Remember:

f ↗ ↘

f' + −

Likewise

f' ↗ ↘

f'' + −

CONCLUSIONS ABOUT FUNCTIONS FROM THEIR DERIVATIVES

1. If $f'(x) > 0$ on the interval (a,b), then the function is increasing on (a,b).
2. If $f'(x) < 0$ on the interval (a,b), then the function is decreasing on (a,b).
3. If $f''(x) < 0$ on the interval (a,b), then the function is concave down on (a,b). ∩
4. If $f''(x) > 0$ on the interval (a,b), then the function is concave up on (a,b). ∪

A **critical point** is defined as a point on the graph where the derivative is either equal to zero or does not exist.

f ↗ ↘

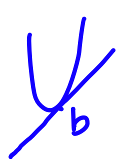
If $f'(x)$ changes from positive to negative around a critical point c, then there is a **relative maximum** point at x = c.

f ↘ ↗

If $f'(x)$ changes from negative to positive around a critical point c, then there is a **relative minimum** point at x = c.

A **point of inflection** is a point on the graph where either $f''(x) = 0$ or does not exist, **and there is a change of concavity at that point.**

concave up



f is concave up at b if there is an interval around b so that the curve lies above the tangent line at b.

concave down

f is concave down at b if there is an interval around b so that the curve lies below the tangent line at b

Do Now: #1

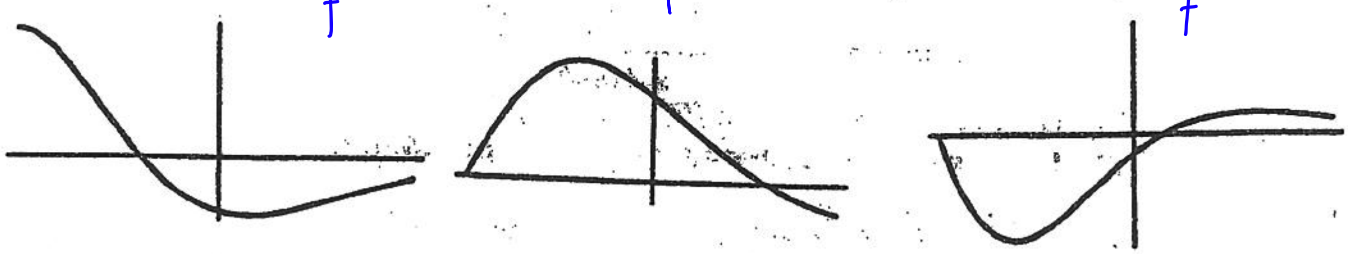
Name: ______________________________ Date: __________

AP Calculus AB Using Graphs of Derivatives

1.

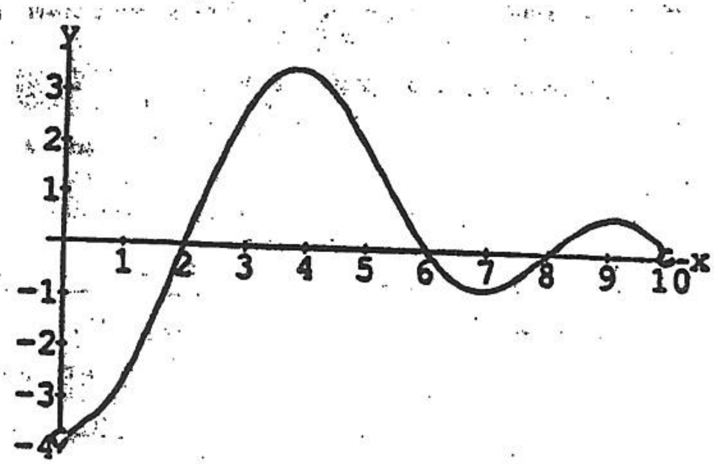
The graphs (i), (ii), and (iii) given below are the graphs of a function f and its first two derivatives f' and f'' (though not necessarily in that order). Identify which of these graphs is the graph of f, which is that of f' and which is that of f''. Justify your answer.

(i) f' (ii) f (iii) f''



2.

