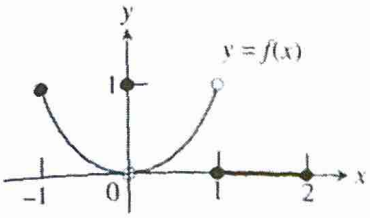
Finding Limits Graphically

For # 1 – 2, tell whether the statements are true or false.

(1)



a, b, d, e, f are TRUE
c, g, h, i, j are FALSE

(a) $\lim_{x \to -1^+} f(x) = 1$

(b) $\lim_{x \to 0^-} f(x) = 0$

(c) $\lim_{x \to 0^-} f(x) = 1$

(d) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

(e) $\lim_{x \to 0} f(x)$ exists

(f) $\lim_{x \to 0} f(x) = 0$

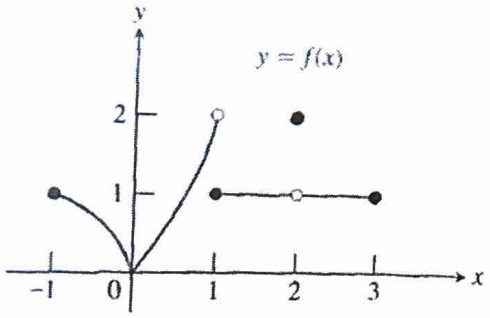
(g) $\lim_{x \to 0} f(x) = 1$

(h) $\lim_{x \to 1} f(x) = 1$

(i) $\lim_{x \to 1} f(x) = 0$

(j) $\lim_{x \to 2^-} f(x) = 2$

(2)



a, d, e, f, g, h, i are TRUE
b, c are FALSE

(a) $\lim_{x \to -1^+} f(x) = 1$

(b) $\lim_{x \to 2} f(x)$ does not exist.

(c) $\lim_{x \to 2} f(x) = 2$

(d) $\lim_{x \to 1^-} f(x) = 2$

(e) $\lim_{x \to 1^+} f(x) = 1$

(f) $\lim_{x \to 1} f(x)$ does not exist.

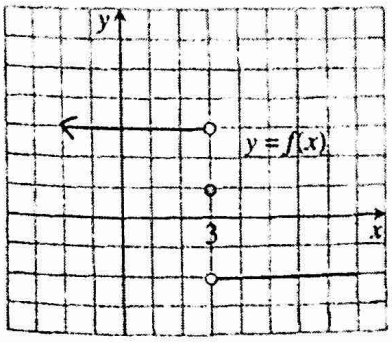
(g) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

(h) $\lim_{x \to c} f(x)$ exists at every c in $(-1, 1)$.

(i) $\lim_{x \to c} f(x)$ exists at every c in $(1, 3)$.

For # 3 – 8, use the graph to estimate the limits and the value of the function or explain why the limits do not exist.

3.



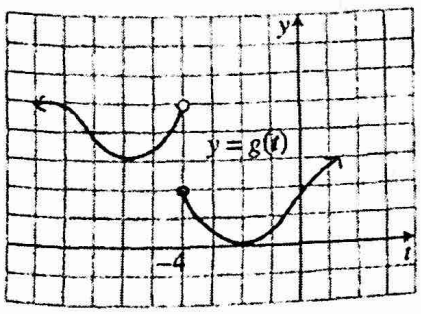
(a) $\lim_{x \to 3^-} f(x) = 3$

(b) $\lim_{x \to 3^+} f(x) = -2$

(c) $\lim_{x \to 3} f(x)$ = DNE

(d) $f(3) = 1$

4.



