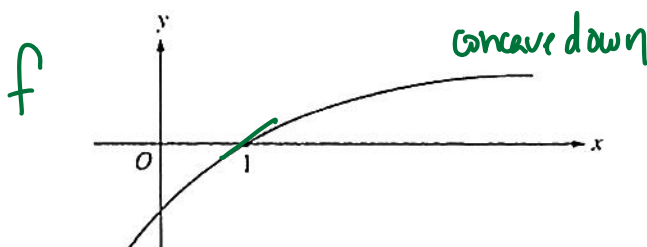
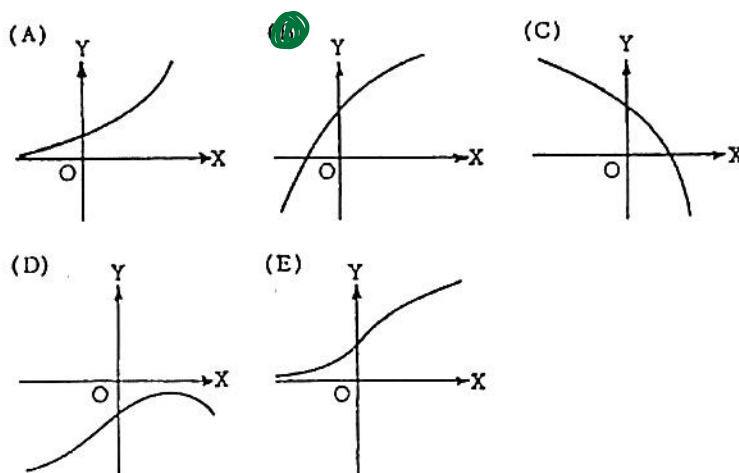


$y \rightarrow y' \downarrow y$ concave down

1. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x , which of the following could be part of the graph of $y = f(x)$?



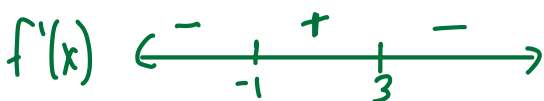
2. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

- (A) $f(1) < f'(1) < f''(1)$
 (B) $f(1) < f''(1) < f'(1)$
 (C) $f'(1) < f(1) < f''(1)$
☒ (D) $f''(1) < f(1) < f'(1)$
 (E) $f''(1) < f'(1) < f(1)$

$$\begin{aligned} f(1) &= 0 \\ f'(1) &> 0 \\ f''(1) &< 0 \end{aligned}$$

3. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

- (A) There are no such values of x .
 (B) $x < -1$ and $x > 3$
 (C) $-3 < x < 1$
☒ (D) $-1 < x < 3$
 (E) All values of x



$$\begin{aligned} f'(x) &= -(x^2 - 3)e^{-x} + e^{-x}(2x) \\ f'(x) &= e^{-x}(-x^2 + 3 + 2x) \\ f'(x) &= e^{-x}(-x^2 + 2x + 3) \\ f'(x) &= -e^{-x}(x^2 - 2x - 3) \\ f'(x) &= -e^{-x}(x - 3)(x + 1) \end{aligned}$$

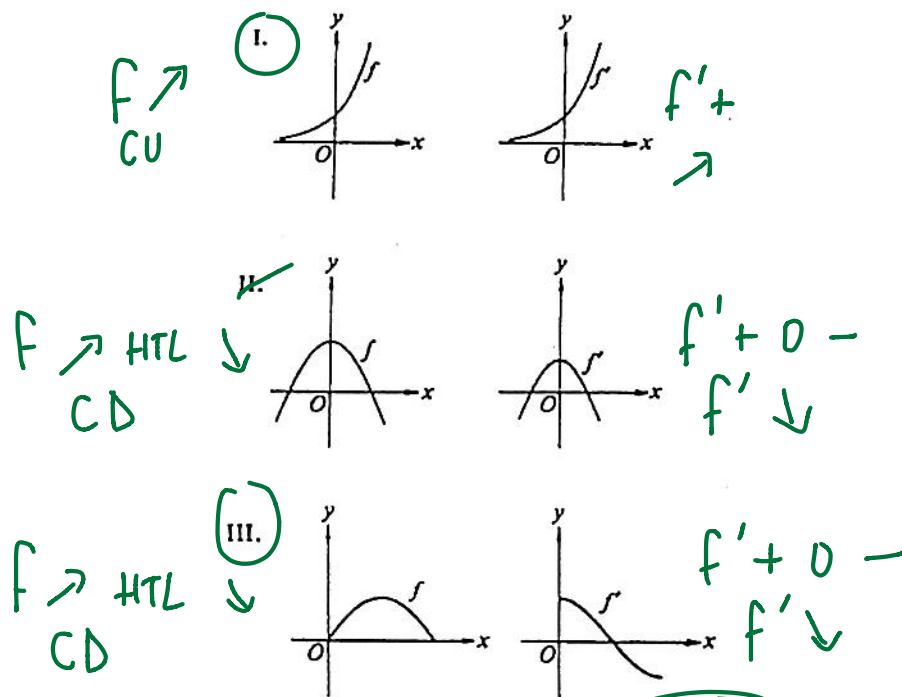
$y = -5(x-2)^{-1}$
 $y' = 5(x-2)^{-2}$
 $y'' = -10(x-2)^{-3}$

$y'' = \frac{-10}{(x-2)^3}$
 $y'' \neq 0$ when $x=2$
 The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that

$\leftarrow + \quad - \rightarrow y''$
 $cv \quad 2 \quad cd \quad f$

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

5. Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?