The graph below is the graph of the *derivative* of a function f. Use this graph to answer the following questions about f on the interval $(0, 10)$. In each case be sure to justify your answer.



a. On what subinterval(s) is f increasing? $(2,6) \cup (8,10)$ b/c f' is ⊕ in those intervals.

b. On what subinterval(s) is f decreasing? $(0,2) \cup (6,8)$ b/c f' is ⊖ in those intervals

∪ c. Find the x-coordinates of all relative minima of f. f ↘ ↗ , f' − to + $x=2,8$

∩ d. Find the x-coordinates of all relative maxima of f. f ↗ ↘ f' + to − $x=6$

e. On what subinterval(s) is f concave up? f'' ⊕ f' ↗ $(0,4) \cup (7,9)$

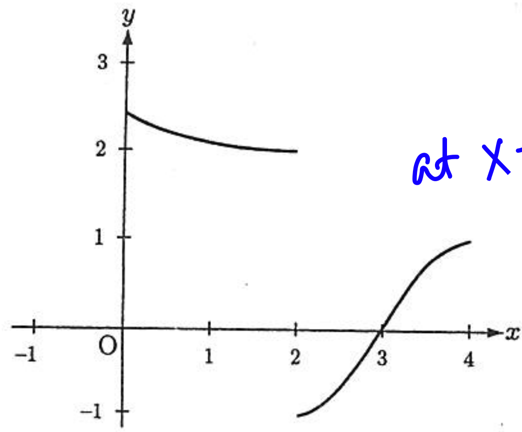
f. On what subinterval(s) is f concave down? f'' ⊖ f' ↘ $(4,7) \cup (9,10)$

g. Find the x-coordinates of all points of inflection of f.

f'' + to − or f'' − to + $x=4,7,9$

f' ↗ ↘ or ↘ ↗

3.



at $x=2$ there has to be a sharp edge
It can't be a hole or asym

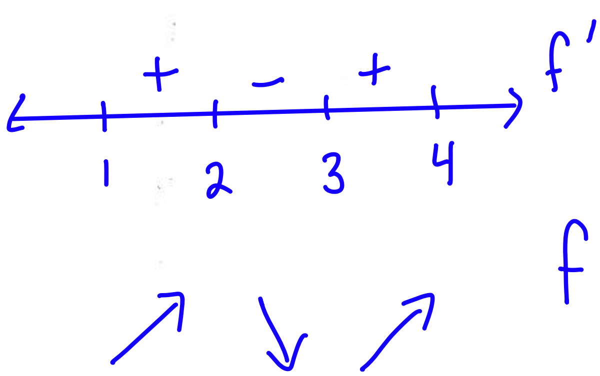
The figure above shows the graph of the derivative of a continuous function f for $0 \le x \le 4$.

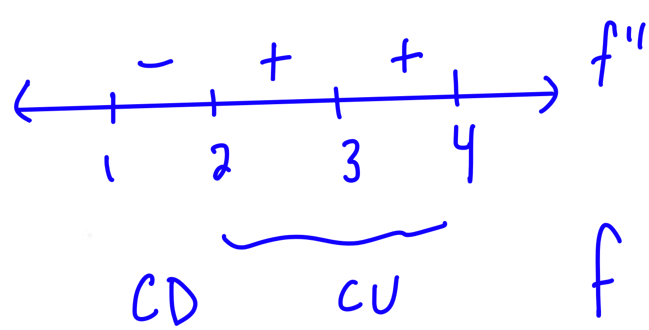
(a) For what values of x is f increasing? Justify your answer. $(3,4) \cup (0,2)$ b/c f' is +

