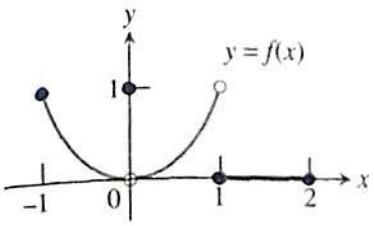
Finding Limits Graphically

For # 1 – 2, tell whether the statements are true or false.



(a) $\lim_{x \to -1^+} f(x) = 1$

(b) $\lim_{x \to 0^-} f(x) = 0$

(c) $\lim_{x \to 0^-} f(x) = 1$

(d) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

(e) $\lim_{x \to 0} f(x)$ exists

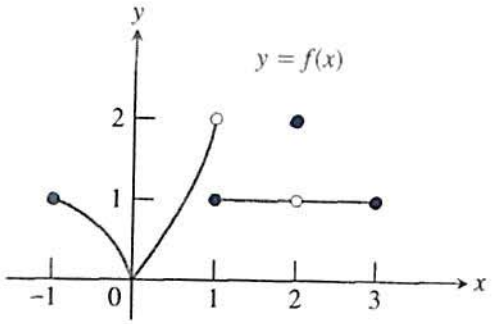
(f) $\lim_{x \to 0} f(x) = 0$

(g) $\lim_{x \to 0} f(x) = 1$

(h) $\lim_{x \to 1} f(x) = 1$

(i) $\lim_{x \to 1} f(x) = 0$

(j) $\lim_{x \to 2^-} f(x) = 2$



(a) $\lim_{x \to -1^+} f(x) = 1$

(b) $\lim_{x \to 2} f(x)$ does not exist.

(c) $\lim_{x \to 2} f(x) = 2$

(d) $\lim_{x \to 1^-} f(x) = 2$

(e) $\lim_{x \to 1^+} f(x) = 1$

(f) $\lim_{x \to 1} f(x)$ does not exist.

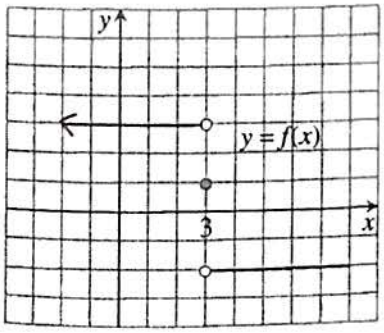
(g) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

(h) $\lim_{x \to c} f(x)$ exists at every c in $(-1, 1)$.

(i) $\lim_{x \to c} f(x)$ exists at every c in $(1, 3)$.

For # 3 – 8, use the graph to estimate the limits and the value of the function or explain why the limits do not exist.

3.



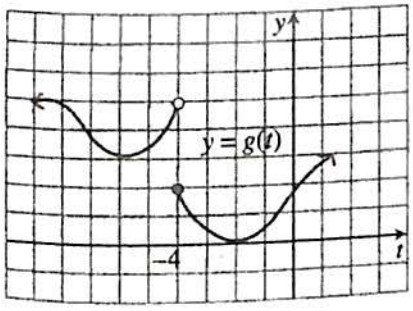
(a) $\lim_{x \to 3^-} f(x)$

(b) $\lim_{x \to 3^+} f(x)$

(c) $\lim_{x \to 3} f(x)$

(d) $f(3)$

4.



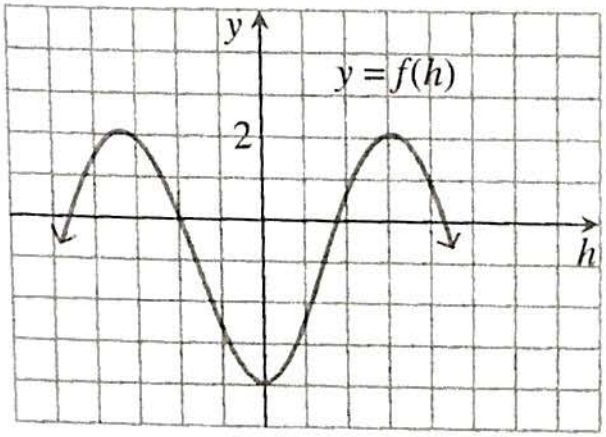
(a) $\lim_{t \to -4^-} g(t)$

(b) $\lim_{t \to -4^+} g(t)$

(c) $\lim_{t \to -4} g(t)$

(d) $g(-4)$

5.