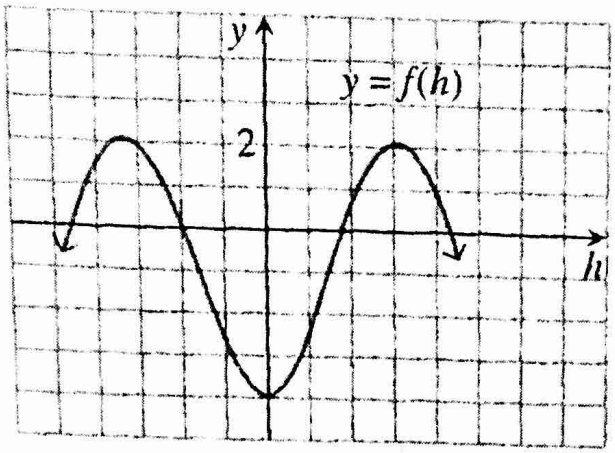
(a) $\lim_{t \to -4^-} g(t) = 5$

(b) $\lim_{t \to -4^+} g(t) = 2$

(c) $\lim_{t \to -4} g(t)$ = DNE

(d) $g(-4) = 2$

5.



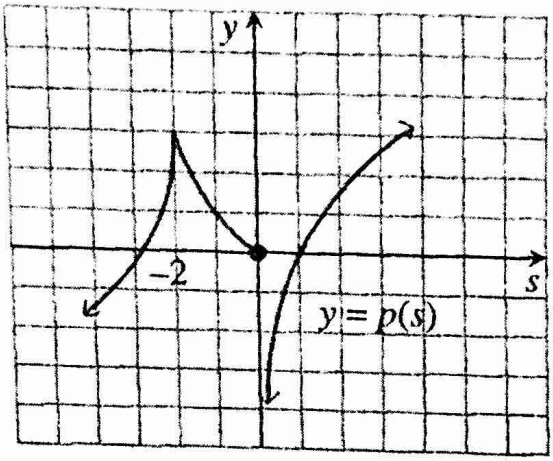
(a) $\lim_{h \to 0^-} f(h) = -4$

(b) $\lim_{h \to 0^+} f(h) = -4$

(c) $\lim_{h \to 0} f(h) = -4$

(d) $f(0) = -4$

6.



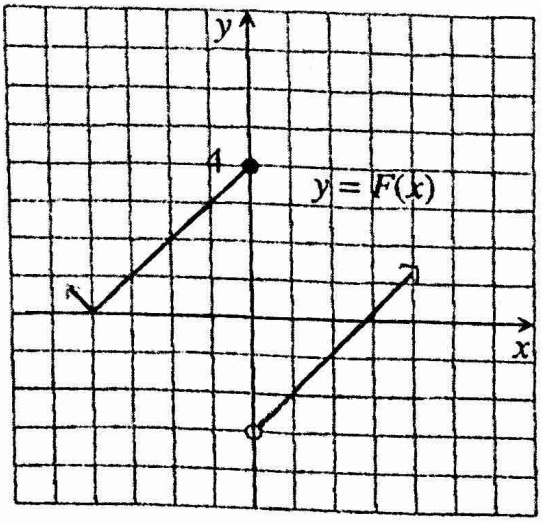
(a) $\lim_{s \to -2^-} p(s) = 3$

(b) $\lim_{s \to -2^+} p(s) = 3$

(c) $\lim_{s \to -2} p(s) = 3$

(d) $p(-2) = 3$

7.



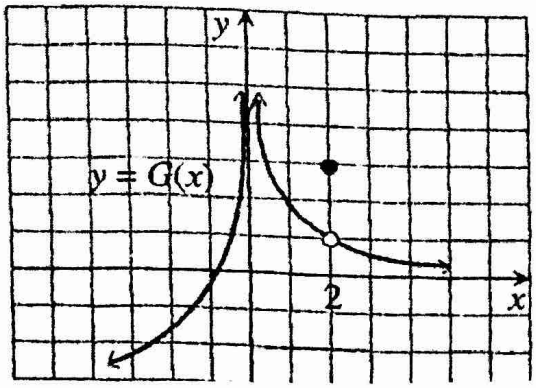
(a) $\lim_{x \to 0^-} F(x) = 4$

(b) $\lim_{x \to 0^+} F(x) = -3$

(c) $\lim_{x \to 0} F(x)$ = DNE

(d) $F(0) = 4$

8.



