Name: ______________________________ Date: __________

AP Calculus AB Ms. Loughran

Do Now:

If $f(-x) = f(x)$, then $f(x)$ is even
- symmetric with respect to the y-axis

If $f(-x) = -f(x)$, then $f(x)$ is odd
- symmetric with respect to the origin

* $\sin x$ is an odd function symmetric with respect to the origin

$\cos x$ is an even function symmetric with respect to the y-axis

1984 AB 2

Let f be the function defined by $f(x) = \dfrac{x + \sin x}{\cos x}$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(a) State whether f is an even function or an odd function. Justify your answer.

(b) Find $f'(x)$.

(c) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

(a) $f(-x) = \dfrac{-x + \sin(-x)}{\cos(-x)} = \dfrac{-x - \sin x}{\cos x} = \dfrac{-(x + \sin x)}{\cos x} = -f(x)$

f is odd b/c $f(-x)$ is $= -f(x)$

(b) $f'(x) = \dfrac{\cos x(1 + \cos x) - (x + \sin x)(-\sin x)}{\cos^2 x}$

(c) $f(0) = \dfrac{0 + \sin 0}{\cos 0} = \dfrac{0}{1} = 0$ $(0, 0)$

$f'(0) = \dfrac{\cos 0(1 + \cos 0) - (0 + \sin 0)(-\sin 0)}{\cos^2(0)}$

$f'(0) = \dfrac{2 - 0}{1} = 2$

$y - 0 = 2(x - 0)$

$y = 2x$

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AP Calc AB: More Local Linear Approximations Ms. Loughran

1. Approximately how much less than 4 is $\sqrt[3]{63}$?

(a) $\frac{1}{48}$ (b) $\frac{1}{16}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ (e) 1

2. The best linear approximation for $f(x) = \tan x$ near $x = \frac{\pi}{4}$ is

(a) $1 + \frac{1}{2}(x - \frac{\pi}{4})$ (b) $1 + (x - \frac{\pi}{4})$ (c) $1 + \sqrt{2}(x - \frac{\pi}{4})$

(d) $1 + 2(x - \frac{\pi}{4})$ (e) $2 + 2(x - \frac{\pi}{4})$

$f(\frac{\pi}{4}) = \tan \frac{\pi}{4} = 1$

$y - 1 = 2(x - \frac{\pi}{4})$

$f'(x) = \sec^2 x \quad f'(\frac{\pi}{4}) = \left(\frac{2}{\sqrt{2}}\right)^2 = \frac{4}{2} = 2$

4. If $f(6) = 30$ and $f'(x) = \frac{x^2}{x+3}$, then an estimate of $f(6.02)$, using the local linearization, is

(a) 29.92 (b) 30.02 (c) 30.08 (d) 34.00 (e) none of these

$f'(6) = \frac{6^2}{6+3} = 4$

$y - 30 = 4(x - 6)$

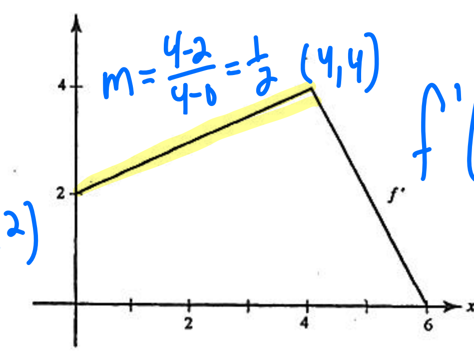
$y - 30 = 4(6.02 - 6)$

$y - 30 = 4(.02)$

$y - 30 = .08$

6. The graph of f' is shown below. If we know that $f(2) = 10$, then the local linearization of f at $x = 2$ is $f(x) \approx$

$f'(2) = \frac{1}{2}(2) + 2 = 3$



$f'(x) = \begin{cases} \frac{1}{2}x + 2 & 0 \le x \le 4 \\ \end{cases}$

don't need to keep going here we just need $f'(2)$

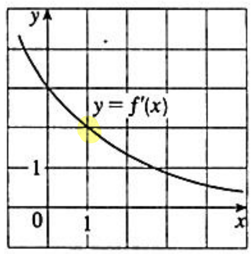
(a) $\frac{x}{2} + 2$ (b) $\frac{x}{2} + 9$ (c) $3x - 3$ (d) $3x + 4$ (e) $10x - 17$

$y - 10 = 3(x - 2)$

$y - 10 = 3x - 6$

$y = 3x + 4$

7. Suppose that the only information we have about a function, f, is that $f(1) = 5$ and the graph of its derivative is as shown.
   (a) Use a linear approximation to estimate $f(0.9)$ and $f(1.1)$.
   (b) Are your estimates in part (a) too large or too small? Explain.