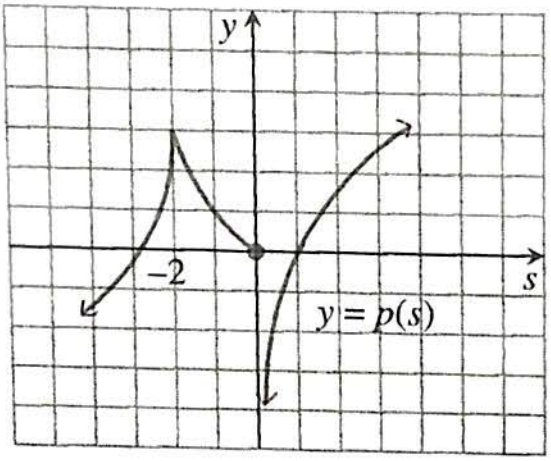
(a) $\lim_{h \to 0^-} f(h)$

(b) $\lim_{h \to 0^+} f(h)$

(c) $\lim_{h \to 0} f(h)$

(d) $f(0)$

6.



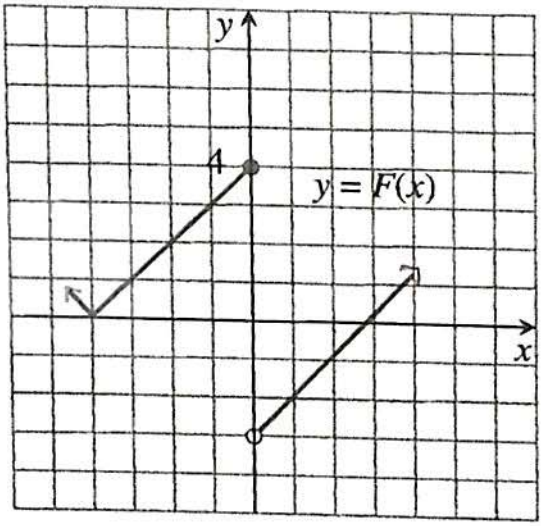
(a) $\lim_{s \to -2^-} p(s)$

(b) $\lim_{s \to -2^+} p(s)$

(c) $\lim_{s \to -2} p(s)$

(d) $p(-2)$

7.



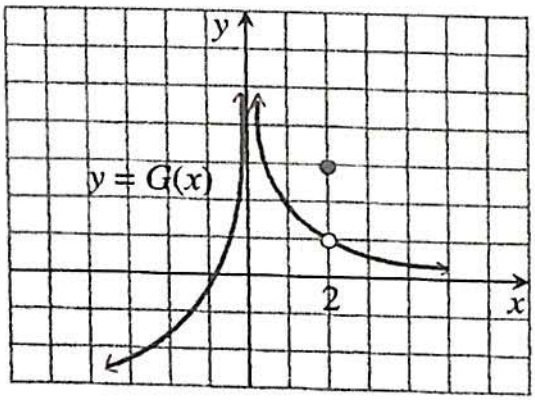
(a) $\lim_{x \to 0^-} F(x)$

(b) $\lim_{x \to 0^+} F(x)$

(c) $\lim_{x \to 0} F(x)$

(d) $F(0)$

8.



(a) $\lim_{x \to 2^-} G(x)$

(b) $\lim_{x \to 2^+} G(x)$

(c) $\lim_{x \to 2} G(x)$

(d) $G(2)$

Evaluating Limits

Find the numerical value of each limit or state that the limit does not exist or is best described as being $+\infty$ or $-\infty$.