(a) $\lim_{x \to 2^-} G(x) = 1$

(b) $\lim_{x \to 2^+} G(x) = 1$

(c) $\lim_{x \to 2} G(x) = 1$

(d) $G(2) = 3$

Evaluating Limits

Find the numerical value of each limit or state that the limit does not exist or is best described as being $+\infty$ or $-\infty$.

1. $\lim_{x\to -\frac{1}{2}} 3x^2(2x-1) = -\frac{3}{2}$
2. $\lim_{x\to 1}(x^3+3x^2-2x-17) = -15$
3. $\lim_{y\to -3}\frac{y^2+4y+3}{y^2-3} = 0$
4. $\lim_{x\to -2}(x-6)^{\frac{2}{3}} = 4$
5. $\lim_{x\to 2}\sqrt{x+3} = \sqrt{5}$
6. $\lim_{y\to 2}\frac{y^2+5y+6}{y+2} = 5$
7. $\lim_{x\to -4}(x+3)^{1998} = 1$
8. $\lim_{x\to \frac{1}{2}}[x] = 0$
9. $\lim_{x\to -2}\sqrt{x-2} = \text{DNE}$
10. $\lim_{x\to 0}\frac{|x|}{x} = \text{DNE}$
11. $\lim_{x\to 0}\frac{1}{x^2} = \infty$
12. $\lim_{x\to 0}\frac{(4+x)^2-16}{x} = 8$
13. $\lim_{x\to 1}\frac{x-1}{x^2-1} = \frac{1}{2}$
14. $\lim_{x\to 0}\frac{5x^3+8x^2}{3x^4-16x^2} = -\frac{1}{2}$
15. $\lim_{x\to 0}\frac{(2+x)^3-8}{x} = 12$
16. $\lim_{t\to 2}\frac{t^2-3t+2}{t^2-4} = \frac{1}{4}$
17. $\lim_{x\to 0}\frac{\frac{1}{2+x}-\frac{1}{2}}{x} = -\frac{1}{4}$
18. $\lim_{x\to 1}\frac{x^2-4}{x-1} = \text{DNE}$
19. $\lim_{x\to 2}\frac{x+1}{x^2-4} = \text{DNE}$
20. $\lim_{x\to 0^-}[x] = -1$
21. $\lim_{x\to 2^-}[x] = 1$
22. $\lim_{x\to 0^+}[x] = 0$
23. $\lim_{x\to 0^-}\frac{x}{|x|} = -1$
24. $\lim_{x\to 0.01}[x] = 0$
25. $\lim_{x\to 0^+}\frac{x}{|x|} = 1$
26. Find $\lim_{x\to -\infty} f(x)$, $\lim_{x\to \infty} f(x)$, $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$ if $f(x)=\begin{cases}\frac{x-2}{x-1}, x\le 0\\ \frac{1}{x^2}, x>0\end{cases}$

$\lim_{x\to -\infty} f(x) = 1$ $\lim_{x\to 0^-} f(x)=2$

$\lim_{x\to \infty} f(x) = 0$ $\lim_{x\to 0^+} f(x)=\infty$

Evaluating Limits as $x \to \pm\infty$

For # 1 – 6, find $\lim_{x\to\infty} f(x)$ (a) and $\lim_{x\to-\infty} f(x)$ (b).

