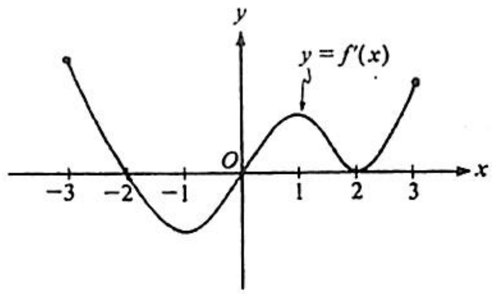
Name: ____ Date: ____________

AP Calculus AB: Analyzing Graphs Ms. Loughran

Do Now: 1985 AB 6

1985 AB 6



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of the function f is the set of all x such that $-3 \leq x \leq 3$.

(a) For what values of x, $-3 < x < 3$, does f have a relative maximum? A relative minimum? Justify your answer.

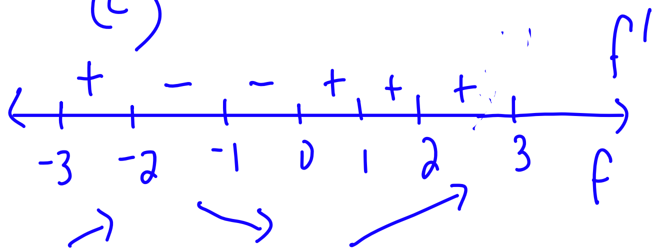
(b) For what values of x is the graph of f concave up? Justify your answer.

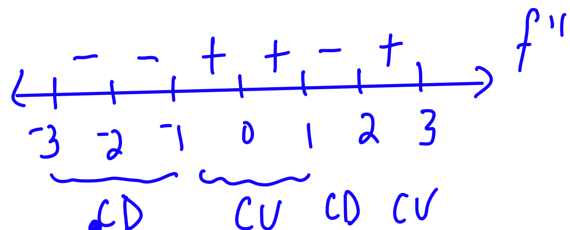
(c) Use the information found in parts (a) and (b) and the fact that $f(-3) = 0$ to sketch a possible graph of f on the axes provided below.

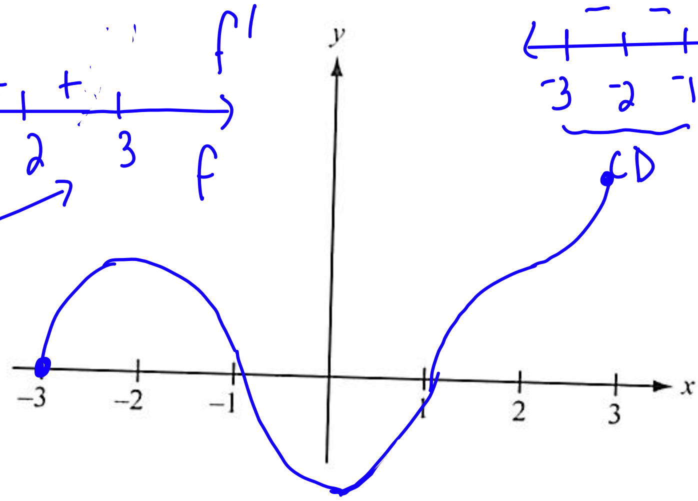
(a) At $x = -2$ there is a rel. max b/c f' goes from + to − at $x = -2$.
At $x = 0$ there a rel min b/c f' goes from − to + at $x = 0$

(b) $(-1, 1)$ and $(2, 3)$ b/c f' is increasing over those intervals.

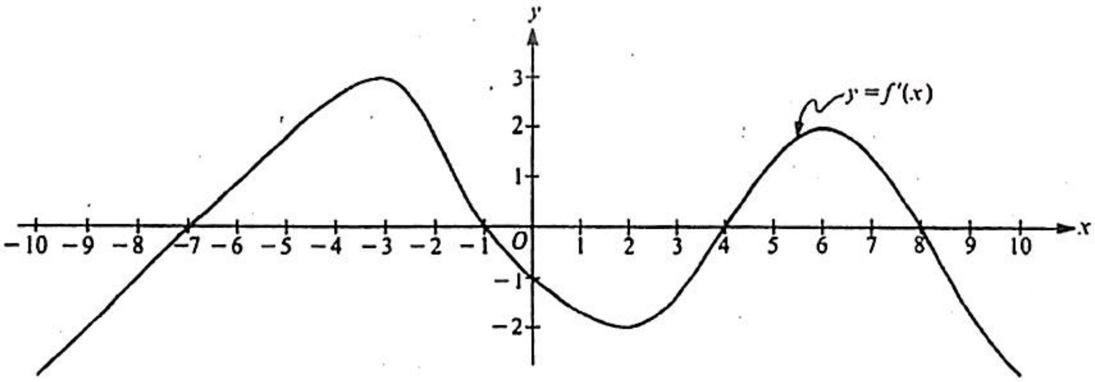
(c)







1989 AB 5



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \leqq x \leqq 10$.

