Name: ____________________ Date: ________

AP Calculus AB Analysis of Functions

In Exercises 1–4, find the first derivative of the function.

1. $f(x) = \sqrt{4 - x}$ $f'(x) = \frac{1}{2}(4-x)^{-\frac{1}{2}} \cdot -1 = -\frac{1}{2\sqrt{4-x}}$

2. $f(x) = \frac{2}{\sqrt{9 - x^2}}$ $2(9-x^2)^{-\frac{1}{2}}$ $f'(x) = -1(9-x^2)^{-3/2} \cdot -2x = \frac{2x}{(\sqrt{9-x^2})^3}$

3. $g(x) = \cos(\ln x)$ $g'(x) = -\sin(\ln x) \cdot \frac{1}{x} = \frac{-\sin(\ln x)}{x}$

4. $h(x) = e^{2x}$ $h'(x) = 2e^{2x}$

In Exercises 5–8, match the table with a graph of $f(x)$.

5. C

x	$f'(x)$
a	0
b	0
c	5

6. B

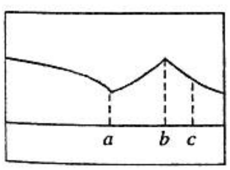
x	$f'(x)$
a	0
b	0
c	−5

7. D

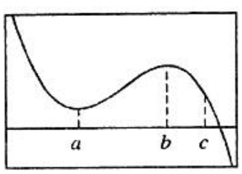
x	$f'(x)$
a	does not exist
b	0
c	−2

8. A

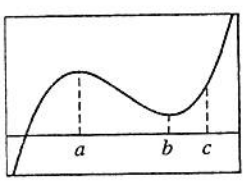
x	$f'(x)$
a	does not exist
b	does not exist
c	−1.7



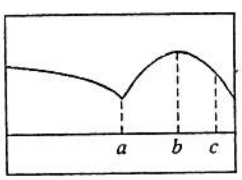
(a)



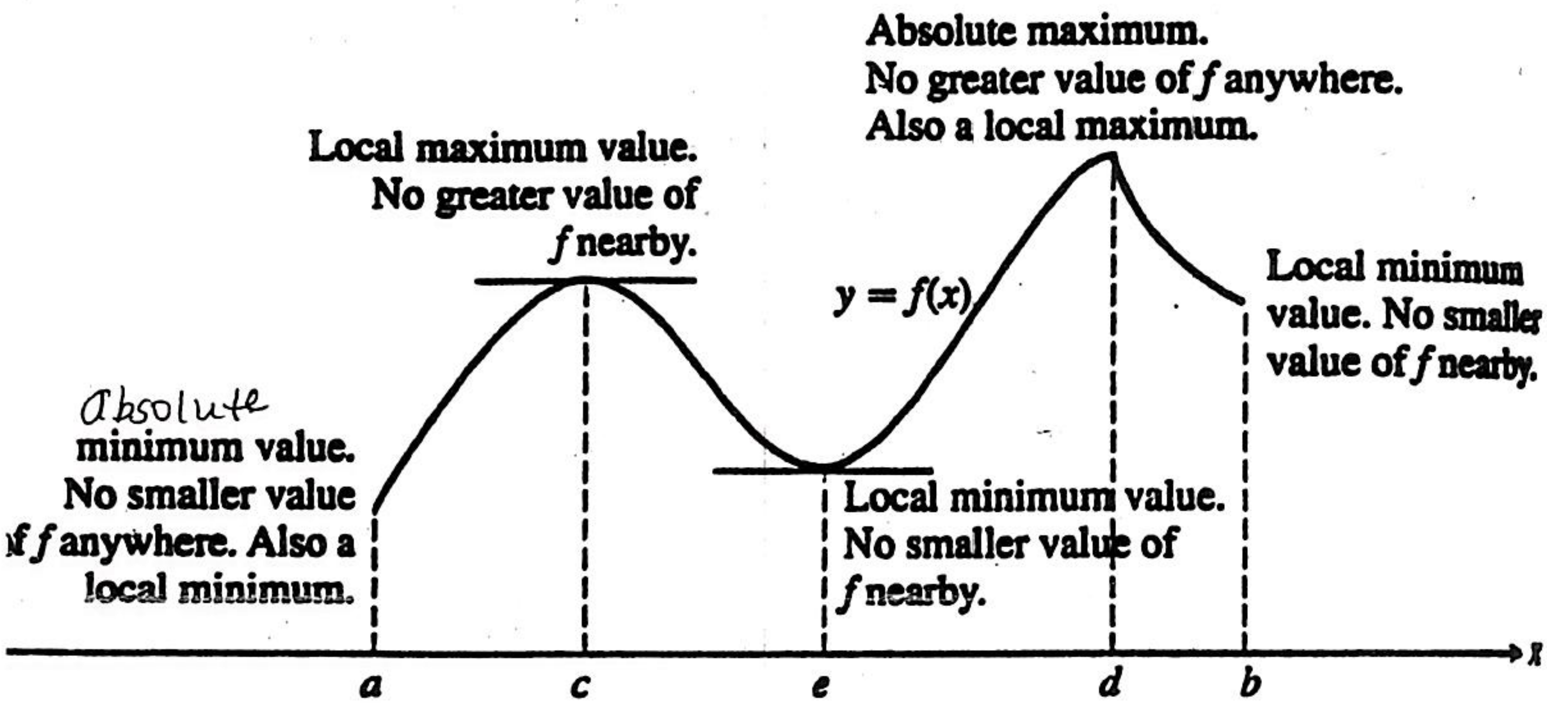
(b)



