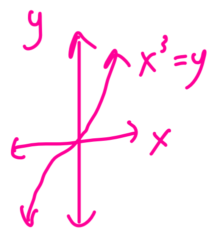
Do Now:

Determine if each of the following statements is true or false. If you decide a statement is false, provide a counterexample to show why it is false and then rewrite the statement in order to make it true. Unless otherwise specified, assume each function is defined and continuous for all real numbers.

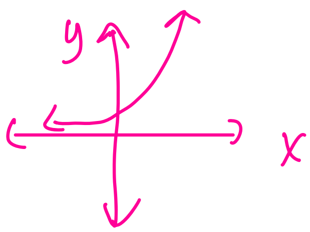
1. A critical point (or critical number) of a function f of a variable x is the x-coordinate of a relative maximum or minimum value of the function. False
2. A continuous function on a closed interval can have only one maximum value. True
3. If $f''(x)$ is always positive, then the function f must have a relative minimum value. False
4. If a function f has a local minimum value at $x = c$, then $f'(c) = 0$. False



① $f(x) = x^3$ has a critical pt at $x=0$, but there is no rel min or max at $x=0$.

② A maximum can occur at several x values but there is only one absolute maximum on a closed interval (not talking about rel. maxs)

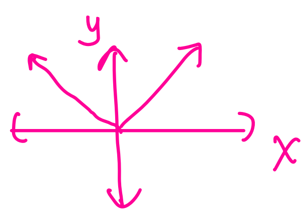
③ $f(x) = e^x$



, $f''(x)>0$ but there is no rel min.

If $f'(c)=0$ and $f''(c)>0$ then there must be a rel min at c.

④ $f(x) = |x|$



has a rel min at $x=0$
but $f'(0) \neq 0$
$f'(0)$ is undefined

Connecting f' and f'' with the graph of f

EXTREMA
(rel mins/maxs)

Find critical pts (c)
$f'(x) = 0$ or
$f'(x)$ undefined

rel min

FTD f' − to +

STD $f''(c) > 0$

rel max

FTD f' + to −

STD $f''(c) < 0$

CONCAVITY
(second derivative)

concave up

$f'' > 0$
or
$f' \nearrow$

concave down

$f'' < 0$
or
$f' \searrow$

POINTS OF INFLECTION
(second derivative)

$f'' = 0$
or
f'' undefined

f'' has to change sign from + to − or − to +
or
f' $\nearrow \searrow$ or $\searrow \nearrow$

IF THERE IS NO SIGN CHANGE, THEN THERE IS NO INFLECTION POINT.

* IMPORTANT: FOR CLOSED INTERVALS CHECK THE ENDPOINTS TO FIND ABSOLUTE EXTREMA *

Homework 12-11

Name: Ky   Date: ____________
AP Calc AB: ~~Second~~ Derivative Test Homework   Ms. Loughran

For 1 and 2, use the graphs of f' shown to estimate all values of x at which f has
(a) relative maxima $\cap$ f' + −
(b) relative minima f' − +
(c) inflection points f' ↑↓ or ↓↑

1.

a $x=0$
b $x=2$
c $x=1,3$

2.

a $x=5$
b $x=1$
c $x=-1,0,3$

For 3 and 4, find the relative extrema using both the first and second derivative tests.

3. $f(x)=1-4x-x^2$

$1+8-4$

$f'(x) = -4-2x$

$-4-2x=0$
$-2x=4$
$x=-2$

$f'(x)$: + | − at -2

rel max of 5 at $x=-2$

$f''(x) = -2$
$f''(-2) < 0$ $\cap$

4. $f(x)=\sin^2 x$, $0<x<2\pi$

$f'(x) = 2\sin x\cos x = \underline{\sin 2x}$

$\sin 2x = 0$
$2x = 0, \pi, 2\pi, 3\pi$
$2x = k\pi$ $k \in \mathbb{Z}$
$x = \frac{k}{2}\pi$ $k \in \mathbb{Z}$

f': + | − | + | − with 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π; $\frac{5\pi}{4}$, $\frac{5\pi}{2}$

$f'' = 2\cos(2x)$ at $\pi/2$ rel max of 1
at π rel min of 0
at $3\pi/2$ rel max of 1

Second Derivative Test
$f''(x) = 2\cos 2x$ at $x=\frac{\pi}{2}$ rel max of 1
$f''(\frac{\pi}{2}) < 0$ rel min at π of 0
$f''(\pi) > 0$
$f''(3\pi/2) < 0$ rel max of 1

(1,0) (1,0)

For 5-8, use any method to find the relative extrema of the function *f*.

5. $f(x) = x^3 + 5x - 2$

$f'(x) = 3x^2 + 5$ always +

$3x^2 + 5 = 0$

$3x^2 = -5$

$x^2 = -5/3$ imaginary

Since $f'(x)$ is always > 0 there are no rel. extrema f is always ↑

6. $f(x) = x(x-1)^2$

$f'(x) = (x-1)^2 + x \cdot 2(x-1) = (x-1)^2 + 2x(x-1) = (x-1)(x-1+2x)$

$(x-1)(3x-1)$

$x = 1, \frac{1}{3}$

$\frac{1}{3}\left(-\frac{2}{3}\right)^2$

$\frac{1}{3} \cdot \frac{4}{9}$

$f''(x) = 3x-1 + (x-1)3 = 3x-1+3x-3 = 6x-4$

$f''(1) > 0$ so f has a rel min of 0 at $x=1$

$f''\left(\frac{1}{3}\right) < 0$ so f has a rel max of $\frac{4}{27}$ at $x = \frac{1}{3}$

Second Derivative Test

7. $f(x) = \dfrac{x^2}{x^2+1}$

$f'(x) = \dfrac{2x(x^2+1) - 2x(x^2)}{(x^2+1)^2} = \dfrac{2x^3+2x-2x^3}{(x^2+1)^2} = \dfrac{2x}{(x^2+1)^2}$

$x = 0$

f: ↘ ↗

f': − +

0

So f has a rel min of 0 at $x=0$

First Derivative test

8. $f(x) = \ln(1+x^2)$

$f'(x) = \dfrac{1}{1+x^2} \cdot 2x = \dfrac{2x}{1+x^2}$

$x = 0$

f': − +

0

f: ↘ ↗

So f has a rel min of 0 at $x = 0$