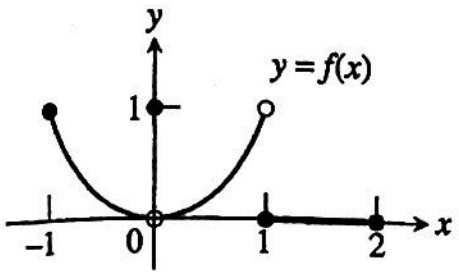
For Students Entering
AP Calculus AB

The attached packet contains optional practice questions on Limits and Continuity. They will not be collected or graded but will help you know what types of problems you can expect to be tested on in AP Calc next year. You can see us for extra help when we return in the fall with any questions.

Finding Limits Graphically

For # 1 – 2, tell whether the statements are true or false.

1.



(a) $\lim_{x \to -1^+} f(x) = 1$

(b) $\lim_{x \to 0^-} f(x) = 0$

(c) $\lim_{x \to 0^-} f(x) = 1$

(d) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

(e) $\lim_{x \to 0} f(x)$ exists

(f) $\lim_{x \to 0} f(x) = 0$

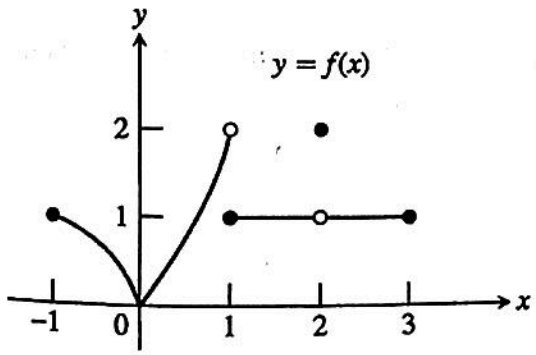
(g) $\lim_{x \to 0} f(x) = 1$

(h) $\lim_{x \to 1} f(x) = 1$

(i) $\lim_{x \to 1} f(x) = 0$

(j) $\lim_{x \to 2^-} f(x) = 2$

2.



(a) $\lim_{x \to -1^+} f(x) = 1$

(b) $\lim_{x \to 2} f(x)$ does not exist.

(c) $\lim_{x \to 2} f(x) = 2$

(d) $\lim_{x \to 1^-} f(x) = 2$

(e) $\lim_{x \to 1^+} f(x) = 1$

(f) $\lim_{x \to 1} f(x)$ does not exist.

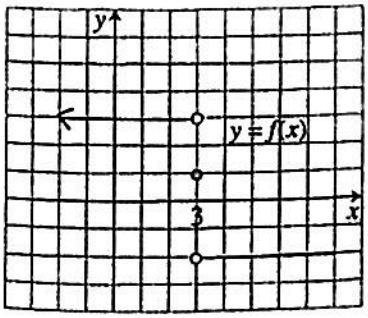
(g) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

(h) $\lim_{x \to c} f(x)$ exists at every c in $(-1, 1)$.

(i) $\lim_{x \to c} f(x)$ exists at every c in $(1, 3)$.

For # 3 – 8, use the graph to estimate the limits and the value of the function or explain why the limits do not exist.

3.



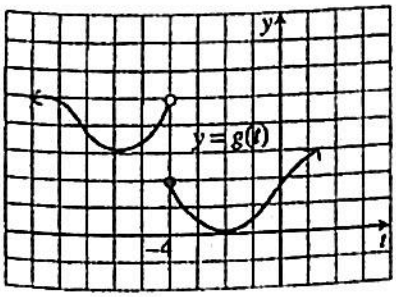
(a) $\lim_{x \to 3^-} f(x)$

(b) $\lim_{x \to 3^+} f(x)$

(c) $\lim_{x \to 3} f(x)$

(d) $f(3)$

4.