(a) For what values of x does the graph of f have a horizontal tangent? $x = -7, -1, 4, 8$

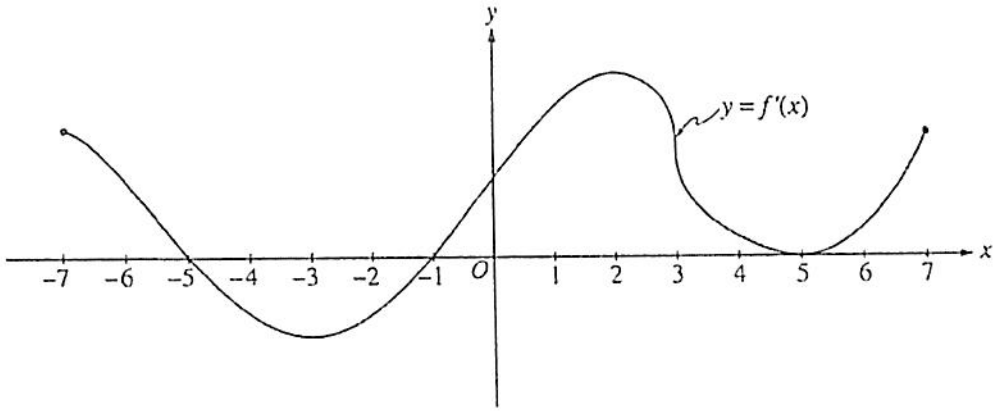
(b) For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer. $x = -1, 8$ b/c f' changes sign from + to −

(c) For what values of x is the graph of f concave downward?

f is CD where $f'' < 0$, $f'' < 0$ where $f' \downarrow$

$(-3, 2) \cup (6, 10)$

2000 AB 3



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

~~(d)~~ At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

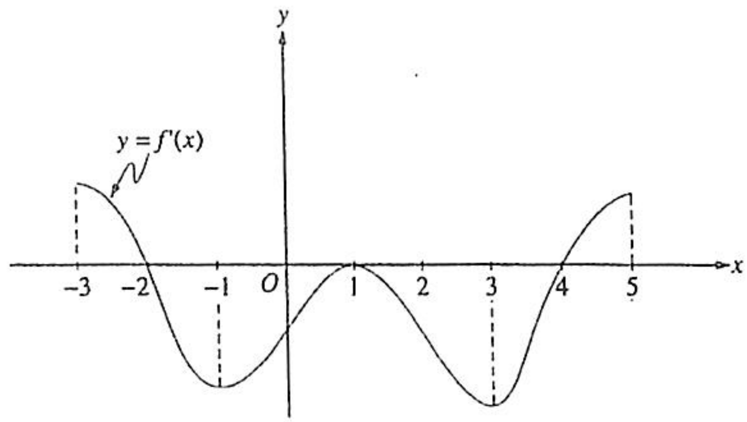
a) $x = -1$ b/c f' goes from − to +

b) $x = -5$ b/c f' goes from + to −

c) $f''(x) < 0$ means the graph of f is concave down. f'' is ⊖ when $f'(x)$ is decreasing so $(-7, -3) \cup (2, 5)$, $x \neq 3$

($x \neq 3$ b/c since f' has a vertical tan line at $x = 3$ there is a pt. of nondifferentiability at $x = 3$ on f)

1996 AB 1



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

(a) For what values of x does f have a relative maximum? Why?

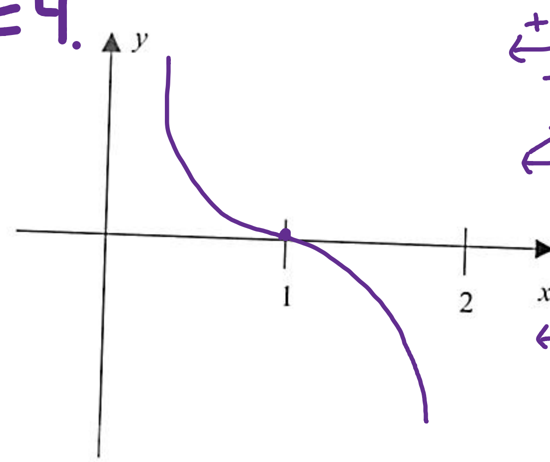
(b) For what values of x does f have a relative minimum? Why?

