Do Now

① At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

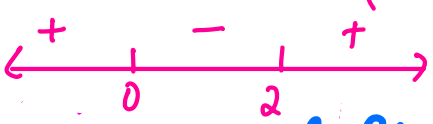
$y = x^{-2} - x^{-3}$

$y' = -2x^{-3} + 3x^{-4}$

$y'' = 6x^{-4} - 12x^{-5} = 6x^{-5}(x-2) = \frac{6(x-2)}{x^5}$

$f''(x) = 0$ at $x = 2$

$f''(x)$ dne $x = 0$



$x=0$ is not a location of a pt of inflection b/c $x=0$ is not in the domain of f

② Find all relative extrema of $f(x) = \ln(1+3x^2)$

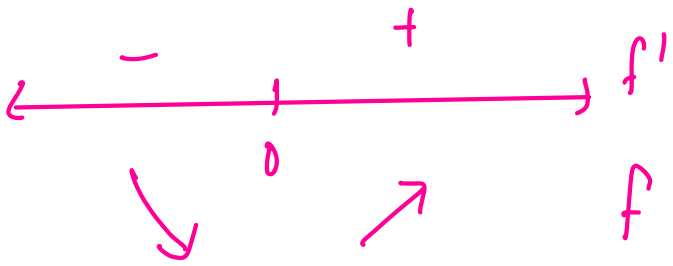
$f'(x) = \frac{6x}{1+3x^2}$ (+)

$f'(x) = 0$ at $x = 0$

$f(0) = \ln(1+3(0)^2) = 0$

There is a rel min of 0 at $x=0$

FDT



SDT $f''(x) = \frac{6(1+3x^2) - 6x(6x)}{(1+3x^2)^2}$

$f''(0) = \frac{6-0}{1} > 0$

f is CU therefore there is a rel min of 0 at $x=0$.

③ How many critical values does $f(x) = (x+6)^7(x+2)^8$ have?

$$f'(x) = 8(x+6)^7(x+2)^7 + 7(x+2)^8(x+6)^6$$

$$0 = 8(x+6)^7(x+2)^7 + 7(x+2)^8(x+6)^6$$

$$0 = (x+6)^6(x+2)^7(8x+48+7x+14)$$

$$0 = (x+6)^6(x+2)^7(15x+62)$$

3

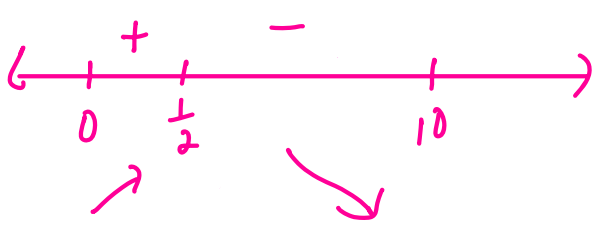
④ Given $f(x) = xe^{-2x}$ with domain $0 \le x \le 10$.

a) find all values of x for which the graph of f is increasing and all values of x for which the graph of f is decreasing.

$$f'(x) = e^{-2x} - 2xe^{-2x} = e^{-2x}(1-2x)$$

$$e^{-2x}(1-2x) = 0$$

$e^{-2x} \neq 0$ | $x = \frac{1}{2}$



increasing: $(0, \frac{1}{2})$

decreasing: $(\frac{1}{2}, 10)$

b) Find the absolute maximum and the absolute minimum.

candidate test

$f(0) = 0$

$f(\frac{1}{2}) = \frac{1}{2}e^{-2(\frac{1}{2})} = \frac{1}{2}e^{-1} = \frac{1}{2e}$

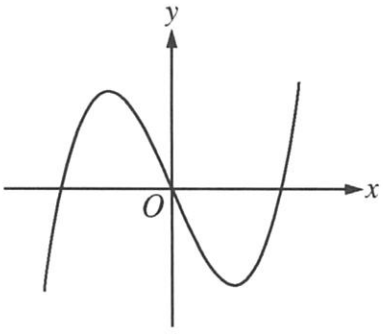
$f(10) = 10e^{-2(10)} = 10e^{-20} = \frac{10}{e^{20}}$

abs min: 0

abs max: $\frac{1}{2e}$

2008

(5)