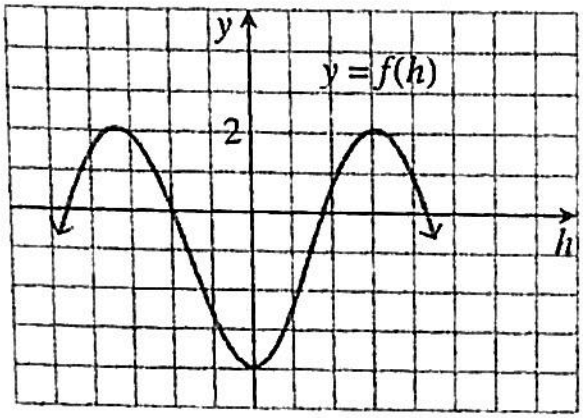
(a) $\lim_{t \to -4^-} g(t)$

(b) $\lim_{t \to -4^+} g(t)$

(c) $\lim_{t \to -4} g(t)$

(d) $g(-4)$

5.



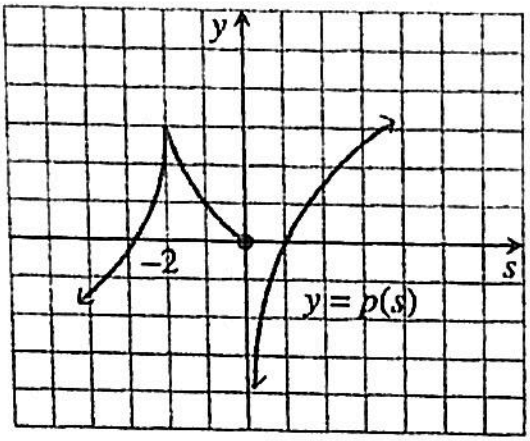
(a) $\lim_{h \to 0^-} f(h)$

(b) $\lim_{h \to 0^+} f(h)$

(c) $\lim_{h \to 0} f(h)$

(d) $f(0)$

6.



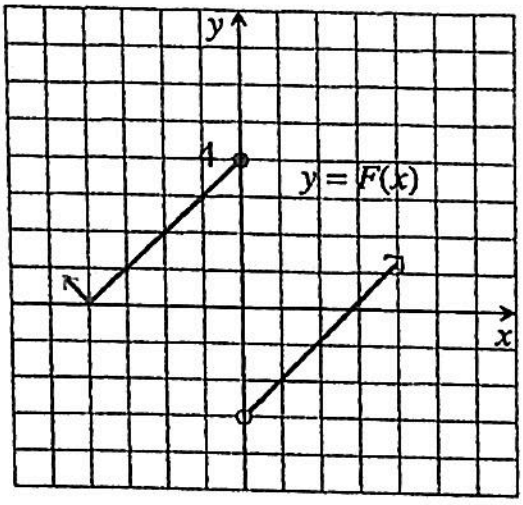
(a) $\lim_{s \to -2^-} p(s)$

(b) $\lim_{s \to -2^+} p(s)$

(c) $\lim_{s \to -2} p(s)$

(d) $p(-2)$

7.



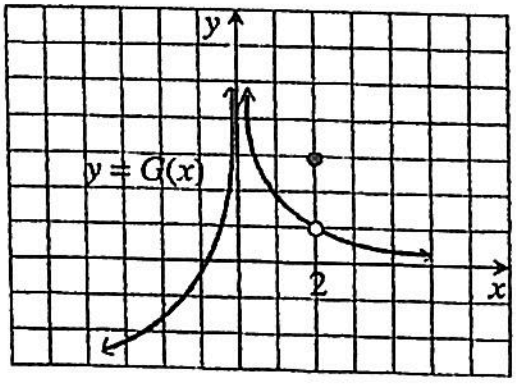
(a) $\lim_{x \to 0^-} F(x)$

(b) $\lim_{x \to 0^+} F(x)$

(c) $\lim_{x \to 0} F(x)$

(d) $F(0)$

8.



