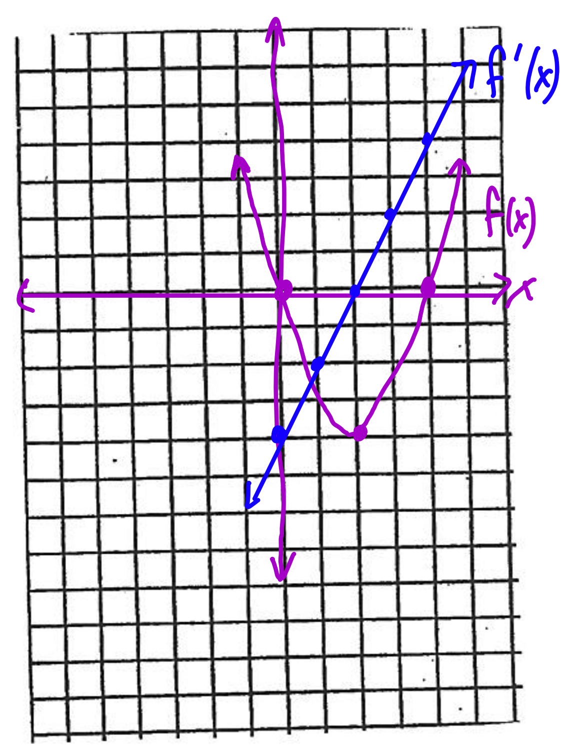


$f(x) = x(x-4)$ $\qquad$ $f(2) = 4 - 4(2) = -4$

2. $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$f''(x) = 2$



When $f(x)$ is increasing, $f'(x) > 0$

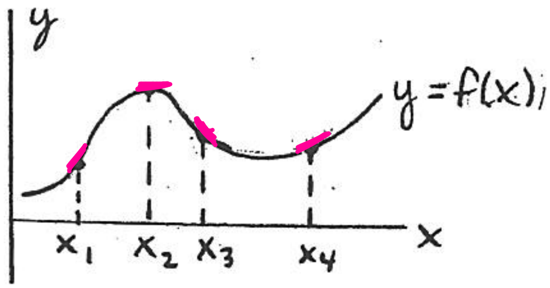
When $f(x)$ is decreasing, $f'(x) < 0$

Where there are horizontal tangent lines to $f(x)$, $f'(x) = 0$

When $f''(x) > 0$, $f(x)$ is concave up $\cup$

When $f''(x) < 0$, $f(x)$ is concave down $\cap$

Visual Estimates of Derivatives



Using the graph of $y = f(x)$, fill in =, >, or <:

$f'(x_2) = 0$

$f'(x_4) > 0$

$f'(x_3) < 0$

$f'(x_1) > 0$

$f'(x_1) > f'(x_4)$

$f'(\overset{-}{x_3}) < f(\overset{+}{x_3})$

$f'(\overset{0}{x_2}) < f'(\overset{+}{x_1})$

$f'(\overset{0}{x_2}) > f'(\overset{-}{x_3})$

$f'(\overset{-}{x_3}) < f'(\overset{+}{x_4})$

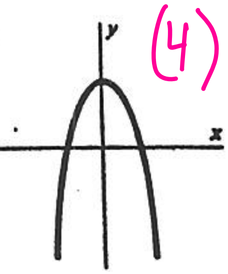
MATCHING QUESTIONS ON GRAPHS OF DERIVATIVES

Each of the Figures (1)-(5) is the graph of the derivative of one of the functions graphed in Figures (a)-(e). Match each function with its derivative.

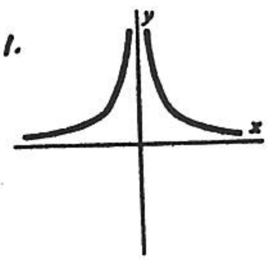
Functions Derivatives

f ↗ HTL ↘
f′ + 0 −

a. (4)

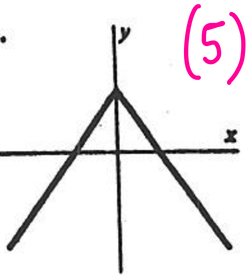


1.

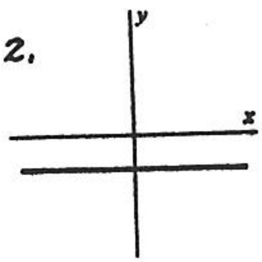


f linear ↗ corner linear ↘
f′ positive constant not differentiable negative constant

b. (5)

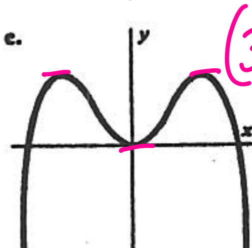


2.