- (A) I only (B) II only (C) III only (D) I and III (E) II and III

6. Let $f(x) = x \ln x$. The minimum value attained by f is

(A) $-\frac{1}{e}$

(B) 0

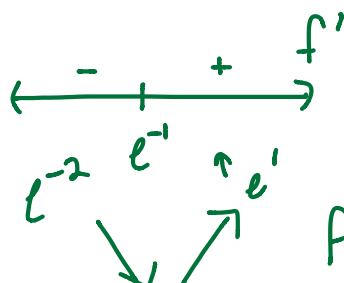
(C) $\frac{1}{e}$

(D) -1

(E) There is no minimum.

$f'(x) = x \cdot \frac{1}{x} + \ln x$
 $f'(x) = 1 + \ln x$
 $1 + \ln x = 0$
 $\ln x = -1$
 $x = e^{-1}$

$1 + \ln(e^{-1})$
 $1 - 1 = 0$
 $1 + \ln e^{-1} = 0$



$f(e^{-1}) = e^{-1} \cdot \ln e^{-1}$
 $= e^{-1} \cdot (-1)$
 $= -e^{-1}$

7. What is the x -coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

Write an equation for the normal line to the curve at its point of inflection

- (A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

$y' = x^2 + 10x$
 $y'' = 2x + 10$

$2x + 10 = 0$
 $x = -5$

so at $x = -5$ there is a poi.

$y(-5) = \frac{322}{3}$

$y'(-5) = (-5)^2 + 10(-5) = -25$

$y - \frac{322}{3} = \frac{1}{25}(x + 5)$

1980 AB5/BC2

Solution

(a) $f(x) = \cos x \cdot (1 - \cos x)$ $\cos x = 1$

Either $\cos x = 0$ or $1 - \cos x = 0$, so the x -intercepts are $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$, and $x = 0$.

(b) $f'(x) = -\sin x + 2 \sin x \cos x$

$0 = \sin x \cdot (-1 + 2 \cos x)$

Either $\sin x = 0$ or $\cos x = \frac{1}{2}$, so the candidates are $x = \pm\pi$, $x = 0$, and $x = \pm\frac{\pi}{3}$.

The relative maximum points are at $\left(\pm\frac{\pi}{3}, \frac{1}{4}\right)$.

Justification:

(i) $f''(x) = -\cos x + 2 \cos 2x$

$f''(\pm\pi) = 3 \Rightarrow$ relative minimum

$f''(0) = 1 \Rightarrow$ relative minimum

$f''\left(\pm\frac{\pi}{3}\right) = -\frac{3}{2} \Rightarrow$ relative maximum

Second
Derivative
Test

or

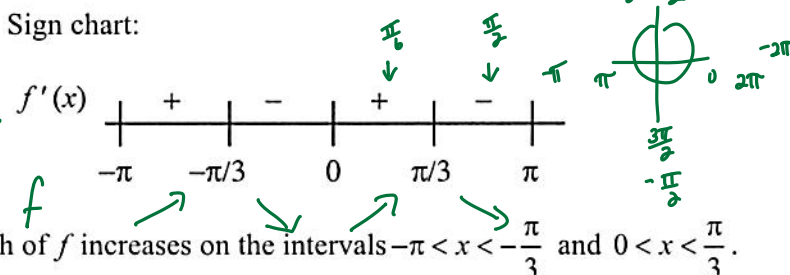
(ii) Selecting critical values:

x	$-\pi$	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$	π
$f(x)$	-2	$\frac{1}{4}$	0	$\frac{1}{4}$	-2

or

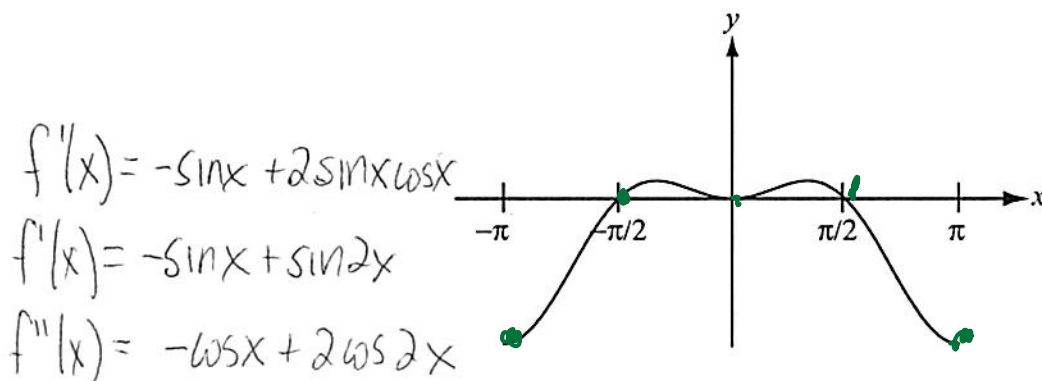
(iii) Sign chart:

First
Derivative
Test



(c) Graph of f increases on the intervals $-\pi < x < -\frac{\pi}{3}$ and $0 < x < \frac{\pi}{3}$.

(d)



$f'(x) = -\sin x + 2 \sin x \cos x$

$f'(x) = -\sin x + \sin 2x$

$f''(x) = -\cos x + 2 \cos 2x$

x	$f(x)$	f''
$-\pi$	-2	+ CU
$-\frac{\pi}{3}$	$\frac{1}{4}$	- CD
0	0	+ CU
$\frac{\pi}{3}$	$\frac{1}{4}$	- CD
π	-2	+ CU

$f(\pi) = -2$
 $f(-\pi) = -2$

$f\left(\frac{\pi}{3}\right) = \frac{1}{4}$ $f\left(-\frac{\pi}{3}\right) = \frac{1}{4}$