(a) $\lim_{x \to 2^-} G(x)$

(b) $\lim_{x \to 2^+} G(x)$

(c) $\lim_{x \to 2} G(x)$

(d) $G(2)$

Evaluating Limits

Find the numerical value of each limit or state that the limit does not exist or is best described as being $+\infty$ or $-\infty$.

1. $\lim_{x\to -\frac{1}{2}} 3x^2(2x-1)$
2. $\lim_{x\to 1}\left(x^3+3x^2-2x-17\right)$
3. $\lim_{y\to -3}\dfrac{y^2+4y+3}{y^2-3}$
4. $\lim_{x\to -2}(x-6)^{\frac{2}{3}}$
5. $\lim_{x\to 2}\sqrt{x+3}$
6. $\lim_{y\to 2}\dfrac{y^2+5y+6}{y+2}$
7. $\lim_{x\to -4}(x+3)^{1998}$
8. $\lim_{x\to \frac{1}{2}}[x]$
9. $\lim_{x\to -2}\sqrt{x-2}$
10. $\lim_{x\to 0}\dfrac{|x|}{x}$
11. $\lim_{x\to 0}\dfrac{1}{x^2}$
12. $\lim_{x\to 0}\dfrac{(4+x)^2-16}{x}$
13. $\lim_{x\to 1}\dfrac{x-1}{x^2-1}$
14. $\lim_{x\to 0}\dfrac{5x^3+8x^2}{3x^4-16x^2}$
15. $\lim_{x\to 0}\dfrac{(2+x)^3-8}{x}$
16. $\lim_{t\to 2}\dfrac{t^2-3t+2}{t^2-4}$
17. $\lim_{x\to 0}\dfrac{\dfrac{1}{2+x}-\dfrac{1}{2}}{x}$
18. $\lim_{x\to 1}\dfrac{x^2-4}{x-1}$
19. $\lim_{x\to 2}\dfrac{x+1}{x^2-4}$
20. $\lim_{x\to 0^-}[x]$
21. $\lim_{x\to 2^-}[x]$
22. $\lim_{x\to 0^+}[x]$
23. $\lim_{x\to 0^-}\dfrac{x}{|x|}$
24. $\lim_{x\to 0.01}[x]$
25. $\lim_{x\to 0^+}\dfrac{x}{|x|}$
26. Find $\lim_{x\to -\infty} f(x)$, $\lim_{x\to \infty} f(x)$, $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$ if $f(x)=\begin{cases}\dfrac{x-2}{x-1}, x\le 0\\ \dfrac{1}{x^2}, x>0\end{cases}$

Evaluating Limits as $x \to \pm\infty$

For # 1 – 6, find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

1. $f(x)=3x^2-2x+1$
2. $f(x)=-4x^3+x^2-2x$
3. $f(x)=\dfrac{3x^2-x+5}{x^2-4}$
4. $f(x)=\dfrac{x-2}{2x^2+3x-5}$
5. $f(x)=\dfrac{-x^4+2x^2+x-3}{x^2-4}$
6. $f(x)=\dfrac{-4x^3-2x+1}{x-2}$

Additional Limits Practice

For # 7 – 17, find the numerical value of each limit or state that the limit does not exist or is best described as being $+\infty$ or $-\infty$.

7. $\lim_{x\to2^+}\dfrac{1}{x-2}$
8. $\lim_{x\to-3^-}\dfrac{1}{x+3}$
9. $\lim_{x\to2^-}\dfrac{x}{x-2}$
10. $\lim_{x\to-3^+}\dfrac{x}{x+3}$
11. $\lim_{x\to3}\dfrac{1}{(x-3)^2}$
12. $\lim_{x\to-6}\dfrac{1}{(x+6)^3}$
13. $\lim_{x\to1}\dfrac{x^2-1}{\sqrt{x}-1}$
14. $\lim_{h\to0}\dfrac{(3+h)^{-1}-3^{-1}}{h}$
15. $\lim_{h\to0}\dfrac{\sqrt{1+h}-1}{h}$
16. $\lim_{x\to7}\dfrac{\sqrt{x+2}-3}{x-7}$
17. Find $\lim_{x\to-\infty} f(x)$, $\lim_{x\to\infty} f(x)$, $\lim_{x\to0^-} f(x)$ and $\lim_{x\to0^+} f(x)$ if $f(x)=\begin{cases}\dfrac{1}{x}, x<0\\ -1, x\geq0\end{cases}$