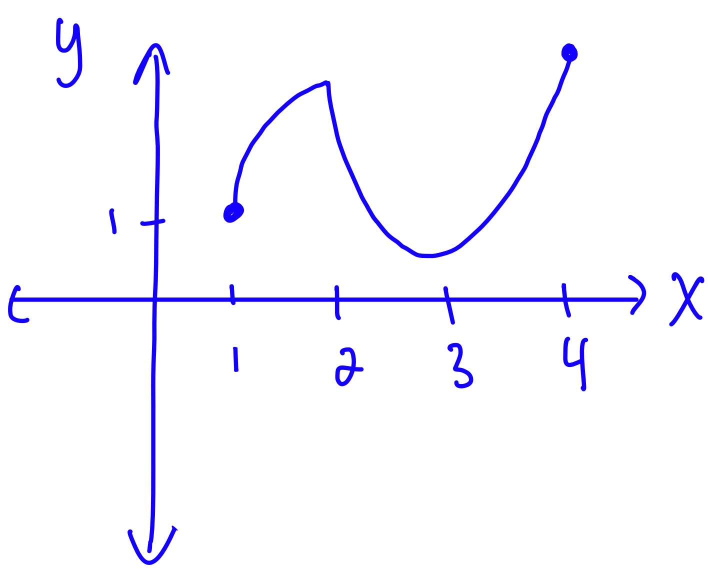
(b) For what values of x does f have its relative minimum value? Justify your answer.
$x=3$ b/c f' changes sign from − to +

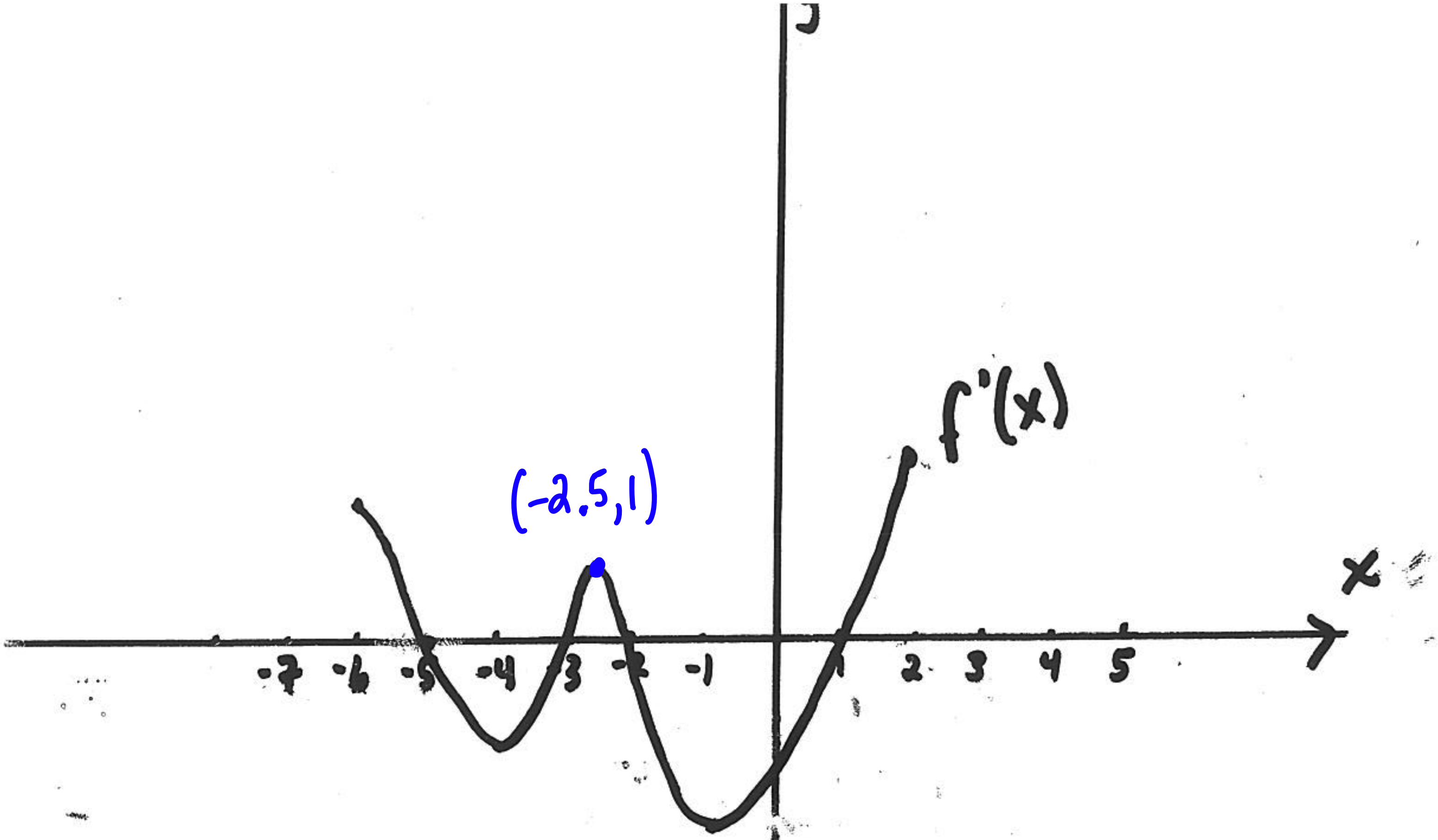
(c) For what values of x does f have its relative maximum value? Justify your answer.
$x=2$ b/c f' changes sign from + to −