1. $f(x) = 3x^2 - 2x + 1$
(a) ∞ (b) ∞

2. $f(x) = -4x^3 + x^2 - 2x$
(a) $-\infty$ (b) ∞

3. $f(x) = \dfrac{3x^2 - x + 5}{x^2 - 4}$
(a) 3 (b) 3

4. $f(x) = \dfrac{x-2}{2x^2 + 3x - 5}$ (a) 0 (b) 0

5. $f(x) = \dfrac{-x^4 + 2x^2 + x - 3}{x^2 - 4}$ (a) $-\infty$ (b) $-\infty$

6. $f(x) = \dfrac{-4x^3 - 2x + 1}{x - 2}$
(a) $-\infty$ (b) $-\infty$

Additional Limits Practice

For # 7 – 17, find the numerical value of each limit or state that the limit does not exist or is best described as being $+\infty$ or $-\infty$.

7. $\lim_{x\to2^+} \dfrac{1}{x-2} = \infty$

8. $\lim_{x\to-3^-} \dfrac{1}{x+3} = -\infty$

9. $\lim_{x\to2^-} \dfrac{x}{x-2} = -\infty$

10. $\lim_{x\to-3^+} \dfrac{x}{x+3} = -\infty$

11. $\lim_{x\to3} \dfrac{1}{(x-3)^2} = \infty$

12. $\lim_{x\to-6} \dfrac{1}{(x+6)^3} = \text{DNE}$

13. $\lim_{x\to1} \dfrac{x^2-1}{\sqrt{x}-1} = 4$

14. $\lim_{h\to0} \dfrac{(3+h)^{-1} - 3^{-1}}{h} = -\dfrac{1}{9}$

15. $\lim_{h\to0} \dfrac{\sqrt{1+h}-1}{h} = \dfrac{1}{2}$

16. $\lim_{x\to7} \dfrac{\sqrt{x+2}-3}{x-7} = \dfrac{1}{6}$

17. Find $\lim_{x\to-\infty} f(x)$, $\lim_{x\to\infty} f(x)$, $\lim_{x\to0^-} f(x)$
$= 0$ $= -1$ $= -\infty$

and $\lim_{x\to0^+} f(x)$ if $f(x) = \begin{cases} \dfrac{1}{x}, x < 0 \\ -1, x \geq 0 \end{cases}$
$= -1$

AP Calc: Limits involving Trig

Evaluate each of the following.

1. $\lim_{x\to\frac{\pi}{4}} \sin 2x$ $= 1$

2. $\lim_{x\to\frac{\pi}{2}} \tan x$ $= DNE$

3. $\lim_{x\to 0} \frac{\sin 3x}{x}$ $= 3$

4. $\lim_{x\to 0} \frac{\tan 5x}{3x}$ $= \frac{5}{3}$

5. $\lim_{x\to 0} \frac{\sin x \cos x}{x}$ $= 1$

6. $\lim_{x\to 0} \frac{\sin^2 x}{x}$ $= 0$

7. $\lim_{x\to 0} \frac{\cos x}{x^2}$ $= \infty$

8. $\lim_{x\to\infty} \frac{\tan x}{x}$ $= DNE$

9. $\lim_{x\to 0} \frac{3\sin 4x}{\sin 3x}$ $= 4$

10. $\lim_{x\to 0} \frac{\sin x}{2x^2 - x}$ $= -1$

11. $\lim_{x\to 0} \frac{x + \sin x}{x}$ $= 2$

12. $\lim_{x\to\infty} x(\sin\frac{1}{x})$ $= 1$

13. $\lim_{x\to 0} x(\sin\frac{1}{x})$ $= 0$

14. $\lim_{x\to 0} \frac{1-\cos x}{x}$ $= 0$

15. $\lim_{x\to 0} \frac{\sin x}{x^2}$ $= DNE$

16. $\lim_{x\to 0} \frac{\sin x}{x^3}$ $= \infty$

17. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$ $= \frac{1}{2}$

18. $\lim_{x\to 0} \frac{x}{\cos x}$ $= 0$

19. $\lim_{x\to\infty} x^2 \sin(\frac{1}{x})$ $= \infty$

20. $\lim_{x\to 0} \frac{\sin x}{x(x+2)}$ $= \frac{1}{2}$

21. $\lim_{x\to\infty} \frac{5x + \sin x}{x}$ $= 5$

AP Calc: Continuity

1. For what value(s) of x is each of the following functions discontinuous?

(a) $f(x)=\dfrac{1}{(x+2)^2}$ at $x=-2$

(c) $h(x)=\sqrt[3]{2x-1}$ none

(b) $g(x)=\dfrac{x+1}{x^2-4x+3}$ at $x=1$ and $x=3$

(d) $f(x)=\ln(x+1)$ $x \le -1$

For each of the following, find a value for the constant k, that will make the function continuous.