(c)



(d)



How to classify maxima and minima.

Name: ______________________________________ Date: ____________

AP Calculus AB: Extreme Value Theorem Ms. Loughran

Extreme Value Theorem: (EVT)

If a function f is continuous on a CLOSED interval [a,b] then f has both an absolute minimum and an absolute maximum on the interval.

Let f be a function with domain D that is continuous, then f(c) is the:

absolute maximum on D if and only if $f(x) \le f(c)$ for all x in D.

absolute minimum on D if and only if $f(x) \ge f(c)$ for all x in D

In a closed interval, extreme values occur at critical points or at endpoints. [Candidate Test]

To find extrema on a closed interval:

1. find the critical points of f on [a,b]
2. evaluate f at those critical points pts in [a,b]
3. evaluate f at the endpoints
4. the least of these values is the absolute (global) minimum and the greatest of these values is the absolute (global) maximum

Suppose that *f* is continuous and has exactly one relative minimum or exactly one relative maximum on an interval *I*, then that value is the absolute minimum/ absolute maximum on that interval.

For the following, find the extreme values of *f* and where they occur.

1. $f(x) = 2x^3 - 3x^2 - 36x \qquad [1,5]$

$f'(x) = 6x^2 - 6x - 36$

$6x^2 - 6x - 36 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, \cancel{-2}$ not in [1,5]

candidate tests

$f(1) = 2 - 3 - 36 = -37$

$f(3) = 54 - 27 - 108 = -81$

$f(5) = 2(5)^3 - 3(5)^2 - 36(5) = 250 - 75 - 180 = -5$

abs max: −5 at x=5

abs min: −81 at x=3

2. $f(x)=6x^{\frac{4}{3}}-3x^{\frac{1}{3}}$ $[-1,1]$

$f'(x) = 8x^{\frac{1}{3}} - x^{-2/3}$

$f'(x) = x^{-2/3}(8x-1) = \dfrac{8x-1}{x^{2/3}}$

f' is 0 or f' is not defined

$8x-1=0$

$x=\frac{1}{8}$

$x=0$

CT

$f(-1) = 9$

$f(0) = 0$

$f(\frac{1}{8}) = \frac{-9}{8}$

$f(1) = 3$

abs max: 9 at $x=-1$

abs min: $-\frac{9}{8}$ at $x=\frac{1}{8}$

3. $f(x)=\ln(x+1)$ $[0,3]$

$f'(x) = \dfrac{1}{x+1}$

$\dfrac{1}{x+1} \neq 0$

no place where $f'(x)=0$

f' is undefined $x=-1$ so that is a critical pt

ignore it b/c it's not in $[0,3]$

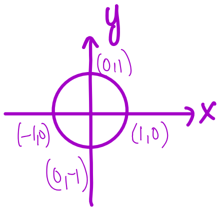
$f(0) = \ln 1 = 0$

$f(3) = \ln 4$

abs max: $\ln 4$ at $x=3$

abs min: 0 at $x=0$

4. $f(x)=\sin\left(x+\dfrac{\pi}{4}\right)$ $\left[0,\dfrac{7\pi}{4}\right]$



$f'(x) = \cos(x+\frac{\pi}{4})$

$\cos(\underbrace{x+\frac{\pi}{4}}_{A}) = 0$

$A = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

$x+\frac{\pi}{4} = \frac{\pi}{2}$ $x+\frac{\pi}{4} = \frac{3\pi}{2}$ $x+\frac{\pi}{4} = \frac{5\pi}{2}$