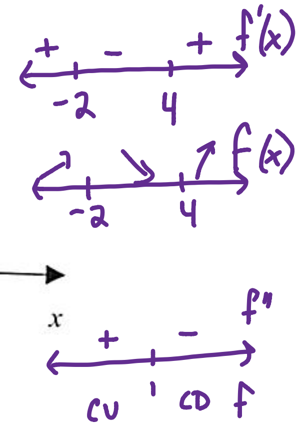
(c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

(d) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

a) f has a relative maximum when f' changes from + to − so at $x=-2$.

b) f has a relative minimum when f' changes from − to + so at $x=4$.





c) f is CU when f'' is ⊕
f'' is ⊕ when f' is ↗
so $(-1,1)$ and $(3,5)$

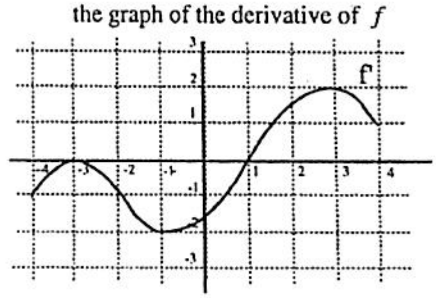
1979 AB 3, BC 3

Find the maximum volume of a box that can be made by cutting out squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer.

Review Book Question

The graph of the **derivative** of f is shown in the figure.

the graph of the derivative of f



(a) Suppose that $f(3) = 1$. Find an equation of the line tangent to the graph of f at the point (3, 1).

(b) Where does f have a local minimum? Explain briefly.

(c) Estimate $f''(2)$.

(d) Where does f have an inflection point? Explain briefly.

(e) Where does f achieve its maximum on the interval $[1, 4]$?

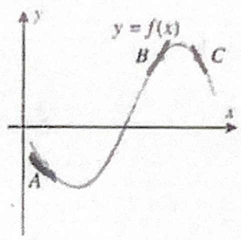
Homework 12-06

Name: Key　　　　　　　　　　　　　　Date: ____________

AP Calc AB: Testing for relative extrema and points of inflection homework

1. Use the graph of the equation $y = f(x)$ in the accompanying diagram to find the signs of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the points *A*, *B*, and *C*.

	$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$	
A	−	+	concave up
B	+	−	concave down
C	−	−	

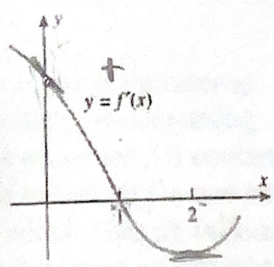


f(x)

(6)

2. Use the graph of $y = f'(x)$ in the accompanying figure to replace the question marks with <, =, or >, as appropriate. Explain your reasoning.

f(x)



b/w (0,1) $f'(x) > 0$ so $f(x)\uparrow$

(0,1) f' + $f\uparrow$

(1,2) $f'(x)$ − $f(x)\downarrow$

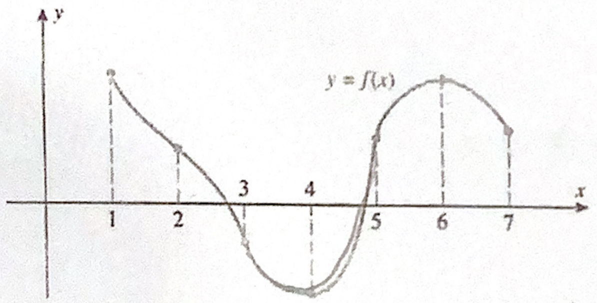
this is the graph of f'　$f'(0)$ +

(a) $f(0)$ < $f(1)$　　(b) $f(1)$ > $f(2)$　　(c) $f'(0)$ > 0 (+)

(d) $f'(1)$ = 0 (0)　　(e) $f''(0)$ < 0 (−)　　(f) $f''(2)$ = 0

the derivative of the derivative

3. In each part, use the graph of $y = f(x)$ in the accompanying figure to find the requested information.
   (a) Find the intervals on which f is increasing. (4,6)
   (b) Find the intervals on which f is decreasing. (1,4), (6,7)
   (c) Find the open intervals on which f is concave up. (1,2), (3,5)
   (d) Find the open intervals on which f is concave down. (2,3), (5,7)
   (e) Find all values of x at which f has an inflection point. x=2, 3, 5



y=f(x)

For questions 4-6,
   (a) Find the intervals on which f is increasing.
   (b) Find the intervals on which f is decreasing. f'
   (c) Find the open intervals on which f is concave up.
   (d) Find the open intervals on which f is concave down. f''
   (e) Find all values of x at which f has an inflection point.
   (f) Find the x values of any relative minimums.
   (g) Find the x values of any relative maximums. f'

