(KEY)

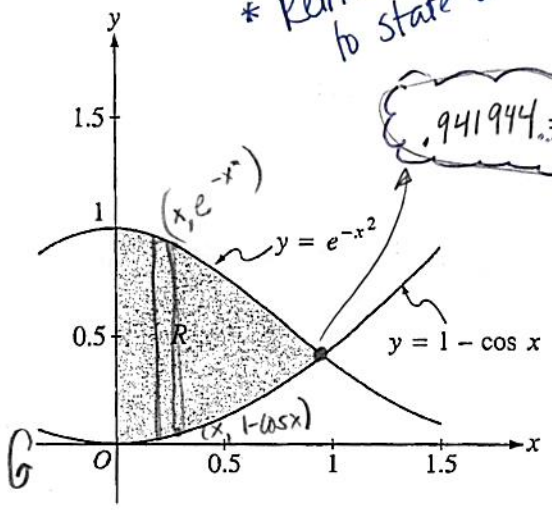
AP CALCULUS AB

AREA & VOLUME

2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



* Remember you need to state what A is equal to!

$e^{-x^2} = 1 - \cos x$

.941944... = A

A = .9419

$R = e^{-x^2} - 0 = e^{-x^2}$

$r = 1 - \cos x - 0 = 1 - \cos x$

1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.
   - (a) Find the area of the region R.
   - (b) Find the volume of the solid generated when the region R is revolved about the x-axis.
   - (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

$\int_0^A Y_1 - Y_2$

(a) $A = \int_0^A e^{-x^2} - (1 - \cos x)\,dx$ = fnInt$(Y_1 - Y_2, X, 0, A) = .5909 \approx .591$ or .590

(b) $V = \pi \int_0^A (e^{-x^2})^2 - (1 - \cos x)^2\,dx = .5559\pi = 1.7464 \approx 1.746$ or 1.747

(c) $A_{sq} = s^2$

$s = e^{-x^2} - (1 - \cos x)$

$V = \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx = .461$

fnInt$((e^{\wedge}(-X^2) - (1 - \cos(X)))^2, X, 0, A)$

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CALCULUS AB
SECTION II, Part A
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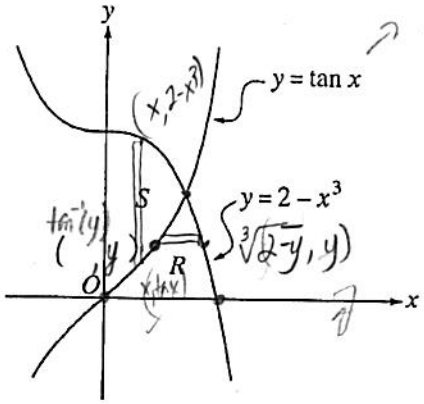
$y = 2 - x^3$

$x^3 = 2 - y$

$x = \sqrt[3]{2-y}$

$y = \tan x$

$x = \tan^{-1}(y)$



$\tan^{-1}(y) = x$

$2 - x^3 = \tan x$

$y = 2 - x^3$

$y - 2 = -x^3$

$\sqrt[3]{2-y} = x$

1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.
   - (a) Find the area of R.
   - (b) Find the area of S.
   - (c) Find the volume of the solid generated when S is revolved about the x-axis.

b. pt. intersection (.902, 1.266)

$A = .902\ldots$
$B = 1.266\ldots$

Alpha Y Store B
(Don't move cursor off point of intersection!!!)

OR can express B as $B = 2 - A^3$ or $B = \tan A$

(a) $\int_0^B \left(\sqrt[3]{2-y} - \tan^{-1} y\right) dy = .72933 \approx .729$

(b) $\int_0^A \left(2 - x^3 - \tan x\right) dx = 1.16054 \approx 1.161$ or 1.160

(c) $R = 2 - x^3 - 0 = 2 - x^3$

$r = \tan x - 0 = \tan x$

$V = \pi \int_0^A \left(2 - x^3\right)^2 - \tan^2 x \, dx$

$8.3318 \approx 8.332$ or 8.331 or 2.652π

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CALCULUS AB
SECTION II, Part A
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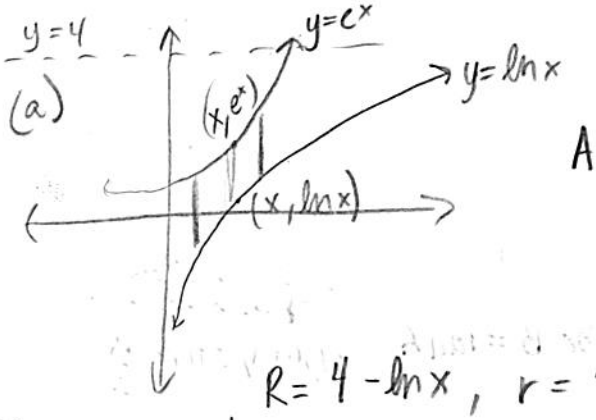
1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