1. $\lim_{x\to -\frac{1}{2}} 3x^2(2x-1)$
2. $\lim_{x\to 1}\left(x^3+3x^2-2x-17\right)$
3. $\lim_{y\to -3}\dfrac{y^2+4y+3}{y^2-3}$
4. $\lim_{x\to -2}(x-6)^{\frac{2}{3}}$
5. $\lim_{x\to 2}\sqrt{x+3}$
6. $\lim_{y\to 2}\dfrac{y^2+5y+6}{y+2}$
7. $\lim_{x\to -4}(x+3)^{1998}$
8. $\lim_{x\to \frac{1}{2}}[x]$
9. $\lim_{x\to -2}\sqrt{x-2}$
10. $\lim_{x\to 0}\dfrac{|x|}{x}$
11. $\lim_{x\to 0}\dfrac{1}{x^2}$
12. $\lim_{x\to 0}\dfrac{(4+x)^2-16}{x}$
13. $\lim_{x\to 1}\dfrac{x-1}{x^2-1}$
14. $\lim_{x\to 0}\dfrac{5x^3+8x^2}{3x^4-16x^2}$
15. $\lim_{x\to 0}\dfrac{(2+x)^3-8}{x}$
16. $\lim_{t\to 2}\dfrac{t^2-3t+2}{t^2-4}$
17. $\lim_{x\to 0}\dfrac{\dfrac{1}{2+x}-\dfrac{1}{2}}{x}$
18. $\lim_{x\to 1}\dfrac{x^2-4}{x-1}$
19. $\lim_{x\to 2}\dfrac{x+1}{x^2-4}$
20. $\lim_{x\to 0^-}[x]$
21. $\lim_{x\to 2^-}[x]$
22. $\lim_{x\to 0^+}[x]$
23. $\lim_{x\to 0^-}\dfrac{x}{|x|}$
24. $\lim_{x\to 0.01}[x]$
25. $\lim_{x\to 0^+}\dfrac{x}{|x|}$
26. Find $\lim_{x\to -\infty} f(x)$, $\lim_{x\to \infty} f(x)$, $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$ if $f(x)=\begin{cases}\dfrac{x-2}{x-1}, x\le 0\\ \dfrac{1}{x^2}, x>0\end{cases}$

Evaluating Limits as $x \to \pm\infty$

For # 1 – 6, find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

1. $f(x) = 3x^2 - 2x + 1$
2. $f(x) = -4x^3 + x^2 - 2x$
3. $f(x) = \dfrac{3x^2 - x + 5}{x^2 - 4}$
4. $f(x) = \dfrac{x - 2}{2x^2 + 3x - 5}$
5. $f(x) = \dfrac{-x^4 + 2x^2 + x - 3}{x^2 - 4}$
6. $f(x) = \dfrac{-4x^3 - 2x + 1}{x - 2}$

Additional Limits Practice

For # 7 – 17, find the numerical value of each limit or state that the limit does not exist or is best described as being $+\infty$ or $-\infty$.

7. $\lim_{x\to2^+} \dfrac{1}{x-2}$
8. $\lim_{x\to-3^-} \dfrac{1}{x+3}$
9. $\lim_{x\to2^-} \dfrac{x}{x-2}$
10. $\lim_{x\to-3^+} \dfrac{x}{x+3}$
11. $\lim_{x\to3} \dfrac{1}{(x-3)^2}$
12. $\lim_{x\to-6} \dfrac{1}{(x+6)^3}$
13. $\lim_{x\to1} \dfrac{x^2-1}{\sqrt{x}-1}$
14. $\lim_{h\to0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$
15. $\lim_{h\to0} \dfrac{\sqrt{1+h}-1}{h}$
16. $\lim_{x\to7} \dfrac{\sqrt{x+2}-3}{x-7}$
17. Find $\lim_{x\to-\infty} f(x)$, $\lim_{x\to\infty} f(x)$, $\lim_{x\to0^-} f(x)$ and $\lim_{x\to0^+} f(x)$ if $f(x) = \begin{cases} \dfrac{1}{x}, x < 0 \\ -1, x \geq 0 \end{cases}$

AP Calc: Limits involving Trig

Evaluate each of the following.

1. $\lim_{x\to\frac{\pi}{4}} \sin 2x$

2. $\lim_{x\to\frac{\pi}{2}} \tan x$

3. $\lim_{x\to 0} \frac{\sin 3x}{x}$

4. $\lim_{x\to 0} \frac{\tan 5x}{3x}$

5. $\lim_{x\to 0} \frac{\sin x \cos x}{x}$

6. $\lim_{x\to 0} \frac{\sin^2 x}{x}$

7. $\lim_{x\to 0} \frac{\cos x}{x^2}$

8. $\lim_{x\to\infty} \frac{\tan x}{x}$

9. $\lim_{x\to 0} \frac{3\sin 4x}{\sin 3x}$

10. $\lim_{x\to 0} \frac{\sin x}{2x^2 - x}$

11. $\lim_{x\to 0} \frac{x + \sin x}{x}$

12. $\lim_{x\to\infty} x(\sin\frac{1}{x})$

13. $\lim_{x\to 0} x(\sin\frac{1}{x})$

14. $\lim_{x\to 0} \frac{1-\cos x}{x}$

15. $\lim_{x\to 0} \frac{\sin x}{x^2}$

16. $\lim_{x\to 0} \frac{\sin x}{x^3}$

17. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

18. $\lim_{x\to 0} \frac{x}{\cos x}$

19. $\lim_{x\to\infty} x^2 \sin(\frac{1}{x})$

20. $\lim_{x\to 0} \frac{\sin x}{x(x+2)}$

21. $\lim_{x\to\infty} \frac{5x + \sin x}{x}$

AP Calc: Continuity

1. For what value(s) of x is each of the following functions discontinuous?

(a) $f(x)=\dfrac{1}{(x+2)^2}$

(b) $g(x)=\dfrac{x+1}{x^2-4x+3}$

(c) $h(x)=\sqrt[3]{2x-1}$

(d) $f(x)=\ln(x+1)$

For each of the following, find a value for the constant k, that will make the function continuous.