2. $f(x)=\begin{cases} x^2-1, & x<3 \\ 2kx, & x\ge 3 \end{cases}$ $k=\dfrac{4}{3}$

3. $g(x)=\begin{cases} 4-x^2, & x<-1 \\ kx^2-1, & x\ge -1 \end{cases}$ $k=4$

4. $h(x)=\begin{cases} \dfrac{\sin x}{2x}, & x\neq 0 \\ k, & x=0 \end{cases}$ $k=\dfrac{1}{2}$

5. $f(x)=\begin{cases} \dfrac{5-\sqrt{x}}{x-25}, & x\neq 25 \\ k, & x=25 \end{cases}$ $k=-\dfrac{1}{10}$

6. Given $g(x)=\begin{cases} 1, & x\le -1 \\ -x, & -1<x<0 \\ 1, & x=0 \\ -x, & 0<x<1 \\ 1, & x\ge 1 \end{cases}$ Is g continuous at $x=-1$? 0? 1? Explain.

at $x=-1$?
continuous because $\lim_{x\to -1} g(x)=g(-1)$

at $x=0$?
not continuous because $\lim_{x\to 0} g(x)\neq g(0)$

at $x=1$?
not continuous because $\lim_{x\to 1} g(x)$ DNE

In exercises 7 and 8, use the graph of the function with domain $-1 \le x \le 3$.

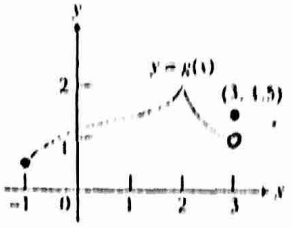
7. Determine

(a) $\lim_{x \to 1^-} g(x)$. $= 1$ (b) $g(3)$. $= 1.5$

(c) whether $g(x)$ is continuous at $x = 3$. no because $\lim_{x \to 3^-} g(x) \neq g(3)$

(d) the points of discontinuity of $g(x)$.

$x = 3$



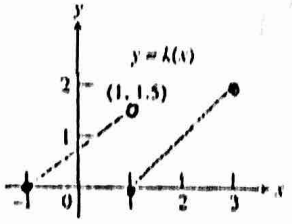
8. Determine

(a) $\lim_{x \to 1^-} k(x)$. $= 1.5$ (b) $\lim_{x \to 1^+} k(x)$. $= 0$ (c) $k(1)$. $= 0$

(d) whether $k(x)$ is continuous at $x = 1$. no because $\lim_{x \to 1} k(x)$ DNE

(e) the points of discontinuity of $k(x)$.

$x = 1$



Key

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REVIEW OF LIMITS

CALCULUS S. CONRAD

PART I: 1) DEFINE what is meant by the statement $\lim_{x \to 3} (x + 4) = 7$.

2) TRUE or FALSE: If $f(x) = \frac{x^2 + x}{x}$, then $\lim_{x \to 0} \frac{x^2 + x}{x} = f(0)$. False

PART II:

3) $\lim_{x \to -5} (3 - x) = 8$ 4) $\lim_{x \to 3} (\frac{2x}{3} + 5) = 7$ 5) $\lim_{x \to 2} (\frac{6 - 5x}{2}) = -2$

PART III: For each, find the numerical value of each limit, or state that the limit does not exist or is best described as being $+\infty$ or $-\infty$.