(a)



$$A = \int_{\frac{1}{2}}^{1} (e^x - \ln x)\,dx = 1.2229 \approx 1.223 \text{ or } 1.222$$

$R = 4 - \ln x, \quad r = 4 - e^x$

(b) $$V = \pi \int_{\frac{1}{2}}^{1} (4 - \ln x)^2 - (4 - e^x)^2\,dx = 7.5151\pi \text{ or } 23.609$$

(c) $h'(x) = f'(x) - g'(x) = e^x - \frac{1}{x} = 0 \quad x = 0.56714$ critical pt.

Abs min/max occur at C.P. or endpts.

$h(0.56714) = 2.330$

$h(\frac{1}{2}) = 2.3418$

$h(1) = 2.718$

abs. min is 2.330
abs. max is 2.718

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2002 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

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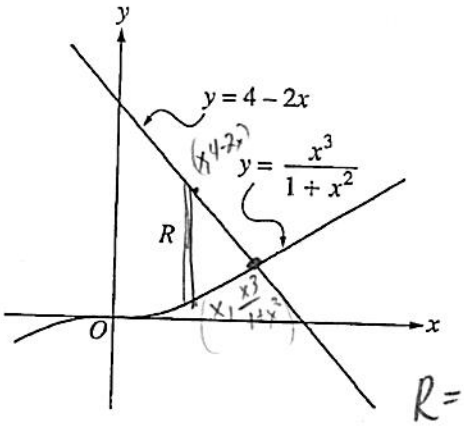

1. Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

2. The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

   (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

   (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

   (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

   (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
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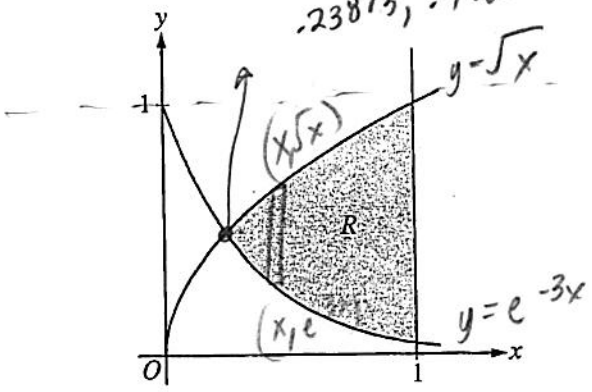
.23873, .47860427



A = .23873...

1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

↓round ↓truncate

(a) $A = \int_A^1 (\sqrt{x} - e^{-3x})\,dx = .44269 \approx$ .443 or .442

(b) $R = 1 - e^{-3x}$
$r = 1 - \sqrt{x}$
$V = \pi \int_A^1 (1 - e^{-3x})^2 - (1 - \sqrt{x})^2\,dx = .453\pi$ or 1.423 or 1.424

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(c) $A_{rec} = bh$ $\quad h = 5b$
$A_{rec} = b \cdot 5b = 5b^2$ $\quad b = \sqrt{x} - e^{-3x}$

$A = \int_A^1 5(\sqrt{x} - e^{-3x})^2\,dx = 1.554$

fnInt(5(√(x)−e^(−3x))^2, X, A, 1)

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SECTION II, Part A
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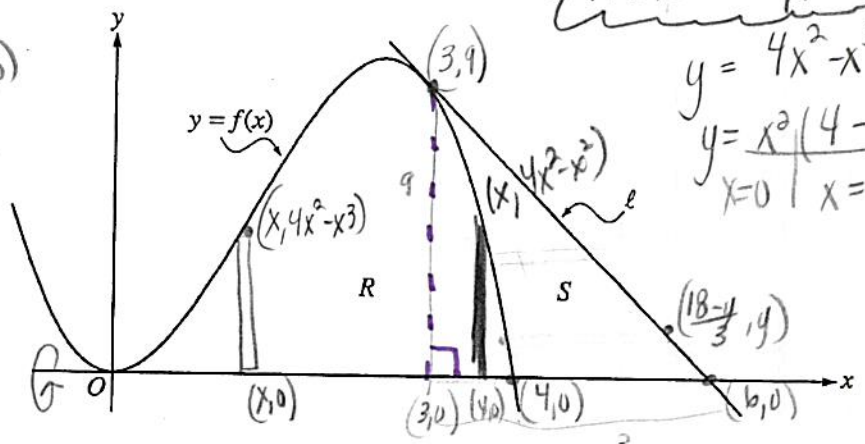

1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.