AP Calc: Limits involving Trig

Evaluate each of the following.

1. $\lim_{x\to\frac{\pi}{4}} \sin 2x$

2. $\lim_{x\to\frac{\pi}{2}} \tan x$

3. $\lim_{x\to 0} \frac{\sin 3x}{x}$

4. $\lim_{x\to 0} \frac{\tan 5x}{3x}$

5. $\lim_{x\to 0} \frac{\sin x \cos x}{x}$

6. $\lim_{x\to 0} \frac{\sin^2 x}{x}$

7. $\lim_{x\to 0} \frac{\cos x}{x^2}$

8. $\lim_{x\to \infty} \frac{\tan x}{x}$

9. $\lim_{x\to 0} \frac{3\sin 4x}{\sin 3x}$

10. $\lim_{x\to 0} \frac{\sin x}{2x^2 - x}$

11. $\lim_{x\to 0} \frac{x + \sin x}{x}$

12. $\lim_{x\to \infty} x(\sin \frac{1}{x})$

13. $\lim_{x\to 0} x(\sin \frac{1}{x})$

14. $\lim_{x\to 0} \frac{1-\cos x}{x}$

15. $\lim_{x\to 0} \frac{\sin x}{x^2}$

16. $\lim_{x\to 0} \frac{\sin x}{x^3}$

17. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

18. $\lim_{x\to 0} \frac{x}{\cos x}$

19. $\lim_{x\to \infty} x^2 \sin(\frac{1}{x})$

20. $\lim_{x\to 0} \frac{\sin x}{x(x+2)}$

21. $\lim_{x\to \infty} \frac{5x + \sin x}{x}$

AP Calc: Continuity

1. For what value(s) of x is each of the following functions discontinuous?

(a) $f(x)=\dfrac{1}{(x+2)^2}$

(b) $g(x)=\dfrac{x+1}{x^2-4x+3}$

(c) $h(x)=\sqrt[3]{2x-1}$

(d) $f(x)=\ln(x+1)$

For each of the following, find a value for the constant k, that will make the function continuous.

2. $f(x)=\begin{cases}x^2-1, & x<3\\ 2kx, & x\geq 3\end{cases}$

3. $g(x)=\begin{cases}4-x^2, & x<-1\\ kx^2-1, & x\geq -1\end{cases}$

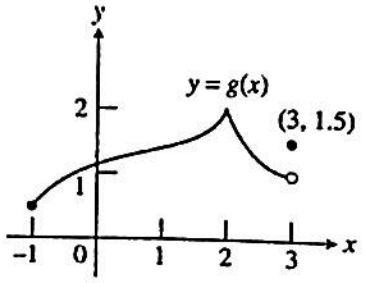
4. $h(x)=\begin{cases}\dfrac{\sin x}{2x}, & x\neq 0\\ k, & x=0\end{cases}$

5. $f(x)=\begin{cases}\dfrac{5-\sqrt{x}}{x-25}, & x\neq 25\\ k, & x=25\end{cases}$

6. Given $g(x)=\begin{cases}1, & x\leq -1\\ -x, & -1<x<0\\ 1, & x=0\\ -x, & 0<x<1\\ 1, & x\geq 1\end{cases}$ Is g continuous at x = -1? 0? 1? Explain.