6) $\lim_{x \to 3} \frac{4}{x - 1} = 2$

7) $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$

8) $\lim_{x \to \frac{1}{2}} \frac{3 + 2x}{5 - x} = 8/9$

9) $\lim_{x \to -3} \frac{x^2 + 5x + 6}{x^2 - x + 12} = \frac{1}{7}$

10) $\lim_{x \to 0} \frac{(3 + x)^2 - 9}{x} = 6$

11) $\lim_{x \to \infty} \frac{4x - 3}{2x + 5} = 2$

12) $\lim_{x \to \infty} \frac{1}{x} = 0$

13) $\lim_{x \to -\infty} x^3 - 5 = -\infty$

14) $\lim_{x \to \infty} \frac{2x^2 - x + 5}{4x^3 - 1} = 0$

15) $\lim_{x \to \infty} \frac{4x^3 + 2x^2 - 5}{9x^3 + x + 2} = \frac{4}{9}$

16) $\lim_{x \to \infty} \frac{2x^3 + 3}{x^2 + 5} = +\infty$

17) $\lim_{x \to 0} \frac{1}{x} = DNE$

18) $\lim_{x \to 0} \frac{1}{x^2} = +\infty$

19) $\lim_{x \to 2} \frac{3}{(x - 2)^2} = +\infty$

20) $\lim_{x \to 2^-} \frac{x + 2}{x^2 - 4} = -\infty$

21) $\lim_{x \to 2} \frac{x + 2}{x^2 - 4} = DNE$

22) $\lim_{x \to 3^+} \frac{2}{3 - x} = -\infty$

23) $\lim_{x \to 0^+} \frac{|x|}{x} = 1$

24) $\lim_{x \to 0^-} \frac{|x|}{x} = -1$

25) $\lim_{x \to 0} \frac{|x|}{x} = DNE$

26) $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = \frac{1}{4}$

27) $\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1} = 0$

28) $\lim_{x \to 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}} = 6$

29) $\lim_{x \to 0^+} \log x = -\infty$

30) $\lim_{x \to 1} \log x = 0$

31) $\lim_{x \to 9} \log_3 x = 2$

32) $\lim_{x \to 0} \frac{\sin 4x}{x} = 4$

33) $\lim_{x \to 0} \frac{x}{\cos x} = 0$

34) $\lim_{x \to 0} \frac{\sin 5x}{\sin 6x} = \frac{5}{6}$

35) $\lim_{x \to \infty} \sin x = DNE$

36) $\lim_{x \to \frac{1}{2}\pi} \frac{\sin x}{x} = \frac{2}{\pi}$

37) $\lim_{x \to 0} \frac{\tan 3x}{x} = 3$

38) $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

HINT: Mult by $\frac{1 + \cos x}{1 + \cos x}$

39) $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

40) $\lim_{x \to 0} \frac{2 \sin x \cos x}{x} = 2$

Key

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CALCULUS

INTRODUCTORY LIMIT PROBLEMS

I: SUBSTITUTION

1) $\lim_{x \to 2} (2x + 3)$ 7

2)* $\lim_{x \to 2} \frac{x^2 - 4}{x + 4}$ 0

3) $\lim_{x \to 0} \frac{3^x - 3^{-x}}{3^x + 3^{-x}}$ 0

4)* $\lim_{x \to 4} \sqrt{25 - x^2}$ 3

II: FACTOR & REDUCE

5) $\lim_{x \to 4} \frac{x - 4}{x^2 - x - 12}$ $\frac{1}{7}$

6)* $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$ -4