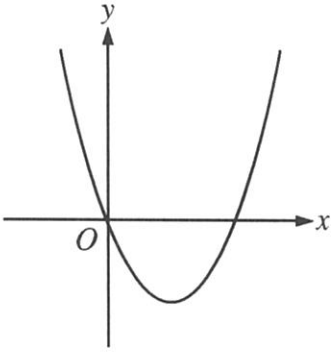


Graph of f

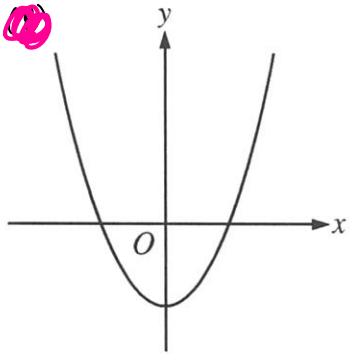
f, ↗ HTL ↘ HTL ↗
f' + 0 − 0 +

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

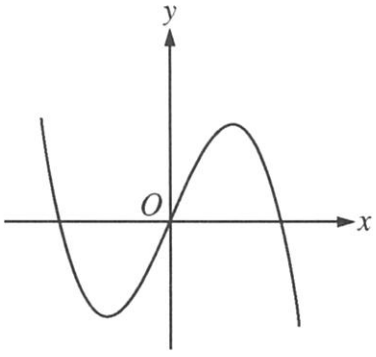
(A)



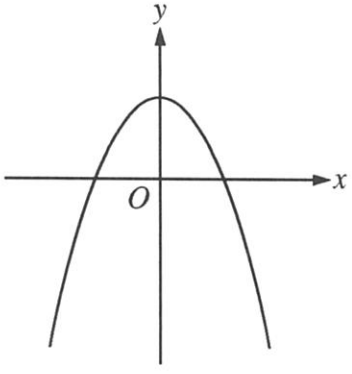
(B)



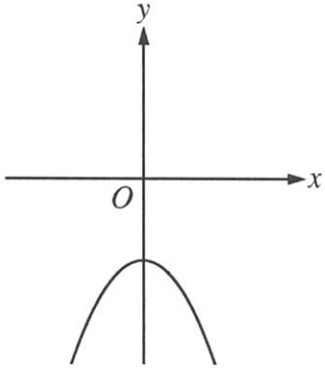
(C)



(D)



(E)



2008

6

x	0	1	2	3
$f''(x)$	5	0	–7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

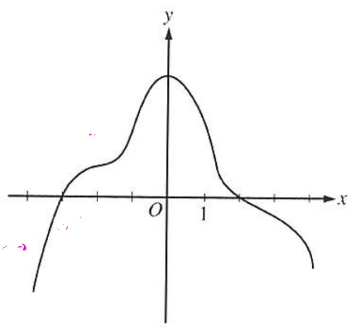
(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

2012

7



Graph of f'

80. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$.

II. The graph of f has a point of inflection at $x = -2$.

III. The graph of f is concave down for $0 < x < 4$.

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

2003

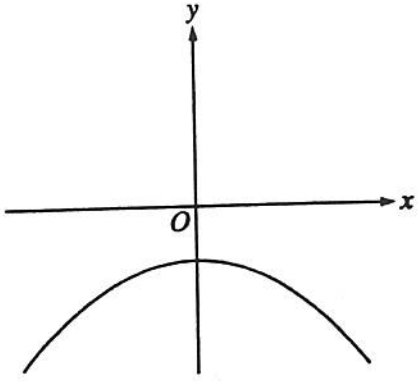
Section I
Part A

$f(x)$ ⊖ below the x-axis

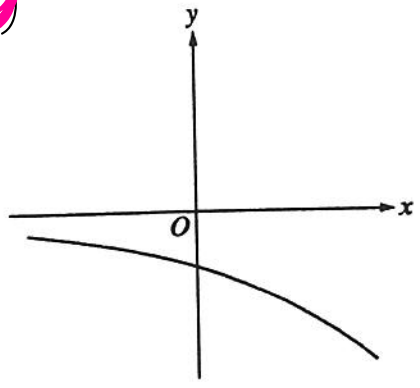
$f'(x)$ ⊖ f ↓ $f''(x)$ ⊖ f CD

10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f ?