4. $f(x) = x^2 - 5x + 6$

5. $f(x) = 3x^4 - 4x^3$

6. $f(x) = \dfrac{x^2}{x^2 + 2}$

(4) $f(x) = x^2 - 5x + 6$

$f'(x) = 2x - 5$

$f''(x) = 2$

$f'(x) = 0$, $x = 5/2$ rel min

f'' $(+)$ so always concave ↑

f': $-$ $\quad \frac{5}{2} \quad$ $+$

(g) none

(f) ($f(x)$ has rel min b/c $f'(x)$ changes from $-$ to $+$)

(a) $(5/2, \infty)$ (c) concave ↑ $(-\infty, \infty)$

(b) $(-\infty, 5/2)$ (d) none

(e) none

5) $f(x) = 3x^4 - 4x^3$

$f'(x) = 12x^3 - 12x^2$

$f''(x) = 36x^2 - 24x$

$36x^2 - 24x = 0$

$12x(3x-2) = 0$

$x = 0 \mid x = 2/3$

$12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

$x = 0 \mid x = 1$

$f'(x)$: $-$ | 0 | $-$ | 1 | $+$

(f) Since $f'(x)$ goes from $-$ to $+$, $f(x)$ has a rel. min at $x = 1$.

f'': $+$ | 0 | $-$ | $\frac{2}{3}$ | $+$

(g) none

(a) inc. $(1, \infty)$

(b) dec. $(-\infty, 1)$

(c) $(-\infty, 0) \cup (2/3, \infty)$

(d) $(0, 2/3)$

(e) $x = 0,\ x = 2/3$

~~5~~ 6) $f(x) = \frac{x^2}{x^2+2}$

$$f'(x) = \frac{(x^2+2)(2x) - x^2(2x)}{(x^2+2)^2}$$

$$f'(x) = \frac{2x^3 + 4x - 2x^3}{(x^2+2)^2} = \frac{4x}{(x^2+2)^2}$$

$$f''(x) = \frac{(x^2+2)^2 \cdot 4 - \overbrace{(4x)\, 2(x^2+2) \cdot 2x}^{16x^2(x^2+2)}}{((x^2+2)^2)^2} = \frac{4(x^2+2)^2 - 16x^2(x^2+2)}{(x+2)^4} = \frac{\cancel{(x^2+2)}(4(x^2+2) - 16x^2)}{(x^2+2)^{\cancel{4}\,3}}$$

$$\frac{4(x^2+2)-16x^2}{(x^2+2)^3} = \frac{4x^2+8-16x^2}{(x^2+2)^3} = \frac{-12x^2+8}{(x^2+2)^3}$$

$f'(x) = 0$

$$\frac{4x}{(x^2+2)^2} = 0$$

$4x = 0$

$x = 0$

X=0 is a rel. min. b/c $f'(x)$ goes from $\ominus$ to $\oplus$

$(x^2+2)^2 \neq 0$ so f' is never undefined

$f'(x)$: $-$ | 0 | $+$

f: $\searrow$ | $\nearrow$

$$\frac{-12x^2+8}{(x^2+2)^3} = 0$$

$(x^2+2)^3 \neq 0$

$x^2+2 \neq 0$

f'': $-$ | $-\sqrt{2/3}$ | $+$ | $+\sqrt{2/3}$ | $-$

f: CD | CU | CD

$-12x^2+8 = 0$

$-12x^2 = -8$

$12x^2 = 8$

$x^2 = 8/12 = 2/3$

$x = \pm\sqrt{2/3} \approx \pm .81649\ldots$

6

(a) inc. $(0, \infty)$

(b) dec. $(-\infty, 0)$

(c) con. ↑ $\left(-\sqrt{2/3}, \sqrt{2/3}\right)$

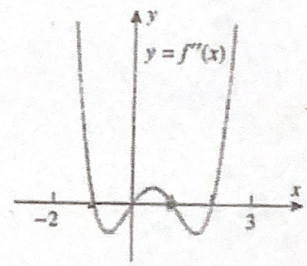
(d) con. ↓ $\left(-\infty, -\sqrt{2/3}\right) \cup \left(\sqrt{2/3}, \infty\right)$

(e) $x = \pm\sqrt{2/3}$

(f) $x=0$ is a rel min b/c $f'(x)$ $-$ $+$

(g) none

7. Use the graph of $y = f''(x)$ in the accompanying figure to determine the x-coordinates of all inflection points of f. Explain your reasoning.



this is f''

$X = 1, -1, 2, 0$