Name:_______________________________________ Date:_______

AP Calc AB: Linear Approximations Ms. Loughran

Do Now

1. Given $f(x) = x^3$
   (a) Write an equation for the line tangent to the curve at $x = 1$
   (b) Using the equation in (a) estimate $f(1.1)$.
   (c) Evaluate $f(1.1)$ using your calculator.
   (d) Compare your findings in (b) and (c), what do you notice?

very close

a) $f'(x) = 3x^2$

$f'(1) = 3$

$f(1) = 1$

$y - 1 = 3(x - 1)$

c) $f(1.1) = (1.1)^3 = 1.331$

b) $y - 1 = 3(1.1 - 1)$

$y = 3(1.1 - 1) + 1 = 1.3$

Tangent line approximation:

To calculate the tangent line at a point $(a, f(a))$ we first have to find the derivative at a, $f'(a)$, then use our point slope formula

$$y - f(a) = f'(a)(x - a)$$

The tangent line approximates the graph of $f(x)$ near $x = a$

Basically it is helping us approximate any function which could be very complicated using a linear function which is very easy to work with.

2.

1995 AB3

Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

(a) Find $\frac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point $(4, -1)$.

(c) There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k.

(d) Write an equation that can be solved to find the actual value of k so that the point $(4.2, k)$ is on the curve.

(e) Solve the equation found in part (d) for the value of k.

$$-16x + 5x\frac{dy}{dx} + 5y + 3y^2\frac{dy}{dx} = 0$$

$$5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 16x - 5y$$

$$\frac{dy}{dx} = \frac{16x-5y}{5x+3y^2}$$

b) $$\left.\frac{dy}{dx}\right|_{(4,-1)} = \frac{16(4)-5(-1)}{5(4)+3(-1)^2} = \frac{69}{23} = 3$$

$$y+1 = 3(x-4)$$

c) $$K+1 = 3(4.2-4)$$

$$K+1 = .6$$

$$K = -.4$$

d) $$-8(4.2)^2 + 5(4.2)K + K^3 = -149$$

$$K = -.373$$