2. $f(x)=\begin{cases} x^2-1, & x<3 \\ 2kx, & x\geq 3 \end{cases}$

3. $g(x)=\begin{cases} 4-x^2, & x<-1 \\ kx^2-1, & x\geq -1 \end{cases}$

4. $h(x)=\begin{cases} \dfrac{\sin x}{2x}, & x\neq 0 \\ k, & x=0 \end{cases}$

5. $f(x)=\begin{cases} \dfrac{5-\sqrt{x}}{x-25}, & x\neq 25 \\ k, & x=25 \end{cases}$

6. Given $g(x)=\begin{cases} 1, & x\leq -1 \\ -x, & -1<x<0 \\ 1, & x=0 \\ -x, & 0<x<1 \\ 1, & x\geq 1 \end{cases}$ Is g continuous at x = -1? 0? 1? Explain.

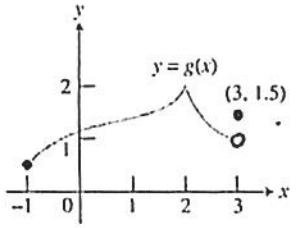
In exercises 7 and 8, use the graph of the function with domain $-1 \le x \le 3$.

7. Determine

(a) $\lim_{x \to 3^-} g(x)$. (b) $g(3)$.

(c) whether $g(x)$ is continuous at $x = 3$.

(d) the points of discontinuity of $g(x)$.

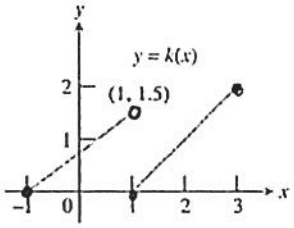


8. Determine

(a) $\lim_{x \to 1^-} k(x)$. (b) $\lim_{x \to 1^+} k(x)$. (c) $k(1)$.

(d) whether $k(x)$ is continuous at $x = 1$.

(e) the points of discontinuity of $k(x)$.



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REVIEW OF LIMITS

CALCULUS S. CONRAD

PART I: 1) DEFINE what is meant by the statement $\lim_{x \to 3} (x + 4) = 7$.

2) TRUE or FALSE: If $f(x) = \frac{x^2 + x}{x}$, then $\lim_{x \to 0} \frac{x^2 + x}{x} = f(0)$.

PART II:

3) $\lim_{x \to -5} (3 - x) =$

4) $\lim_{x \to 3} (\frac{2x}{3} + 5) =$

5) $\lim_{x \to 2} (\frac{6 - 5x}{2}) =$

PART III: For each, find the numerical value of each limit, or state that the limit does not exist or is best described as being $+\infty$ or $-\infty$.