$f'(1) = 2$

$y - 5 = 2(x-1)$

(a) $y - 5 = 2(.9-1)$ $\quad$ $y-5 = 2(1.1-1)$

$y - 5 = -.2$ $\quad$ $y - 5 = .2$

$y = 4.8$ $\quad$ $y = 5.2$

(b) f' ↓

f'' ⊖

f CD

these estimates are over approximations b/c f is concave down

$3y^2 \frac{dy}{dx} - x\frac{dy}{dx} - y = 0$

$\frac{dy}{dx}(3y^2 - x) = y$

$dy/dx = \frac{y}{3y^2 - x}$

$\left.\frac{dy}{dx}\right|_{(7,2)} = \frac{2}{5}$

$y - 2 = \frac{2}{5}(x - 7)$

Homework 12-20

Classwork

6.8	1.92
6.9	1.96
7	2
7.1	2.04
7.2	2.08

1. Make a table of x and approximate y values for the equation $y^3 - xy = -6$ near $x = 7, y = 2$. Your table should include the x values 6.8, 6.9, 7.0, 7.1, and 7.2.

2. Consider the equation $x^3 + y^3 - xy^2 = 5$.

   (a) Find $\frac{dy}{dx}$ by implicit differentiation.

   (b) Give a table of approximate values near $x = 1, y = 2$ for $x = 0.96, 0.98, 1, 1.02, 1.04$.

   (c) Find the y value for $x = 0.96$ by substituting $x = 0.96$ in the equation and solving for y using your calculator. Compare with your answer in part (b).

3. Consider the curve $xe^{5y} = 3y$

   (a) Find $\frac{dy}{dx}$ by implicit differentiation.

   (b) Find the equation of the tangent line to the curve at (0,0).

   (c) If $x = 0.1$, estimate y using the tangent line.

2) $3x^2 + 3y^2\frac{dy}{dx} - \left(y^2 + 2xy\frac{dy}{dx}\right) = 0$

$3x^2 + 3y^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3y^2 - 2xy) = y^2 - 3x^2$

$\frac{dy}{dx} = \frac{y^2 - 3x^2}{3y^2 - 2xy}$

b) $\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{1}{8}$

$y - 2 = \frac{1}{8}(x - 1)$

x	y
.96	1.995
.98	1.9975
1	2
1.02	2.0025
1.04	2.005

c) $(.96)^3 + y^3 - .96y^2 = 5$

$y = 1.994499\ldots$

3. Consider the curve $xe^{5y} = 3y$

(a) Find $\frac{dy}{dx}$ by implicit differentiation.

(b) Find the equation of the tangent line to the curve at (0,0).

(c) If $x = 0.1$, estimate y using the tangent line.

a) $xe^{5y} = 3y$

$$xe^{5y} \cdot 5\frac{dy}{dx} + e^{5y} = 3\frac{dy}{dx}$$

$$5xe^{5y}\frac{dy}{dx} + e^{5y} = 3\frac{dy}{dx}$$

$$e^{5y} = \frac{dy}{dx}\left(3 - 5xe^{5y}\right)$$

$$\frac{e^{5y}}{3-5xe^{5y}} = \frac{dy}{dx}$$

b) $$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{e^{5(0)}}{3-5(0)e^{5(0)}}$$

$$= \frac{1}{3}$$

$$y = \frac{1}{3}x$$

c) $$y = \frac{1}{3}(.1) = \frac{1}{3}\left(\frac{1}{10}\right) = \frac{1}{30}$$

* If you use your calculator to solve for the actual value

$$.1e^{5y} = 3y$$

$$y \approx 0.04$$