In Exercises 7 – 8, use the graphs of the given functions with domain $-1 \le x \le 3$.

7.



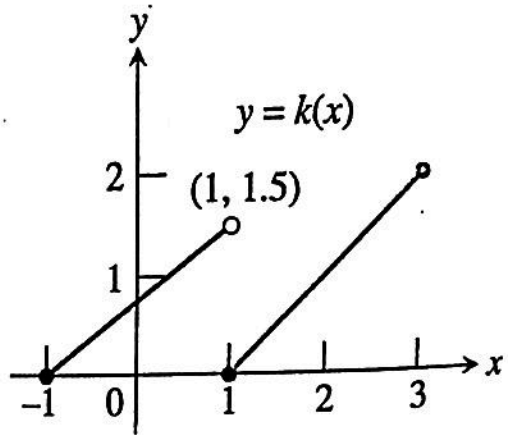
Determine

(a) $\lim_{x \to 3^-} g(x)$ (b) $g(3)$

(c) whether $g(x)$ is continuous at $x = 3$

(d) the points of discontinuity of $g(x)$.

8.



Determine

(a) $\lim_{x \to 1^-} k(x)$ (b) $\lim_{x \to 1^+} k(x)$ (c) $k(1)$

(d) whether $k(x)$ is continuous at $x = 1$

(e) the points of discontinuity of $k(x)$.

Roslyn High School **REVIEW OF LIMITS** CALCULUS S. CONRAD

PART I: 1) DEFINE what is meant by the statement $\lim_{x \to 3} (x + 4) = 7$.

2) TRUE or FALSE: If $f(x) = \frac{x^2 + x}{x}$, then $\lim_{x \to 0} \frac{x^2 + x}{x} = f(0)$.

PART II:

3) $\lim_{x \to -5} (3 - x) =$

4) $\lim_{x \to 3} (\frac{2x}{3} + 5) =$

5) $\lim_{x \to 2} (\frac{6 - 5x}{2}) =$

PART III: For each, find the numerical value of each limit, or state that the limit does not exist or is best described as being $+\infty$ or $-\infty$.