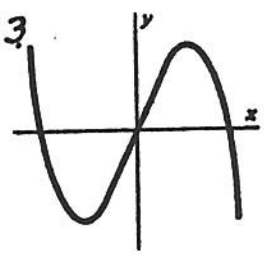


f ↗ HLT ↘ HLT ↗ HLT ↘
f′ + 0 − 0 + 0 −

c. (3)

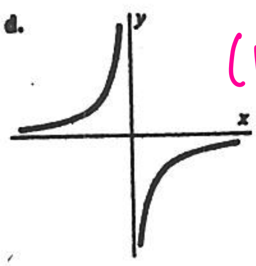


3.

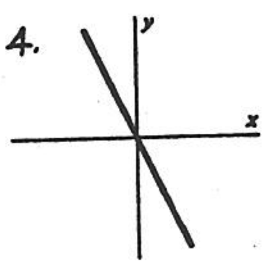


f ↗ VA ↗
f′ + not diff +

d. (1)

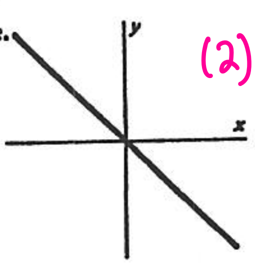


4.

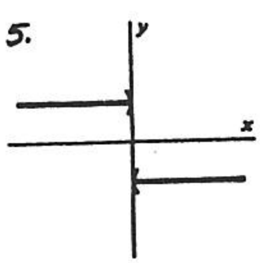


f ↘ linear
f′ ⊖ constant

e. (2)



5.



Homework 11-28

2003

1. If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$ $2(x^3+1) \cdot 3x^2$

(A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

---

2. $\int_0^1 e^{-4x}\,dx =$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4} - 1$ (D) $\frac{1}{4} - \frac{e^{-4}}{4}$ (E) $4 - 4e^{-4}$

---

3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x \to 2} f(x) = \infty$

(E) $\lim_{x \to \infty} f(x) = 2$

$6x+4-(6x+9)$

4. If $y = \dfrac{2x+3}{3x+2}$, then $\dfrac{dy}{dx} = \dfrac{(3x+2)(2)-(2x+3)(3)}{(3x+2)^2}$

(A) $\dfrac{12x+13}{(3x+2)^2}$ (B) $\dfrac{12x-13}{(3x+2)^2}$ (C) $\dfrac{5}{(3x+2)^2}$ (D) $\dfrac{-5}{(3x+2)^2}$ (E) $\dfrac{2}{3}$

GO ON TO THE NEXT PAGE.

5. $\int_0^{\frac{\pi}{4}} \sin x \, dx =$

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2} - 1$ (D) $-\frac{\sqrt{2}}{2} + 1$ (E) $\frac{\sqrt{2}}{2} - 1$

---

6. $\lim_{x \to \infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

(A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1

---

8. $\int x^2 \cos(x^3)\, dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

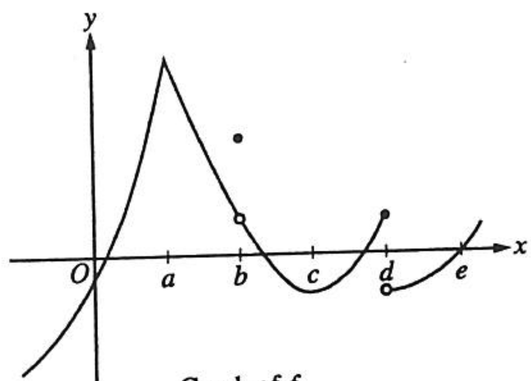
---

9. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

$f'(x) = \frac{1}{x+4+e^{-3x}} \cdot 1-3e^{-3x}$

$f'(0) = \frac{1}{0+4+e^{0}}(1-3e^{0}) = \frac{-2}{5}$



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

---

14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

$x^2\, 2\cos(2x) + 2x\sin 2x$

$2x(x\cos(2x) + \sin 2x)$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

---

15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only

(B) $(-\infty, 0)$

(C) $[-1, 0)$ only

(D) $(0, \sqrt[3]{2}]$

(E) $[\sqrt[3]{2}, \infty)$

---

16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

$$m = \frac{7-(-2)}{1-(-2)} = \frac{9}{3} = 3$$

---

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point (1, 2) ?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