(d) If $f(1)=1$, use your answers to (a), (b), and (c) to sketch the graph of f for $1 \le x \le 4$.









a. On what subinterval(s) is f increasing?

$(-6,-5) \cup (-3,-2) \cup (1,2)$ b/c f' is ⊕ on those intervals

b. On what subinterval(s) is f decreasing?

$(-5,-3) \cup (-2,1)$ b/c f' is ⊖ in those intervals

c. Find the x-coordinates of all relative minima of f.

$x = -3, 1$ b/c f' changes sign $-$ to $+$

d. Find the x-coordinates of all relative maxima of f

$x = -5, -2$ b/c f' changes sign $+$ to $-$

e. On what subinterval(s) is f concave up?

f'' ⊕ $f' \nearrow$ $(-4,-2.5) \cup (-1,2)$

f. On what subinterval(s) is f concave down?

f'' $-$ $f' \downarrow$ $(-6,-4) \cup (-2.5,-1)$

g. Find the x-coordinates of all points of inflection of f.

f'' $+$ to $-$ or $-$ to $+$

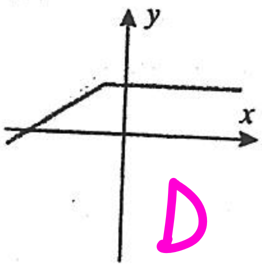
f' $\nearrow \downarrow$ or $\downarrow \nearrow$ $x = -4, -2.5, -1$

Homework 11-29

Match the graphs of the functions shown in (a)–(f) with the graphs of their derivatives in (A)–(F).

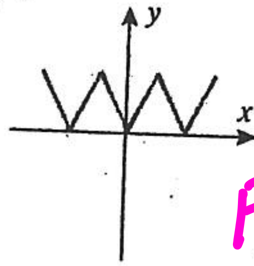
Functions

(a)



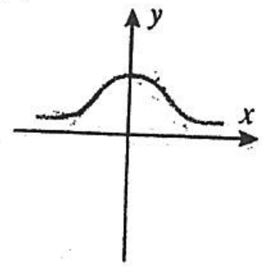
D

(b)



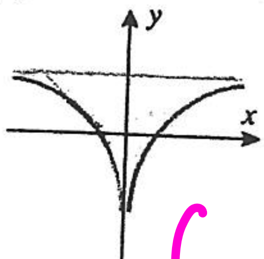
F

(c)



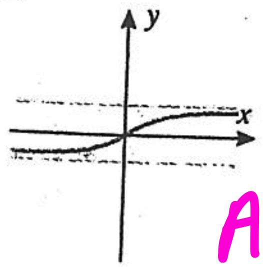
B

(d)



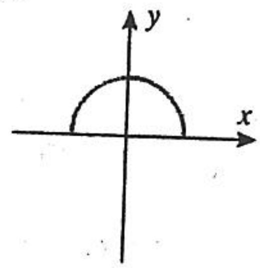
C

(e)



A

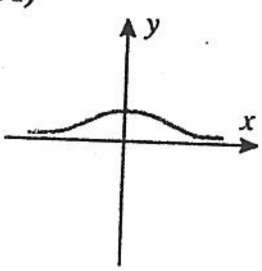
(f)



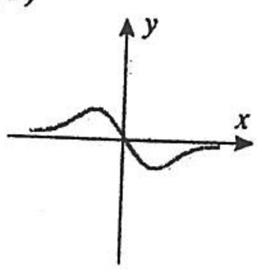
E

Derivatives

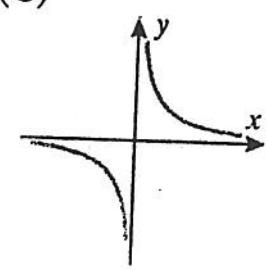
(A)