6) $\lim_{x \to 3} \frac{4}{x - 1}$

7) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

8) $\lim_{x \to \frac{1}{2}} \frac{3 + 2x}{5 - x}$

9) $\lim_{x \to -3} \frac{x^2 + 5x + 6}{x^2 - x - 12}$

10) $\lim_{x \to 0} \frac{(3 + x)^2 - 9}{x}$

11) $\lim_{x \to \infty} \frac{4x - 3}{2x + 5}$

12) $\lim_{x \to \infty} \frac{1}{x}$

13) $\lim_{x \to -\infty} x^3 - 5$

14) $\lim_{x \to \infty} \frac{2x^2 - x + 5}{4x^3 - 1}$

15) $\lim_{x \to \infty} \frac{4x^3 + 2x^2 - 5}{9x^3 + x + 2}$

16) $\lim_{x \to \infty} \frac{2x^3 + 3}{x^2 + 5}$

17) $\lim_{x \to 0} \frac{1}{x}$

18) $\lim_{x \to 0} \frac{1}{x^2}$

19) $\lim_{x \to 2} \frac{3}{(x - 2)^2}$

20) $\lim_{x \to 2^-} \frac{x + 2}{x^2 - 4}$

21) $\lim_{x \to 2} \frac{x + 2}{x^2 - 4}$

22) $\lim_{x \to 3^+} \frac{2}{3 - x}$

23) $\lim_{x \to 0^+} \frac{|x|}{x}$

24) $\lim_{x \to 0^-} \frac{|x|}{x}$

25) $\lim_{x \to 0} \frac{|x|}{x}$

26) $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

27) $\lim_{x \to 1} \frac{x - 1}{\sqrt{x - 1}}$

28) $\lim_{x \to 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}}$

29) $\lim_{x \to 0^+} \log x$

30) $\lim_{x \to 1} \log x$

31) $\lim_{x \to 9} \log_3 x$

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CALCULUS

INTRODUCTORY LIMIT PROBLEMS

I: SUBSTITUTION

1) $\lim_{x \to 2} (2x + 3)$

2)* $\lim_{x \to 2} \frac{x^2 - 4}{x + 4}$

3) $\lim_{x \to 0} \frac{3^x - 3^{-x}}{3^x + 3^{-x}}$

4)* $\lim_{x \to 4} \sqrt{25 - x^2}$

II: FACTOR & REDUCE

5) $\lim_{x \to 4} \frac{x - 4}{x^2 - x - 12}$

6)* $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$

7) $\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$

8)* $\lim_{h \to 0} \frac{(2 + h)^2 - 4}{h}$

9) $\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h}$

10)* $\lim_{x \to -1} \frac{x^2 - 2x - 3}{x^2 - 1}$

III: +∞, −∞, DOESN'T EXIST

11) $\lim_{x \to 1} \frac{1}{(x - 1)^2}$

12)* $\lim_{x \to 1} \frac{1}{x - 1}$

13) $\lim_{x \to 3} \frac{4}{3 - x}$

14)* $\lim_{x \to -2} \frac{-1}{(x + 2)^2}$

15) $\lim_{x \to 1} \frac{-1}{(x - 1)^2}$

16)* $\lim_{x \to 4} \frac{-1}{(4 - x)^2}$

17) $\lim_{x \to 0} \frac{1}{x^2}$

18)* $\lim_{x \to 0} \frac{1}{x}$

19) $\lim_{x \to 0} \frac{1}{3 + 2^{1/x}}$

20)* $\lim_{x \to 0} \frac{1}{2 + 3^{1/x}}$

21) $\lim_{x \to -2} \frac{x^2 + 4x + 3}{x^2 + 3x + 2}$

IV: $x \to \infty$

22) $\lim_{x \to \infty} \frac{3x - 2}{9x + 7}$

23)* $\lim_{x \to \infty} \frac{6x^2 + 2x - 4}{6x^2 - 3x + 4}$

24) $\lim_{x \to \infty} \frac{x^2 + x - 2}{4x^3 - 1}$

25)* $\lim_{x \to \infty} \frac{2x^3}{x^2 + 1}$

26) $\lim_{x \to \infty} \frac{x^2 + 5x + 6}{x + 1}$

27)* $\lim_{x \to \infty} \frac{x + 3}{x^2 + 5x + 6}$

28) $\lim_{x \to \infty} \frac{-x^4 + 2x + 1}{x + 10}$

29)* $\lim_{x \to \infty} \frac{-4x^2}{x}$

V: RATIONALIZING

30) $\lim_{x \to 2} \frac{x - 2}{\sqrt{x^2 - 4}}$

31)* $\lim_{x \to 3} \frac{x - 3}{\sqrt{x^2 - 9}}$

32) $\lim_{x \to 1} \frac{x - 1}{\sqrt{x^2 + 3} - 2}$

33)* $\lim_{x \to 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}}$

34) $\lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x}$

35)* $\lim_{x \to 4} \frac{2 - \sqrt{x}}{4 - x}$

VI: LOGS, TRIG, ABS. VALUE

36) $\lim_{x \to 10} \log x$

37)* $\lim_{x \to 0^+} \log x$

38) $\lim_{x \to \infty} \log x$

39)* $\lim_{x \to 10} x \log x$

40) $\lim_{x \to \frac{1}{2}\pi} \sin x$

41)* $\lim_{x \to \frac{1}{2}\pi} \frac{\sin x}{x}$

42) $\lim_{x \to \pi} \frac{\sin x}{x}$

43) $\lim_{x \to 0} \frac{|x|}{x}$

44) $\lim_{x \to -1} \frac{|x + 1|}{x + 1}$