(c) Find the volume of the solid generated when R is revolved about the x-axis.

To show ℓ is tangent @ $x=3$ find $f'(3)$ and $f(3)$ and use point slope

Find x intercepts:
$y = 4x^2 - x^3$
$y = x^2(4-x)$
$x=0 \mid x=4$

$y = 18-3x$
$3x = 18$
$x = 6$

$y = 18-3x$ $\quad y = 4x^2 - x^3$
$3y = 18 - y$
$x = \frac{18-y}{3}$

(a) $f(x) = 4x^2 - x^3$
$f'(x) = 8x - 3x^2$
$f'(3) = 8(3) - 3(3)^2 = -3$

$x=3,\ y = 4(3)^2 - (3)^3 = 36 - 27 = 9$
$(3,9)$
$m = -3$
$y - 9 = -3(x-3)$
$y = -3x + 9 + 9 = -3x + 18$

(b) $A = \frac{3(9)}{2} - \int_3^4 (4x^2 - x^3)\,dx = 7.917$ or 7.916

(Could have used 2 verticals)

$A = \int_3^4 (18 - 3x - (4x^2 - x^3))\,dx + \int_4^6 (18 - 3x)\,dx$

(*can't use a horizontal strip to find volume of S b/c would need to solve $y = 4x^2 - x^3$ for x & put in terms of y)

(c) $r = 4x^2 - x^3$
$V = \pi \int_0^4 (4x^2 - x^3)^2\,dx$

156.038π or 490.208

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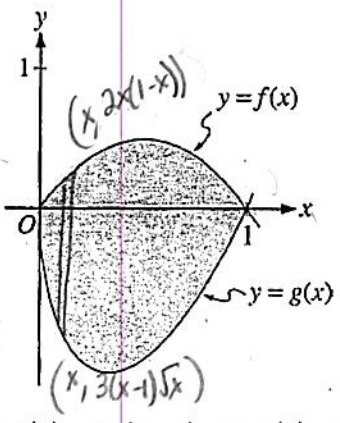

2. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

   (a) Find the area of the shaded region enclosed by the graphs of f and g.

   (b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

   (c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

(a) $A = \int_0^1 \left[2x(1-x) - \left(3(x-1)\sqrt{x}\right)\right] dx = 1.133$

(b) $R = 2 - 3(x-1)\sqrt{x}$

$r = 2 - 2x(1-x)$

$V = \pi \int_0^1 \left[\left(2 - 3(x-1)\sqrt{x}\right)^2 - \left(2 - 2x(1-x)\right)^2\right] dx = 16.179$

(c) $V = \int_0^1 \left(h(x) - g(x)\right)^2 dx$

$15 = \int_0^1 \left(kx(1-x) - 3(x-1)\sqrt{x}\right)^2 dx$

2004 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 3$.

   (c) Find the volume of the solid generated when R is revolved about the vertical line $x = 10$.

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2. For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

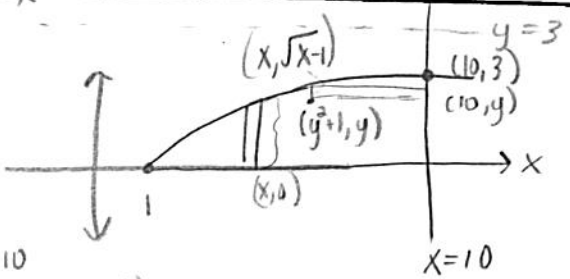
   (a) Show that the number of mosquitoes is increasing at time $t = 6$.

   (b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

   (c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

   (d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

$y=\sqrt{x-1}$
$y^2+1=x$

① 

(a) $A = \int_1^{10} \left(\sqrt{x-1}\right) dx = 18$

(b) $R = 3-0 = 3$ $\quad r = 3-\sqrt{x-1}$ $\quad V = \pi\int_1^{10}\left[3^2-(3-\sqrt{x-1})^2\right]dx$ $\quad 212.058$ or 212.057

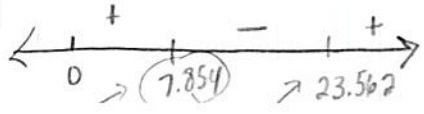
(c) $r = 10-(y^2+1) = -y^2+9$
$V = \pi\int_0^3 (-y^2+9)^2\,dy = 407.150$

② $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t=6$.