$x=\frac{\pi}{4}$ $x=\frac{5\pi}{4}$ $\cancel{x=\frac{9\pi}{4}}$ now I'm outside interval

CT

$f(0) = \frac{\sqrt{2}}{2}$

$f(\frac{\pi}{4}) = 1$

$f(\frac{5\pi}{4}) = -1$

$f(\frac{7\pi}{4}) = 0$

abs min: -1 at $x=\frac{5\pi}{4}$

abs max: 1 at $x=\frac{\pi}{4}$

Classwork/Homework 12- 13

Name: ______________________________
AP Calculus Practice

$y' = x^2 + 10x$

$y'' = 2x+10$

$2x+10=0$

$x=-5$

y'': $+$ $-$ at -5

* Unless noted with a "*" a calculator is **NOT ALLOWED**.

1) What is the x-coordinate of the point of inflection on the graph $y = \frac{1}{3}x^3 + 5x^2 + 24$?

A. 5 B. 0 C. $-\frac{10}{3}$ D. -5 E. -10

2) A particle moves along the x-axis so that its position at time t is given by: $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

A. 1 B. 2 C. 3 D. 4 E. 5

3) If $f''(x) = x(x+1)(x-2)^2$ then the graph of f has inflection points when x =

A. –1 only B. 2 only C. –1 and 0 only D. –1 and 2 only E. –1, 0, and 2 only

4) The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

A. $\left(-\frac{1}{\sqrt{2}}, \infty\right)$ B. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ C. $(0, \infty)$ D. $(-\infty, 0)$ E. $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

5)* The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$.
How many critical values does f have on the open interval (0, 10)?

A. One B. Three C. Four D. Five E. Seven

6) Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$.

On which of the following intervals is f decreasing?

A. $(-\infty, -1)$ only B. $(-\infty, 0)$ C. $(-1, 0)$ only D. $(0, \sqrt[3]{2})$ E. $(\sqrt[3]{2}, 0)$

7) Let f be the function given by $f(x) = 2xe^x$ The graph of f is concave down when

A. $x < -2$ B. $x > -2$ C. $x < -1$ D. $x > -1$ E. $x < 0$

$f(x) = 2xe^x$

$f'(x) = 2xe^x + 2e^x$

$f'(x) = 2e^x(x+1)$

$f''(x) = 2e^x(1) + (x+1)2e^x$

$f''(x) = 2e^x(1 + \underbrace{x+1}_{x+2})$

$f''(x) = 0$

$2e^x(x+2) = 0$

$2e^x \neq 0 \mid x = -2$

f'': $-$, $+$; -2

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

8) -2 2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real number, then g is decreasing on which of the following intervals?

A. $-2 \le x \le 2$ only B. $-1 \le x \le 1$ only C. $x \ge -2$ D. $x \ge 2$ only E. $x \le -2$ or $x \ge 2$

$g(x) \nearrow$ g CU $g' \nearrow$ $m = 6$

9) Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

$(4,12)$ $(5,18)$ $m = \frac{6}{1} = 6$ $(6,21)$ $(6,24)$

A. ~~15~~ B. ~~18~~ C. ~~21~~ D. ~~24~~

E. 27

$m = 9$

$g''(x) > 0$, $g'(x) \nearrow$

$g'(x)$ $+$

$g(x) \nearrow$

10)* A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

A. –2.016 B. –0.677 C. 1.633 D. 1.814 E.
2.978

11)* Let f be the function with derivative given by $f'(x) = \sin\left(x^2 + 1\right)$.

How many relative extrema does f have on the interval $2 < x < 4$?

A. One B. Two C. Three D. Four E. Five

12)* The function f has first derivative given by $f'(x)=\dfrac{\sqrt{x}}{1+x+x^3}$. What is the x-coordinate of the inflection point of the graph of f ?

A. 1.008 B. 0.473 C. 0 D. –0.278 E. the graph has no inflection point

$f \nearrow$ $f' \searrow$

13) For all x in the closed interval $[2,5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f ?

f'' $-$

A.

x	$f(x)$
2	7
3	9
4	12
5	16

$m=2$

$m=3$

$m=4$

B.

x	$f(x)$
2	7
3	11
4	14
5	16

$m=4$

3

2

C.

x	$f(x)$
2	16
3	12
4	9
5	7

D.

x	$f(x)$
2	16
3	14
4	11
5	7

E

x	$f(x)$
2	16
3	13
4	10
5	7

f' $+$ $f \nearrow$

f'' $-$ $f' \searrow$