$x+2 = 4x-7$ @ $x=3$

$5=5$

$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

20. Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists. ✓

II. f is continuous at $x = 3$. ✓

III. f is differentiable at $x = 3$.

$$f'(x) = \begin{cases} 1 & x \le 3 \\ 4 & x > 3 \end{cases}$$

$1 \ne 4$

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

---

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 7x - 3$

(B) $y = 7x + 7$

(C) $y = 7x + 11$

(D) $y = -5x - 1$

(E) $y = -5x - 5$

$f(-1) = 4(-1)^3 - 5(-1) + 3 = -4 + 5 + 3 = 4$

$f'(x) = 12x^2 - 5$

$f'(-1) = 12(-1)^2 - 5 = 7$

$y - 4 = 7(x + 1)$

$y = 7x + 11$

---

25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

$v(t) = 6t^2 - 42t + 72$

$v(t) = 6(t^2 - 7t + 12)$

$v(t) = 6(t-3)(t-4)$

$0 = 6(t-3)(t-4)$

$t = 3, 4$

---

26. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

$(3,2)$

$6y\frac{dy}{dx} - 4x = -2x\frac{dy}{dx} - 2y$

$6(2)\frac{dy}{dx} - 4(3) = -2(3)\frac{dy}{dx} - 2(2)$

$12\frac{dy}{dx} - 12 = -6\, dy/dx - 4$

$18\, dy/dx = 8$

$dy/dx = \frac{8}{18} = \frac{4}{9}$

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

$x^3 + x = 2$

$x^3 + x - 2 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

$x^2 + x + 2 = 0$

$x = \frac{-1 \pm \sqrt{1 - 4(1)(2)}}{2(1)}$ ← imaginary

$g'(2) = \frac{1}{f'(1)} = \frac{1}{4}$

$f'(x) = 3x^2 + 1$

$f'(1) = 3(1)^2 + 1 = 4$

$g: (2,1)$

$f(1, 2)$

$g'(y) = \frac{1}{f'(x)}$ if f and g are inverses and (x, y) is a pt on f

CALCULUS AB
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

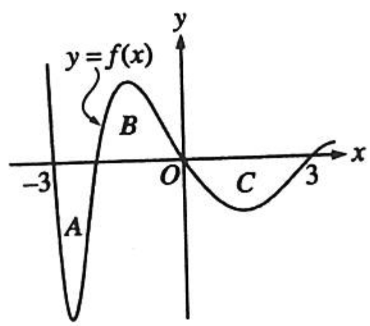
GO ON TO THE NEXT PAGE.

Section I
Part B

76. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.978

$a(4) = v'(4) = 1.6329$



77. The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{-3}^{3} (f(x) + 1)\,dx$?

(A) -2 (B) -1 (C) 4 (D) 7 (E) 12

$\frac{dr}{dt} = .2\text{ m/sec}$ $\quad C = 20\pi$

78. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) $0.04\pi\ \text{m}^2/\text{sec}$

(B) $0.4\pi\ \text{m}^2/\text{sec}$

(C) $4\pi\ \text{m}^2/\text{sec}$

(D) $20\pi\ \text{m}^2/\text{sec}$

(E) $100\pi\ \text{m}^2/\text{sec}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\quad$ * need r

$C = 2\pi r$

$20\pi = 2\pi r$

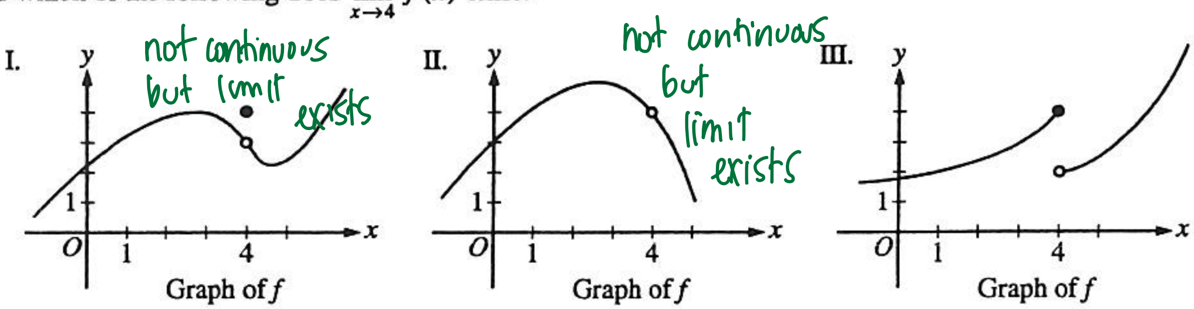
$20 = 2r$

$r = 10$

$\frac{dA}{dt} = 2\pi \cdot 10\,(.2)$

$= 20\pi \cdot \frac{2}{10} = 4\pi\ \text{m}^2/\text{sec}$

79. For which of the following does $\lim_{x \to 4} f(x)$ exist?



(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

89. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

(C) $y - 6 = -5(x - 2)$

(D) $y - 6 = -7(x - 2)$

(E) $y - 6 = -10(x - 2)$

$g(2) = 2 \cdot f(2) = 2 \cdot 3 = 6$

$g'(x) = xf'(x) + f(x)$

$g'(2) = 2 \cdot f'(2) + f(2)$

$= 2 \cdot (-5) + 3 = -10 + 3 = -7$

$y - 6 = -7(x - 2)$

---

90. For all x in the closed interval $[2, 5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	7
3	9
4	12
5	16

(B)

x	$f(x)$
2	7
3	11
4	14
5	16

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

(E)

x	$f(x)$
2	16
3	13
4	10
5	7