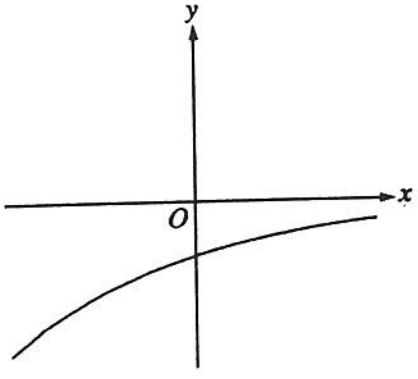
(A)



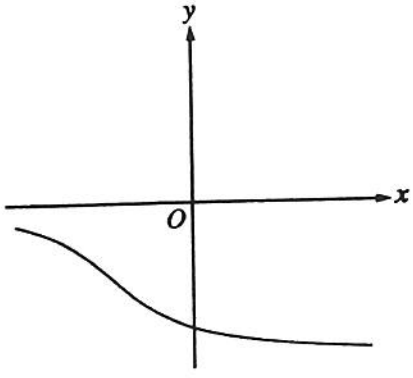
(B)



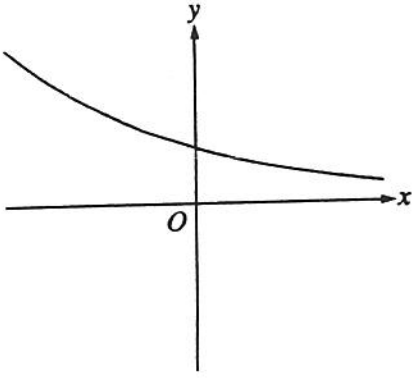
(C)



(D)



(E)



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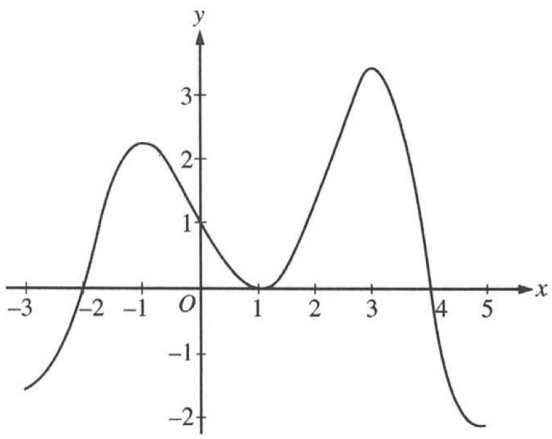
2008 (9)



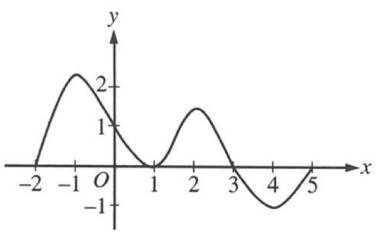
Graph of f'

84. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

(A) -2 only (B) 1 only (C) 4 only (D) -1 and 3 only (E) -2, 1, and 4

f ↗ ↘
f' + −

2008 (10)



Graph of f'

76. The graph of f', the derivative of f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

f' +

(A) $[-2, 1]$ only
(B) $[-2, 3]$
(C) $[3, 5]$ only
(D) $[0, 1.5]$ and $[3, 5]$
(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$

1984 AB4/BC3

A function f is continuous on the closed interval $[-3, 3]$ such that $f(-3) = 4$ and $f(3) = 1$. The functions f' and f'' have the properties given in the table below.

a) candidate test

$f(-3) = 4$ blc f is increasing

$f(-1) > 4$ blw -3 and -1

$f(3) = 1$

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exist	Positive	0	Negative

f(x) ↗ ↘

(a) What are the x-coordinates of all absolute maximum and absolute minimum points of f on the interval $[-3, 3]$? Justify your answer.

Abs max at x=-1 and absolute min at x=3

(b) What are the x-coordinates of all points of inflection of f on the interval $[-3, 3]$? Justify your answer.

f'' + to − or − to +

x=1 blc f'' changes sign from + to − around 1

(c) On the axes provided, sketch a graph that satisfies the given properties of f.