(b) $R'(6) = -1.913 < 0$, the number of mosquitoes is increasing at a decreasing rate at $t=6$

(c) $1000 + \int_0^{31} 5\sqrt{t}\cos\left(\frac{t}{5}\right)dt =$ (initial amount $t=0$)
To the nearest whole number there are 964 mosquitoes

(d) $R(t) = 0$ $\quad t = 0,\ t = 7.854,\ t = 23.562$
$t = 31$
At $t=31$, 964 mos.

2 

At $t=7.854$
$1000 + \int_0^{7.854} 5\sqrt{t}\cos\left(\frac{t}{5}\right)dt = 1039.$
The max # of mos. is 1039 at

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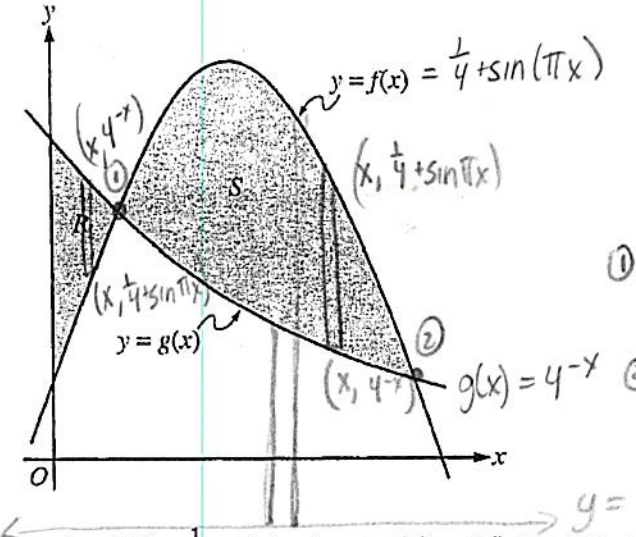

pt of int:
① (.178, .781)
② (1, .25)

A = .178...

1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the area of S.
   (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

(a) $A = \int_0^A \left[4^{-x} - \left(\frac{1}{4} + \sin(\pi x)\right)\right] dx = 0.065$ or 0.064

fnInt $(4\wedge -x - (\frac{1}{4} + \sin(\pi x)), X, 0, A)$

WRITE ALL WORK IN THE TEST BOOKLET.

(b) $A = \int_A^1 \left(\frac{1}{4} + \sin(\pi x) - 4^{-x}\right) dx = .410$

(c) $R = \frac{1}{4} + \sin(\pi x) - (-1) = \frac{5}{4} + \sin(\pi x)$

$r = 4^{-x} - (-1) = 4^{-x} + 1$

$V = \pi \int_A^1 \left[\left(\frac{5}{4} + \sin(\pi x)\right)^2 - (4^{-x} + 1)^2\right] dx = 4.559$ or 4.558

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CALCULUS AB
SECTION II, Part A
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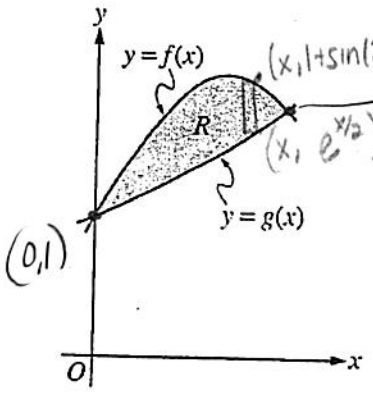

(1.136, 1.764)

A = 1.136...

X = 0
$y = e^{0/2} = 1$

1. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

WRITE ALL WORK IN THE TEST BOOKLET.

$r = \frac{1+\sin(2x) - e^{x/2}}{2}$ ← diameter

(a) $A = \int_0^A \left(1 + \sin 2(x) - e^{x/2}\right) dx = .429$

(b) $R = 1+\sin(2x) - 0 = 1+\sin(2x)$
$r = e^{x/2} - 0 = e^{x/2}$

(c) $A_{semicircle} = \frac{\pi r^2}{2} =$

$V = \pi/2 \int_0^A \left(\frac{1+\sin(2x) - e^{x/2}}{2}\right)^2 dx = .078$ or .077

$\pi/2$ fnInt($\left(\frac{(1+\sin(2x) - e^{\frac{x}{2}})}{2}\right)^2$ X, 0, A)

$V = \pi \int_0^A \left[\left(1+\sin(2x)\right)^2 - \left(e^{x/2}\right)^2\right] dx = 4.267$ 2

4.266