6) $\lim_{x \to 3} \frac{4}{x - 1}$

7) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

8) $\lim_{x \to \frac{1}{2}} \frac{3 + 2x}{5 - x}$

9) $\lim_{x \to -3} \frac{x^2 + 5x + 6}{x^2 - x - 12}$

10) $\lim_{x \to 0} \frac{(3 + x)^2 - 9}{x}$

11) $\lim_{x \to \infty} \frac{4x - 3}{2x + 5}$

12) $\lim_{x \to \infty} \frac{1}{x}$

13) $\lim_{x \to -\infty} x^3 - 5$

14) $\lim_{x \to \infty} \frac{2x^2 - x + 5}{4x^3 - 1}$

15) $\lim_{x \to \infty} \frac{4x^3 + 2x^2 - 5}{9x^3 + x + 2}$

16) $\lim_{x \to \infty} \frac{2x^3 + 3}{x^2 + 5}$

17) $\lim_{x \to 0} \frac{1}{x}$

18) $\lim_{x \to 0} \frac{1}{x^2}$

19) $\lim_{x \to 2} \frac{3}{(x - 2)^2}$

20) $\lim_{x \to 2^-} \frac{x + 2}{x^2 - 4}$

21) $\lim_{x \to 2} \frac{x + 2}{x^2 - 4}$

22) $\lim_{x \to 3^+} \frac{2}{3 - x}$

23) $\lim_{x \to 0^+} \frac{|x|}{x}$

24) $\lim_{x \to 0^-} \frac{|x|}{x}$

25) $\lim_{x \to 0} \frac{|x|}{x}$

26) $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

27) $\lim_{x \to 1} \frac{x - 1}{\sqrt{x - 1}}$

28) $\lim_{x \to 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}}$

29) $\lim_{x \to 0^+} \log x$

30) $\lim_{x \to 1} \log x$

31) $\lim_{x \to 9} \log_3 x$

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CALCULUS

INTRODUCTORY LIMIT PROBLEMS

I: SUBSTITUTION

1) $\lim_{x \to 2} (2x + 3)$

2)* $\lim_{x \to 2} \frac{x^2 - 4}{x + 4}$

3) $\lim_{x \to 0} \frac{3^x - 3^{-x}}{3^x + 3^{-x}}$

4)* $\lim_{x \to 4} \sqrt{25 - x^2}$

II: FACTOR & REDUCE

5) $\lim_{x \to 4} \frac{x - 4}{x^2 - x - 12}$

6)* $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$

7) $\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$

8)* $\lim_{h \to 0} \frac{(2 + h)^2 - 4}{h}$

9) $\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h}$

10)* $\lim_{x \to -1} \frac{x^2 - 2x - 3}{x^2 - 1}$

III: +∞, −∞, DOESN'T EXIST

11) $\lim_{x \to 1} \frac{1}{(x - 1)^2}$

12)* $\lim_{x \to 1} \frac{1}{x - 1}$

13) $\lim_{x \to 3} \frac{4}{3 - x}$

14)* $\lim_{x \to -2} \frac{-1}{(x + 2)^2}$

15) $\lim_{x \to 1} \frac{-1}{(x - 1)^2}$

16)* $\lim_{x \to 4} \frac{-1}{(4 - x)^2}$

17) $\lim_{x \to 0} \frac{1}{x^2}$

18)* $\lim_{x \to 0} \frac{1}{x}$

19) $\lim_{x \to 0} \frac{1}{3 + 2^{1/x}}$

20)* $\lim_{x \to 0} \frac{1}{2 + 3^{1/x}}$

21) $\lim_{x \to -2} \frac{x^2 + 4x + 3}{x^2 + 3x + 2}$

IV: $x \to \infty$

22) $\lim_{x \to \infty} \frac{3x - 2}{9x + 7}$

23)* $\lim_{x \to \infty} \frac{6x^2 + 2x - 4}{6x^2 - 3x + 4}$

24) $\lim_{x \to \infty} \frac{x^2 + x - 2}{4x^3 - 1}$

25)* $\lim_{x \to \infty} \frac{2x^3}{x^2 + 1}$

26) $\lim_{x \to \infty} \frac{x^2 + 5x + 6}{x + 1}$

27)* $\lim_{x \to \infty} \frac{x + 3}{x^2 + 5x + 6}$

28) $\lim_{x \to \infty} \frac{-x^4 + 2x + 1}{x + 10}$

29)* $\lim_{x \to \infty} \frac{-4x^2}{x}$

V: RATIONALIZING

30) $\lim_{x \to 2} \frac{x - 2}{\sqrt{x^2 - 4}}$

31)* $\lim_{x \to 3} \frac{x - 3}{\sqrt{x^2 - 9}}$

32) $\lim_{x \to 1} \frac{x - 1}{\sqrt{x^2 + 3} - 2}$

33)* $\lim_{x \to 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}}$

34) $\lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x}$

35)* $\lim_{x \to 4} \frac{2 - \sqrt{x}}{4 - x}$

VI: LOGS, TRIG, ABS. VALUE

36) $\lim_{x \to 10} \log x$

37)* $\lim_{x \to 0^+} \log x$

38) $\lim_{x \to \infty} \log x$

39)* $\lim_{x \to 10} x \log x$

40) $\lim_{x \to \frac{1}{2}\pi} \sin x$

41)* $\lim_{x \to \frac{1}{2}\pi} \frac{\sin x}{x}$

42) $\lim_{x \to \pi} \frac{\sin x}{x}$

43) $\lim_{x \to 0} \frac{|x|}{x}$

44) $\lim_{x \to -1} \frac{|x + 1|}{x + 1}$