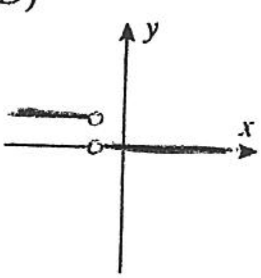
(B)



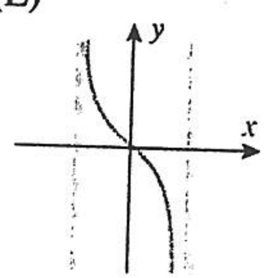
(C)



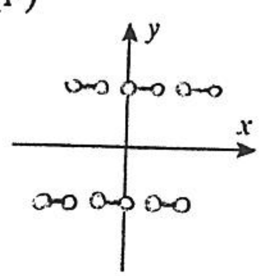
(D)



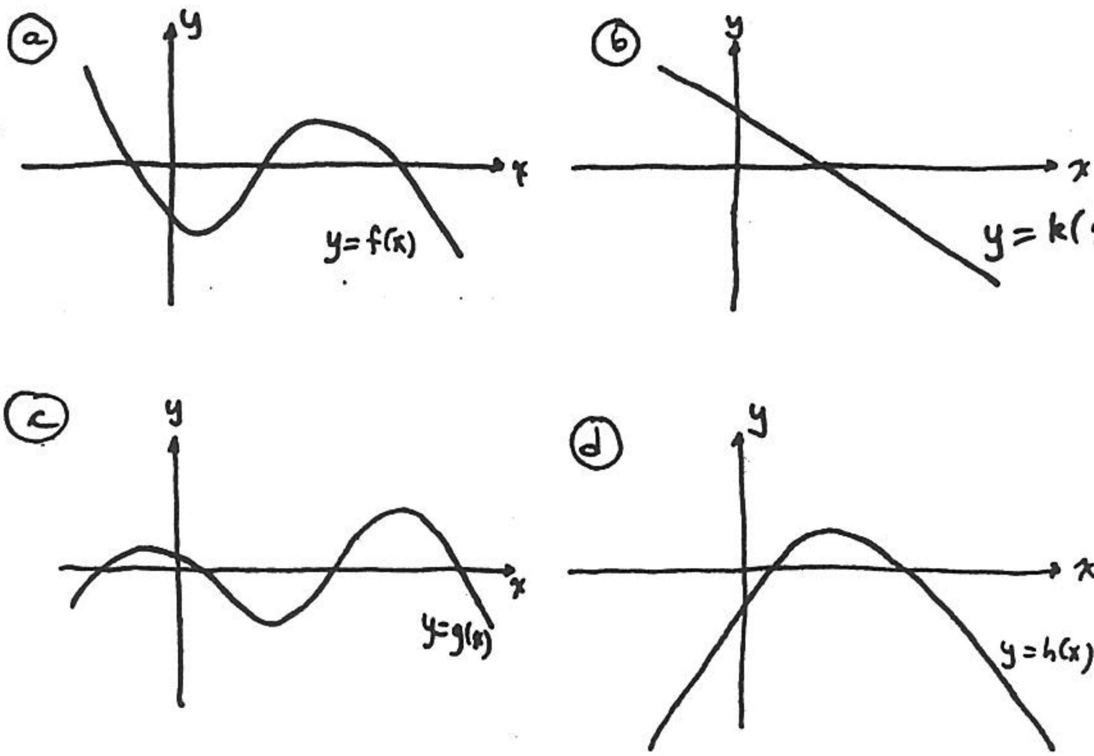
(E)



(F)



GRAPHING THE DERIVATIVE



1. Let the graph in (d) be that of $y = h(x)$.
   Which of the others is the graph of $y = h'(x)$? k(x)

2. Which is the graph of $y = f'(x)$? h(x)

3. Which is the graph of $y = g'(x)$? f(x)

4. Which is the graph of $y = g''(x)$? h(x)

5. Which is the graph of $y = g'''(x)$? k(x)