7) $\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$ $\frac{1}{4}$

8)* $\lim_{h \to 0} \frac{(2 + h)^2 - 4}{h}$ 4

9) $\lim_{h \to 0} \frac{(x + h)^2 - x^2}{h}$ $2x$

10)* $\lim_{x \to -1} \frac{x^2 - 2x - 3}{x^2 - 1}$ 2

III: +∞, −∞, DOESN'T EXIST

11) $\lim_{x \to 1} \frac{1}{(x - 1)^2}$ $+\infty$

12)* $\lim_{x \to 1} \frac{1}{x - 1}$ DNE

13) $\lim_{x \to 3} \frac{4}{3 - x}$ DNE

14)* $\lim_{x \to -2} \frac{-1}{(x + 2)^2}$ $-\infty$

15) $\lim_{x \to 1} \frac{-1}{(x - 1)^2}$ $-\infty$

16)* $\lim_{x \to 4} \frac{-1}{(4 - x)^2}$ $-\infty$

17) $\lim_{x \to 0} \frac{1}{x^2}$ $+\infty$

18)* $\lim_{x \to 0} \frac{1}{x}$ DNE

19) $\lim_{x \to 0} \frac{1}{3 + 2^{1/x}}$ DNE

20)* $\lim_{x \to 0} \frac{1}{2 + 3^{1/x}}$ DNE

21) $\lim_{x \to -2} \frac{x^2 + 4x + 3}{x^2 + 3x + 2}$ DNE

IV: $x \to \infty$

22) $\lim_{x \to \infty} \frac{3x - 2}{9x + 7}$ $\frac{1}{3}$

23)* $\lim_{x \to \infty} \frac{6x^2 + 2x - 4}{6x^2 - 3x + 4}$ 1

24) $\lim_{x \to \infty} \frac{x^2 + x - 2}{4x^3 - 1}$ 0

25)* $\lim_{x \to \infty} \frac{2x^3}{x^2 + 1}$ $+\infty$

26) $\lim_{x \to \infty} \frac{x^2 + 5x + 6}{x + 1}$ $+\infty$

27)* $\lim_{x \to \infty} \frac{x + 3}{x^2 + 5x + 6}$ 0

28) $\lim_{x \to \infty} \frac{-x^4 + 2x + 1}{x + 10}$ $-\infty$

29)* $\lim_{x \to \infty} \frac{-4x^2}{x}$ $-\infty$

V: RATIONALIZING

30) $\lim_{x \to 2} \frac{x - 2}{\sqrt{x^2 - 4}}$ 0

31)* $\lim_{x \to 3} \frac{x - 3}{\sqrt{x^2 - 9}}$ 0

32) $\lim_{x \to 1} \frac{x - 1}{\sqrt{x^2 + 3} - 2}$ 2

33)* $\lim_{x \to 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}}$ 6

34) $\lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x}$ $\frac{1}{2}$

35)* $\lim_{x \to 4} \frac{2 - \sqrt{x}}{4 - x}$ $\frac{1}{4}$

VI: LOGS, TRIG, ABS. VALUE

36) $\lim_{x \to 10} \log x$ 1

37)* $\lim_{x \to 0^+} \log x$ $-\infty$

38) $\lim_{x \to \infty} \log x$ $+\infty$

39)* $\lim_{x \to 10} x \log x$ 10

40) $\lim_{x \to \frac{1}{2}\pi} \sin x$ 1

41)* $\lim_{x \to \frac{1}{2}\pi} \frac{\sin x}{x}$ $\frac{2}{\pi}$

42) $\lim_{x \to \pi} \frac{\sin x}{x}$ 0

43)* $\lim_{x \to 0} \frac{\sin 2x}{2x}$ 1

44) $\lim_{x \to \frac{3\pi}{2}} \frac{\sin x}{x}$ $-\frac{2}{3\pi}$

45)* $\lim_{x \to 0} \frac{\sin 2x}{x}$ 2

46) $\lim_{x \to 0} \frac{\sin^2 x}{x^2}$ 1

47)* $\lim_{x \to 0} \frac{\tan 2x}{x}$ 2

48) $\lim_{x \to 0} \frac{|x|}{x}$ DNE

49) $\lim_{x \to -1} \frac{|x + 1|}{x + 1}$ DNE